Shooting for STAAR!

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Introduce yourself.

Describe your strengths in supporting mathematics.

Describe an opportunity for growth.

Shooting for STAAR!

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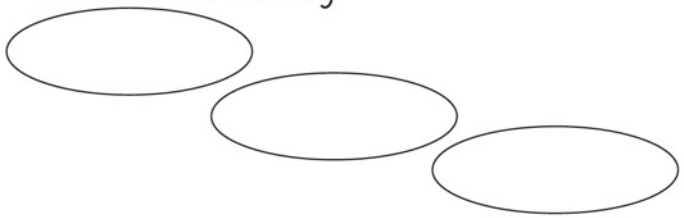
Instructional Platform

Instructional Delivery



Instructional Strategies

Instructional Platform

What's the continuum of mathematics support?

Anita Archer (2019)

Instructional Platform

evidence-based practice

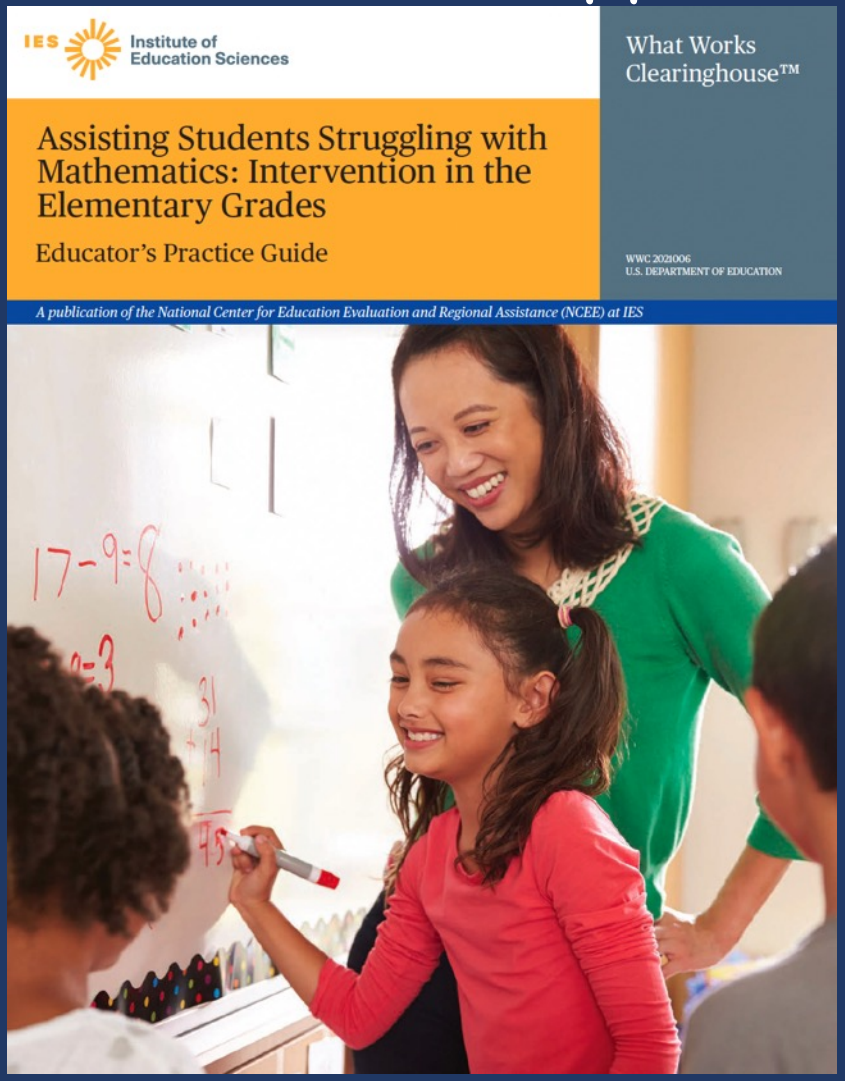
A practice that has shown **consistent and positive** results

evidence-based practice

evidence-based intervention

An intervention (i.e., packaged program) that has shown **consistent and positive** results

What's the continuum of mathematics support?



