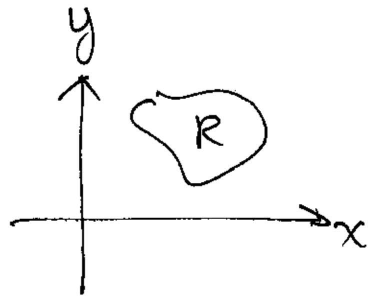
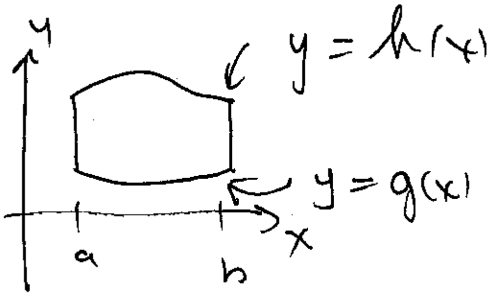
Math 2471 - Calc III

Double Integrals

$$\iint_R f(x,y)\,dA$$



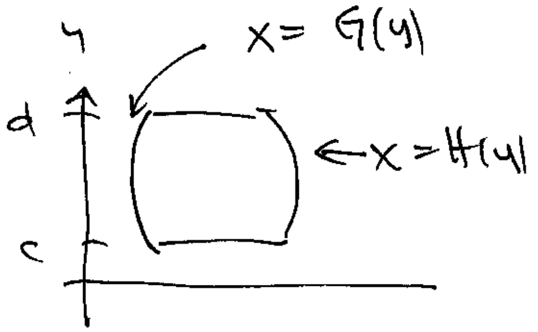
2 Types



$$\int_{x=a}^{b}\int_{y=g(x)}^{h(x)} f(x,y)\,dy\,dx$$

~~top~~ bottom curve → top curve

left pt → right pt



$$\int_{y=c}^{d}\int_{x=G(y)}^{H(y)} f(x,y)\,dx\,dy$$

left curve → right curve

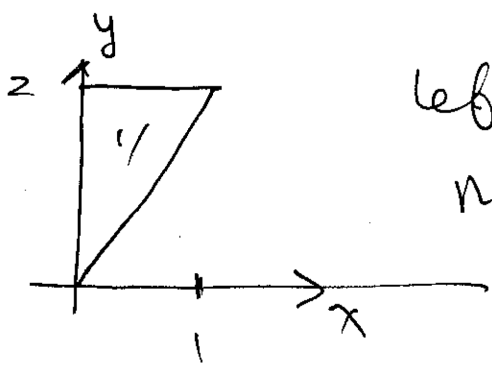
bottom pt → ~~pt~~ top pt.

Sometimes we reverse the order of integration.

Ex! $\displaystyle\int_0^1 \int_{2x}^{2} 4e^{y^2}\,dy\,dx$ This we can't $\int$

so we switch the order of $\int$. first the regi

curve → curve $y=2x \to y=2$ pt → pt $x=0 \to 1$



left curve $x=0$

right curve $y=2x \Rightarrow x=y/2$

$$\int_0^2 \int_0^{y/2} 4e^{y^2}\,dx\,dy = \int_0^2 4e^{y^2} x\Big|_0^{y/2} dy$$

$$= \int_0^2 4e^{y^2}\, y/2\, dy = \int_0^2 2y e^{y^2} dy$$

$u = y^2$ $\qquad = \quad e^{y^2}\Big|_0^2 = e^4 - e^0$

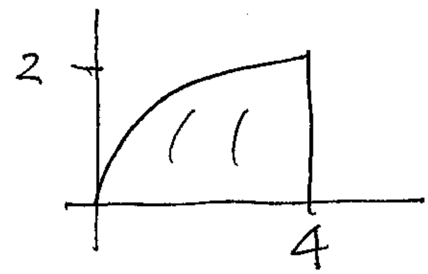
$du = 2y\,dy$

$\int e^u du = e^u$

Ex 2 $\displaystyle\int_0^2 \int_{y^2}^4 \sqrt{x}\sin x \, dx\, dy$ can't $\int$

left curve $x=y^2$ right curve $x=4$

$x=y^2$ or $y=\sqrt{x}$ pt $\to$ pt $0 \to 2$ (in y)



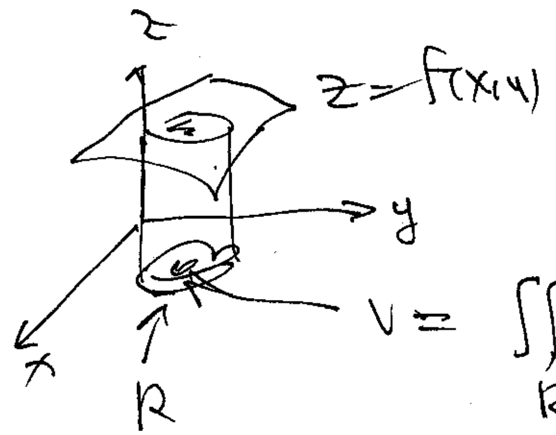
$$\int_0^4 \int_0^{\sqrt{x}} \sqrt{x}\sin x \, dy\, dx = \int_0^4 \sqrt{x}\sin x \, y \Big|_0^{\sqrt{x}} dx$$