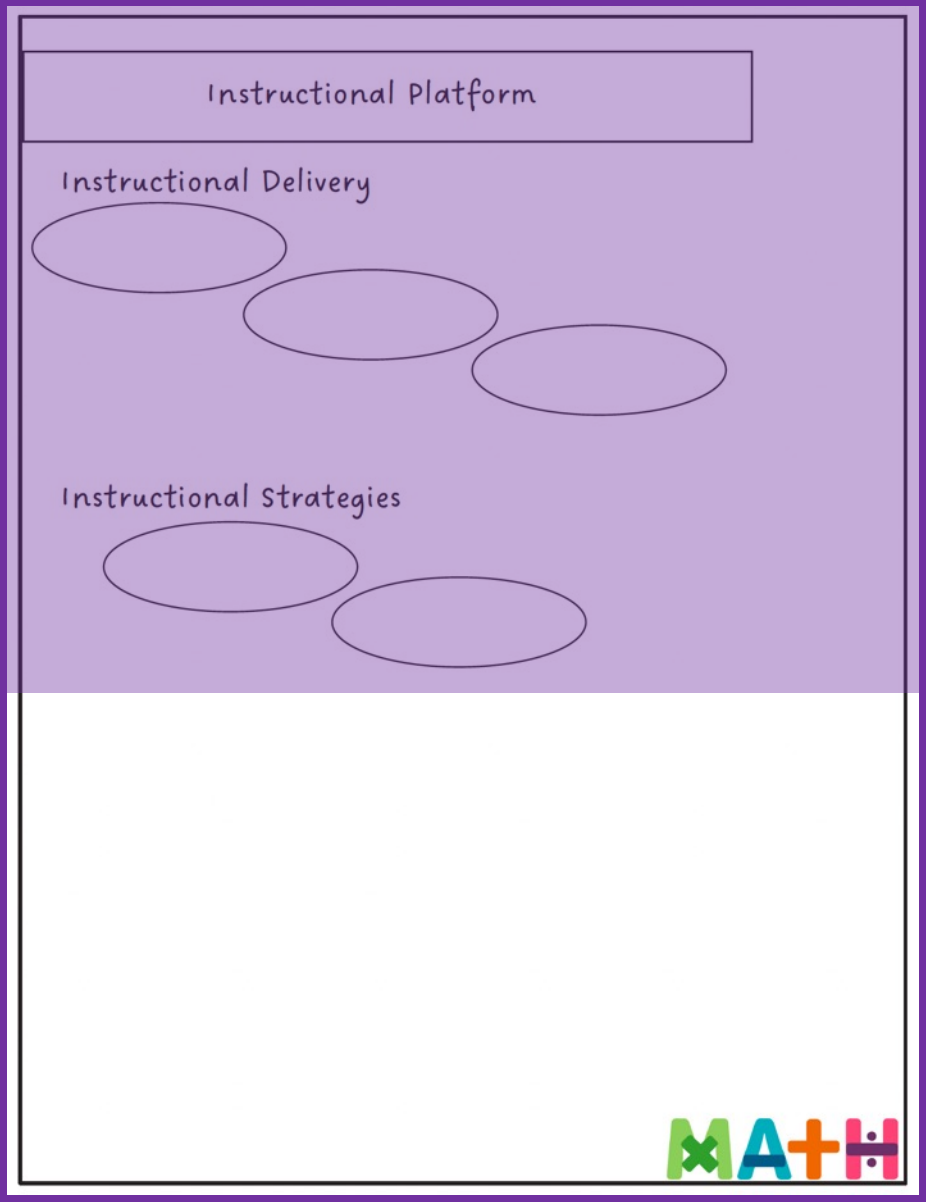
Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.



Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and Practice

INSTRUCTIONAL STRATEGIES

Fluency

Word Problems

Vocabulary

Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

INSTRUCTIONAL STRATEGIES

VOCABULARY

Research and Information

Use Formal Mathematics Language

Instead of that...	Say this...

Significant correlation (r = .49) between mathematics vocabulary and mathematics performance. Mathematics vocabulary appears most important for word-problem performance (r = .58).

(Lin et al., 2021)

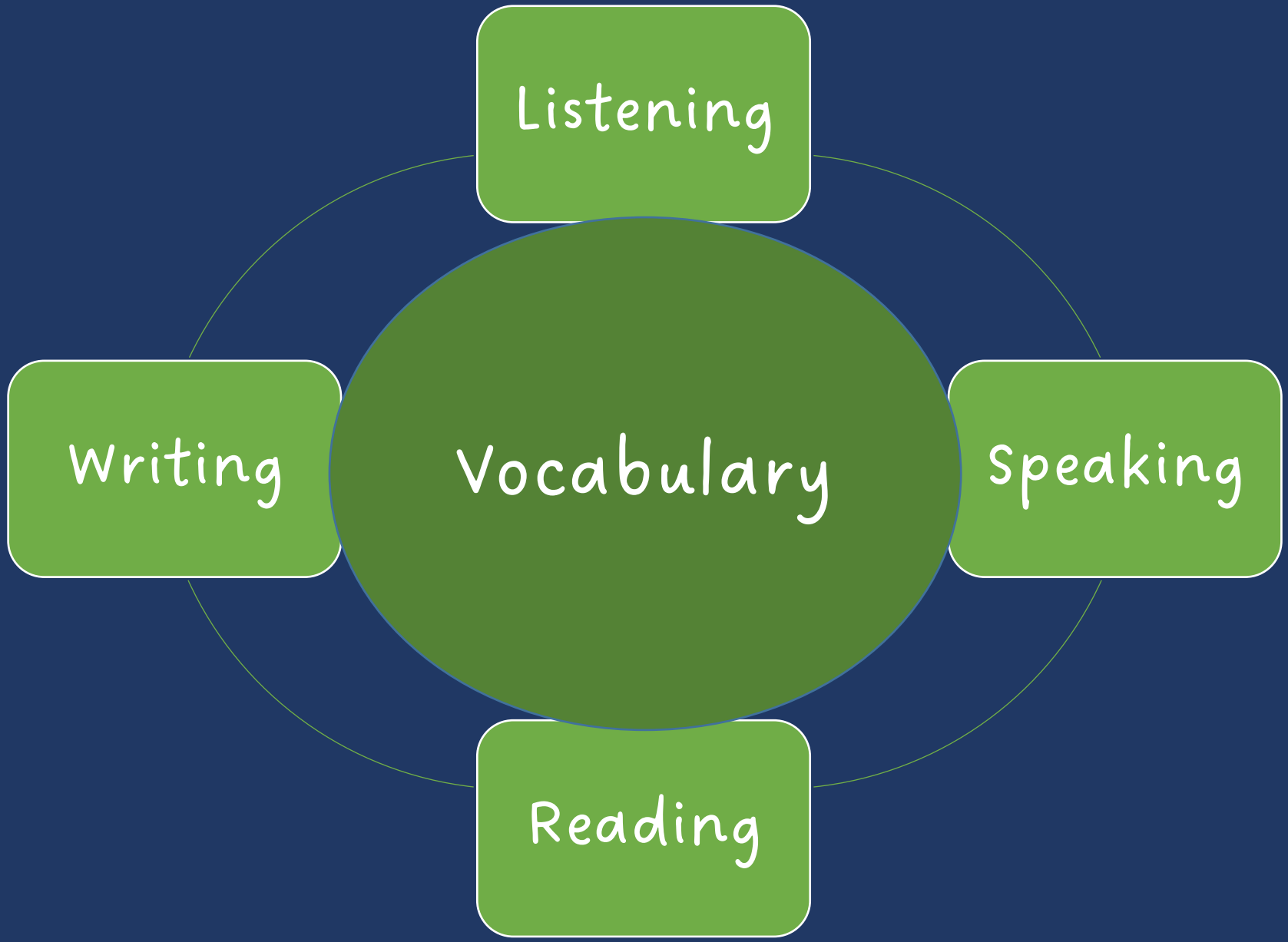
Early mathematics vocabulary related to mathematics and literacy.

(Hornburg et al., 2018; Purpura et al., 2017)

Students who experience difficulty with mathematics demonstrate lower mathematics vocabulary performance.

(Hughes et al., 2020; Powell & Nelson, 2017; Powell et al., 2017; Unal et al., 2021)

Powell, Bos, & Lin (2019)



1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

difference

even

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms have more than one meaning

round

square

second

base

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms have more than one meaning

4. Some math terms are only used in math

trapezoid

numerator

parallelogram

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms have more than one meaning

4. Some math terms are only used in math

5. Some math terms are homophones

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms have more than one meaning

4. Some math terms are only used in math

5. Some math terms are homophones

6. Some math terms are related but have distinct meanings

factor vs. multiple

hundreds vs. hundredths

numerators vs. denominator

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms have more than one meaning

4. Some math terms are only used in math

5. Some math terms are homophones

6. Some math terms are related but have distinct meanings

7. Some math concepts are verbalized in more than one way

skip count vs. multiples

one-fourth vs. one quarter

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms have more than one meaning

4. Some math terms are only used in math

5. Some math terms are homophones

6. Some math terms are related but have distinct meanings

7. Some math concepts are verbalized in more than one way

8 Some informal terms may be used for formal math terms

rhombus vs. diamond

vertex vs. corner

Why might your students have difficulty with math vocabulary?

Use formal math language

Use terms precisely

The alligator eats the bigger number

is less than
OR
is greater than

Why this is important...

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because < and > are two distinct symbols.

What number is in the tens place?

What digit is in the tens place?
What is the value of the digit in the tens place?

135

Why this is important...

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.

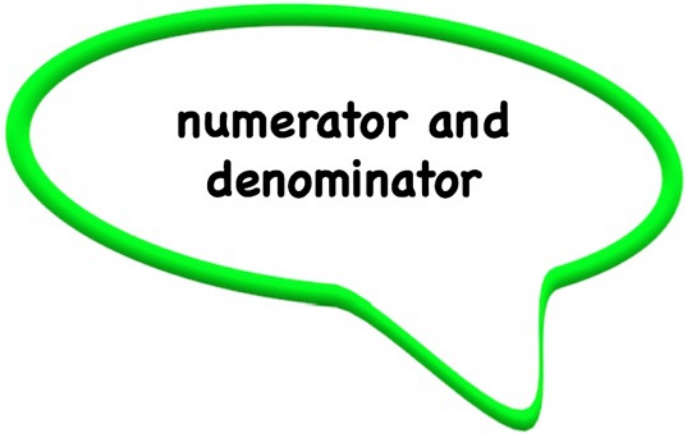
carry OR borrow

regroup OR trade OR exchange

$$\begin{array}{r} 167 \\ +\ 294 \\ \hline \end{array}$$

Why this is important...

- "Carry" or "borrow" is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.



Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.

reduce the fraction

rename OR
find equivalent OR
simplify

Why this is important...

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.

Four point seven
Four point oh seven

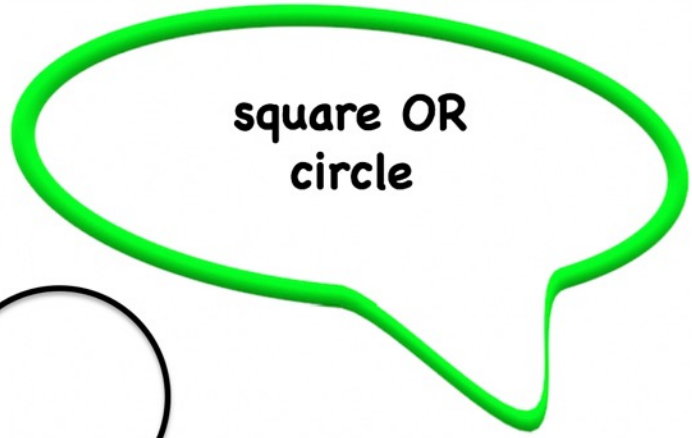
Four and seven tenths
Four and seven hundredths

4.7

4.07

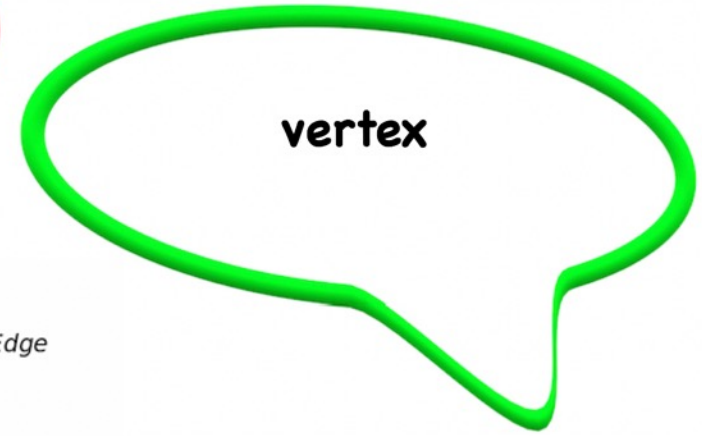
Why this is important...

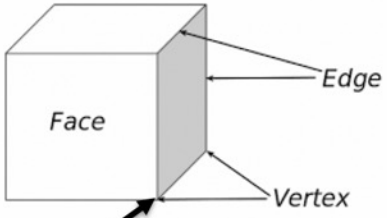
- Accurately shares the magnitude of the decimal.
- Emphasizes place value.