$$= \int_0^4 x\sin x \, dx = \sin x - x\cos x \Big|_0^4$$

$$= \sin 4 - 4\cos 4 - (0-0)$$

$$= \sin 4 - 4\cos 4$$

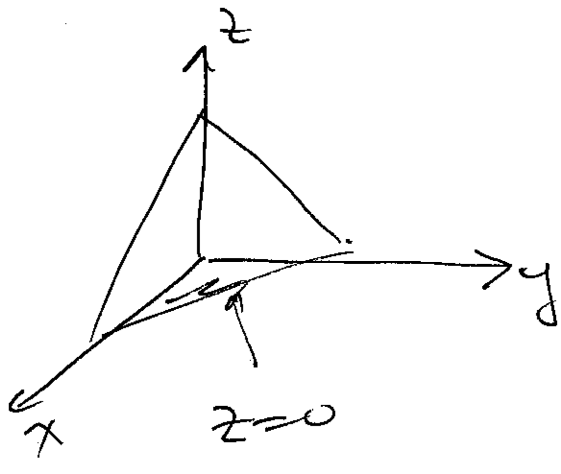
Application – Volumes

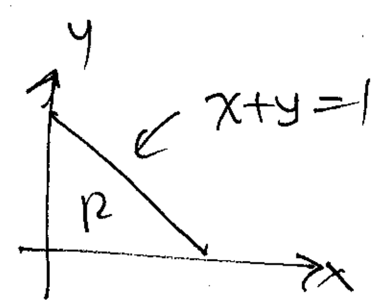


$$V = \iint_R f(x,y)\,dA$$

ex 3 Find the volume of the tetrahedron bound by the planes

$$x=0,\ y=0,\ z=0 \ \ \& \ \ x+y+z=1$$





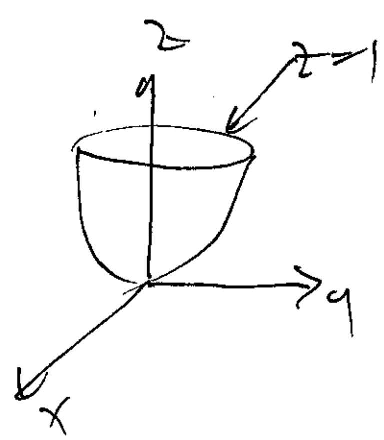
$$V = \int_0^1 \int_0^{1-x} (1-x-y)\,dy\,dx = \int_0^1 y - xy - \frac{y^2}{2}\Big|_0^{1-x} dx$$

$$= \int_0^1 1 - x - x(1-x) - \frac{(1-x)^2}{2}\, dx$$

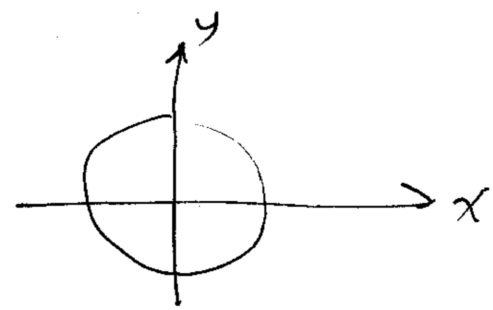
$$= \int_0^1 1 - x - \cancel{x} + x^2 - \frac{1}{2} + \cancel{x} - \frac{x^2}{2}\, dx$$

$$= \int_0^1 \frac{1}{2} - x + \frac{x^2}{2}\, dx = \frac{x}{2} - \frac{x^2}{2} + \frac{x^3}{6}\Big|_0^1 = \frac{1}{2} - \frac{1}{2} + \frac{1}{6} = \frac{1}{6}$$

<u>Ex 4</u> Find the volume bound by $z = x^2 + y^2$ & $z = 1$



intersection $x^2 + y^2 = 1$



$$\iint_R \text{top surface} - \text{bottom surface}\; dA$$

$$4 \int_0^1 \int_0^{\sqrt{1-x^2}} (1 - x^2 - y^2)\, dy\, dx$$

$$4\int_0^1 y - x^2y - \frac{y^3}{3} \Big|_0^{\sqrt{1-x^2}} dx$$

$$4\int_0^1 \underbrace{\sqrt{1-x^2} - x^2\sqrt{1-x^2}} - \frac{1}{3}(1-x^2)^{3/2}\, dx$$

$$4\int_0^1 (1-x^2)\sqrt{1-x^2} - \frac{1}{3}(1-x^2)^{3/2} dx$$

$$4 \cdot \frac{2}{3}\int_0^1 (1-x^2)^{3/2} dx$$ $\Leftarrow$ a Calc 2 prob

trig sub $x = \sin\theta$

$\vdots$

$= \pi/2$

Maybe there's a better way to find this volume - polar coordinates