Why this is important...

- Use the formal language of shapes to confirm informal language.





Why this is important...

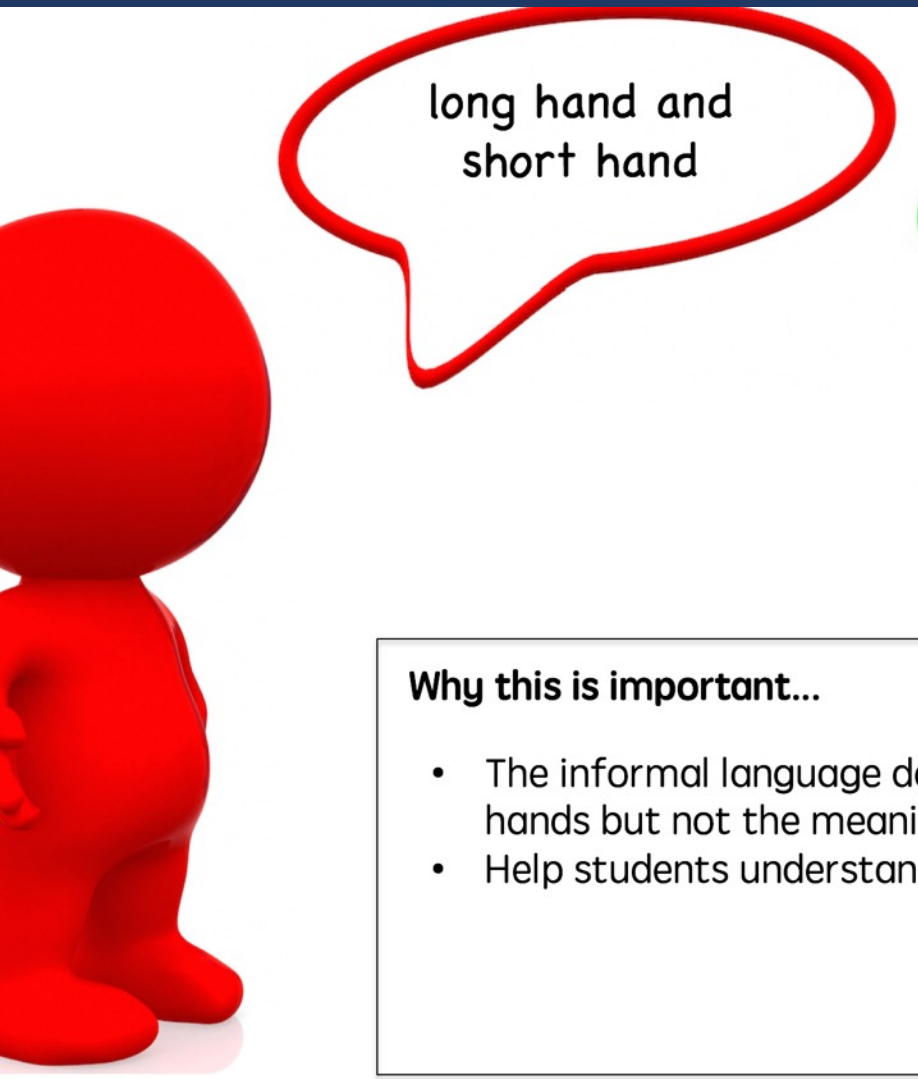
- This is the endpoint where two or more line segments or rays meet.

flips, slides, turns

reflections, translations, rotations

Why this is important...

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.



Why this is important...

- The informal language describes the length of clock hands but not the meaning.
- Help students understand the hours and minutes.

VOCABULARY

Research and Information

Use Formal Mathematics Language

Instead of that...	Say this...

Identify examples of "Instead of ____, say ____."

Use formal math language

Use terms precisely

VOCABULARY

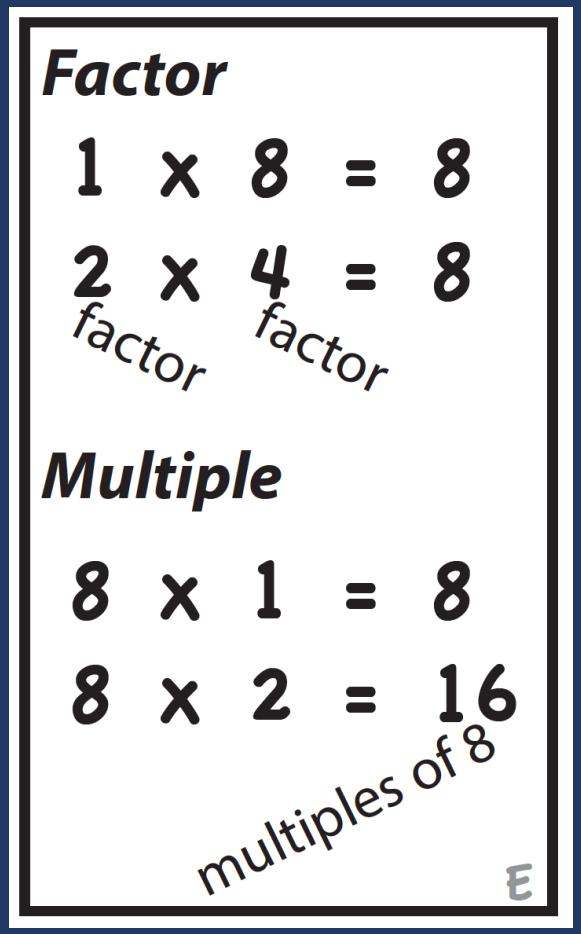
Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?



Improper fraction

$$\frac{8}{5}$$

Proportion

$$\frac{2}{5} = \frac{8}{20}$$

Mixed number

$$1\frac{3}{5}$$

Ratio

4:3

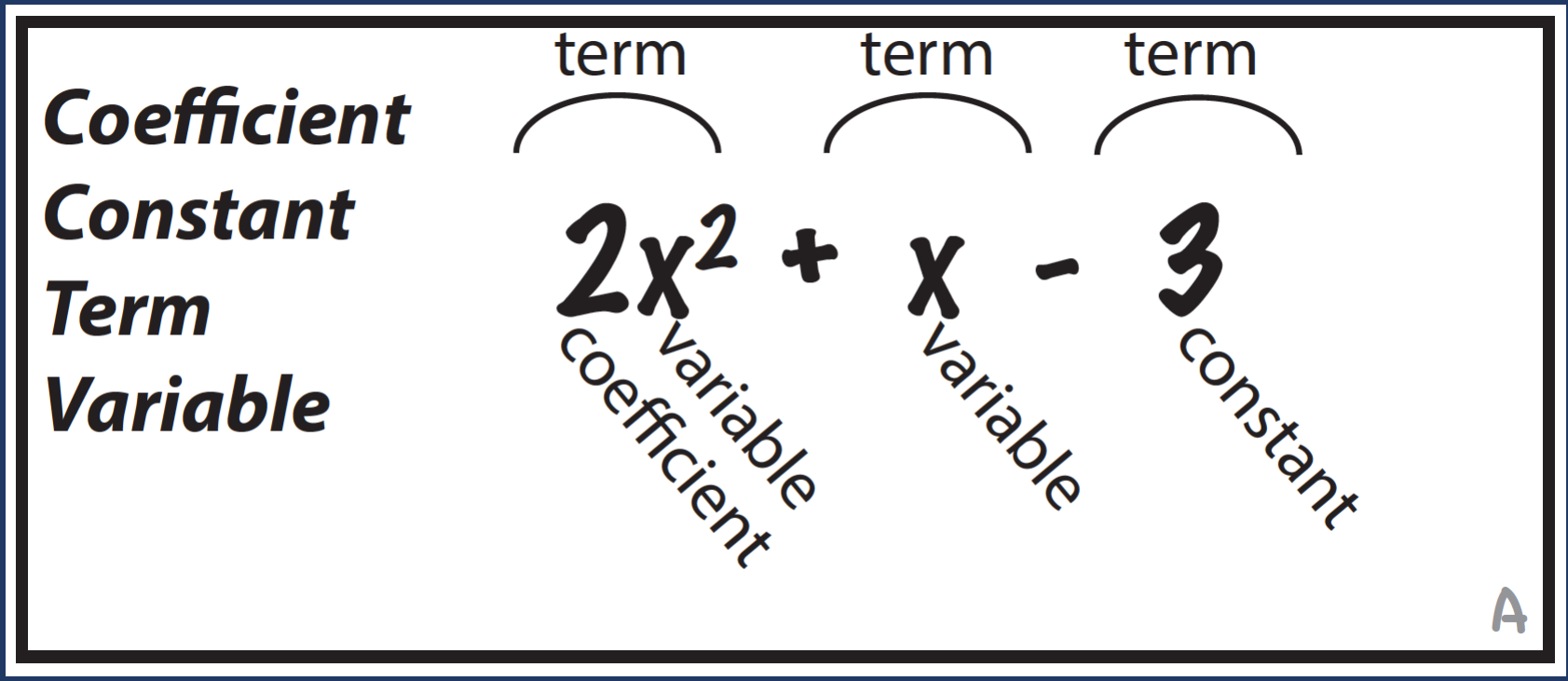
Proper fraction

$$\frac{2}{9}$$

Unit fraction

$$\frac{1}{6}$$

D

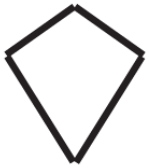


Equation	$9x - 4 = 7x$
Expression	$9x - 4$
Formula	$a^2 + b^2 = c^2$
Function	$f(x)$
Inequality	$9x - 4 > 6x$

C

Quadrilaterals

Kite



Rhombus

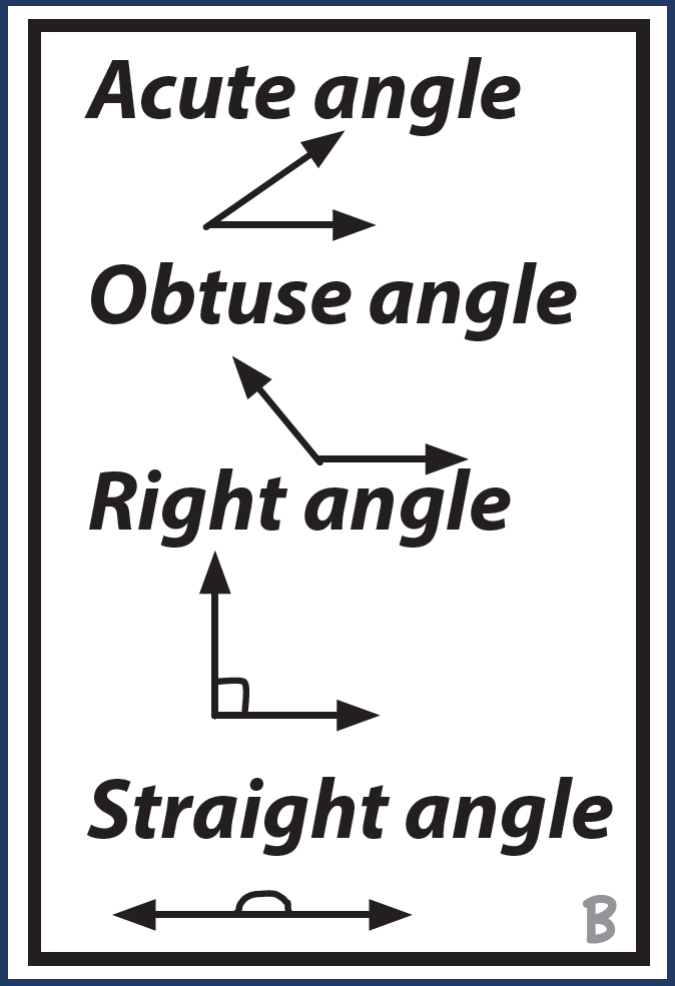
Parallelogram

Square

Rectangle

Trapezoid

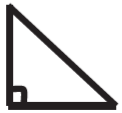
A



Acute triangle

Obtuse triangle

Right triangle



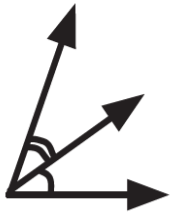
Equilateral triangle

Isosceles triangle

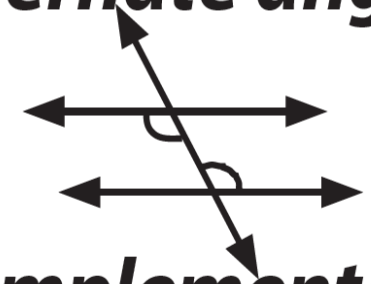
Scalene triangle

C

Adjacent angles

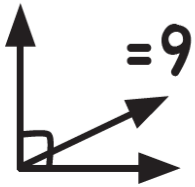


Alternate angles

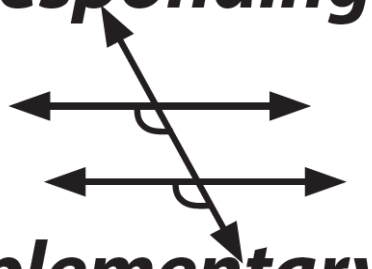


Complementary angles

=90°



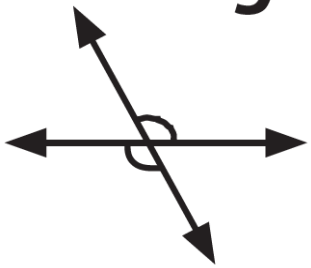
Corresponding angles



Supplementary angles

=180°

Vertical angles

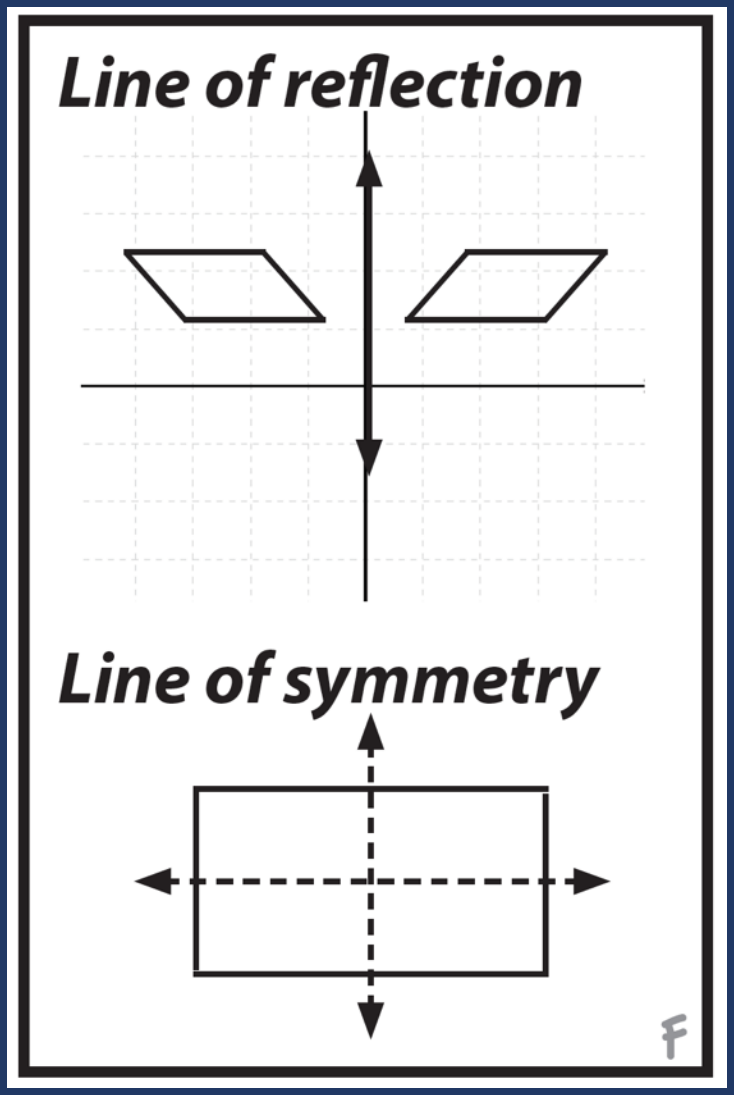


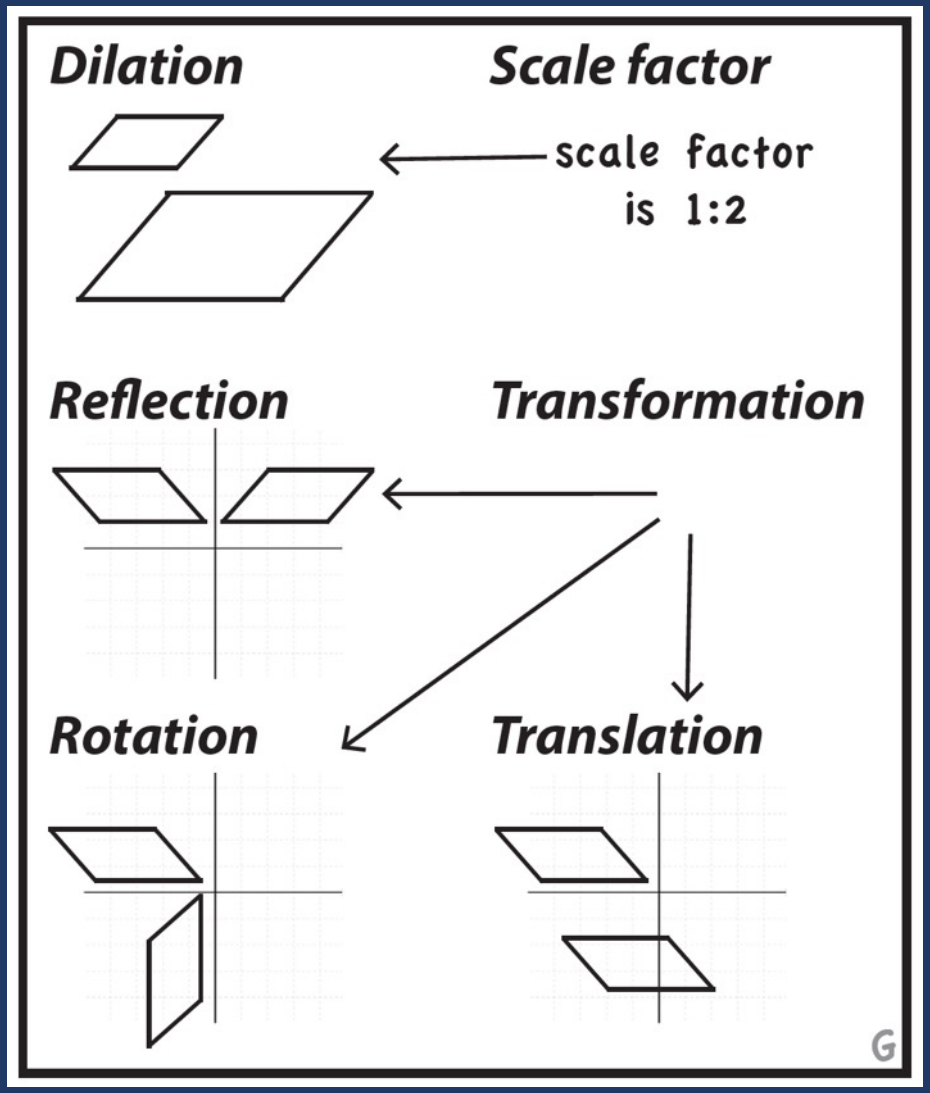
D

Congruent figures

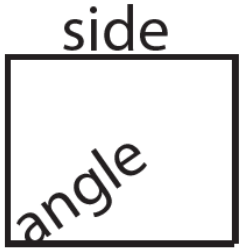
Similar figures

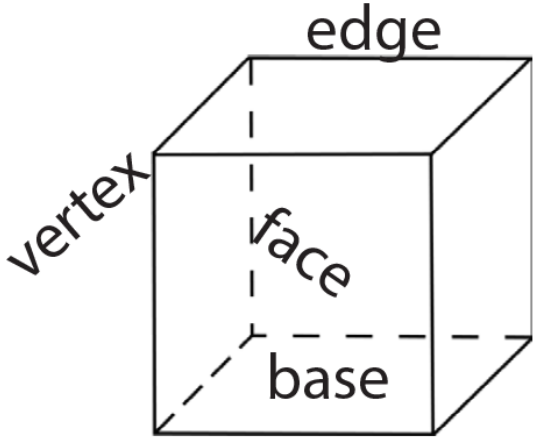
E

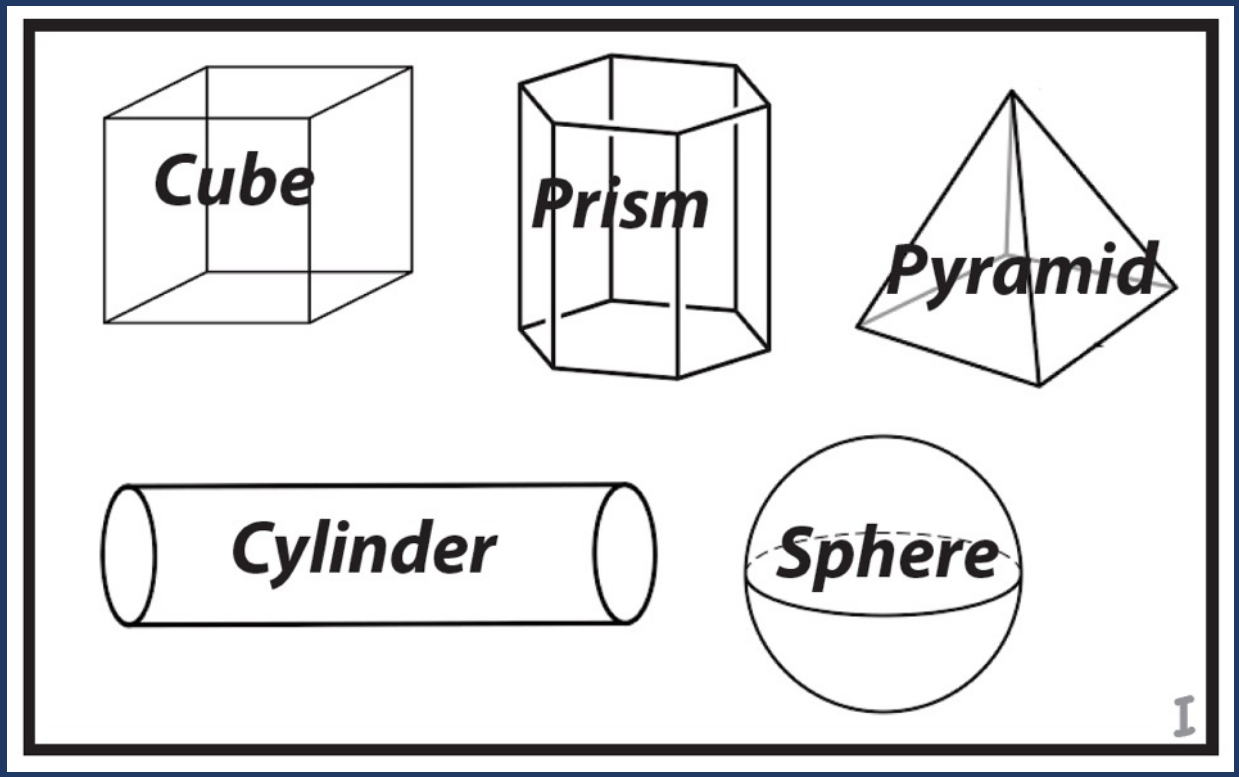




Angle
Base
Edge
Face
Side
Vertex







Coordinate plane

Ordered pair

Quadrants

x

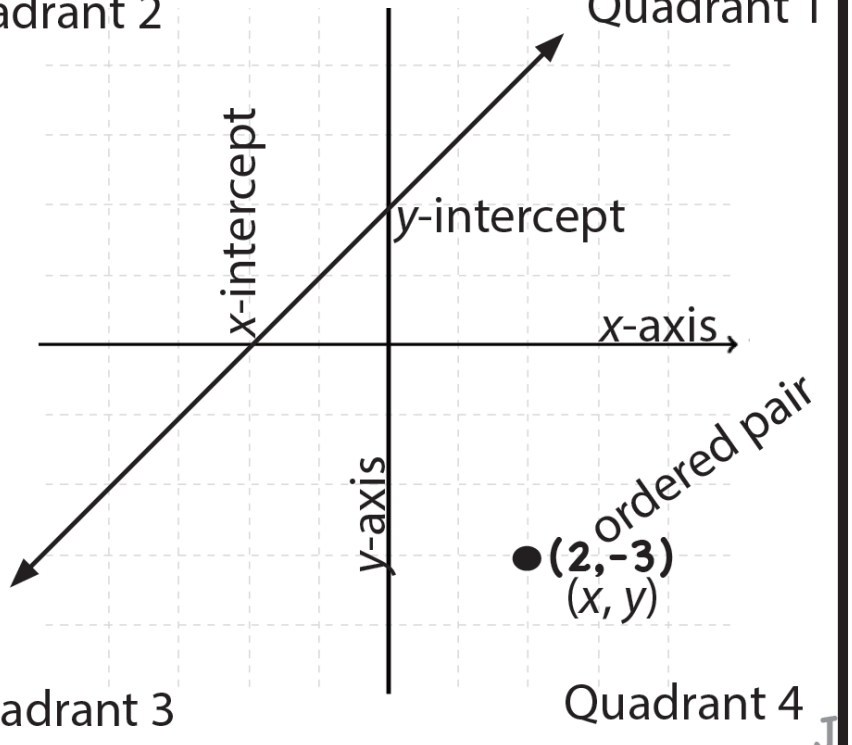
x-axis

x-intercept

y

y-axis

y-intercept



VOCABULARY

Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

Discuss terms you want your students to use with precision.

Use formal math language

Use terms precisely

VOCABULARY

Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

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Use semantic maps

Term	Definition	Example	Nonexample
integer	(... -3, -2, -1, 0, 1, 2, 3...)	15	$\frac{1}{3}$
denominator	The equal parts of a whole or set.	$\frac{5}{8}$ 8 is the denominator 	$\frac{5}{8}$ 
numerator	The equal parts of a given fraction.	$\frac{5}{8}$ 5 is the numerator 	$\frac{5}{8}$ 

(Stevens et al., 2022)

Use word walls

difference

The result of subtracting or the result when comparing two numbers.

5 – 4 = 1

1 is the difference

horizontal line

A straight line that goes from left to right or right to left.

equivalent

Two numbers that have the same value.

$\frac{2}{4}$ **is equivalent to** $\frac{1}{2}$

total

The result or sum when adding numbers.

5 + 7 = 12

(mathspiral.com)

Use flash cards

addend

quotient

divisor

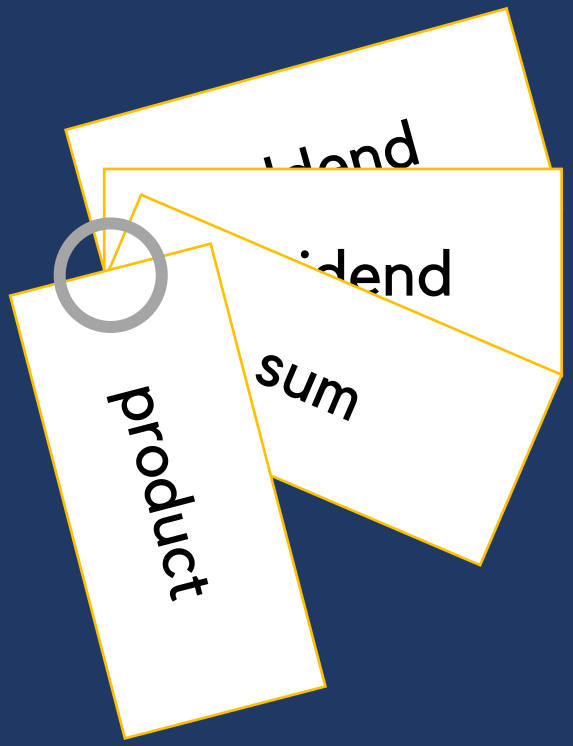
dividend

factor

sum

product

difference

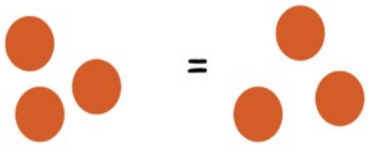


(Petersen-Brown et al., 2019)

Use glossaries

E

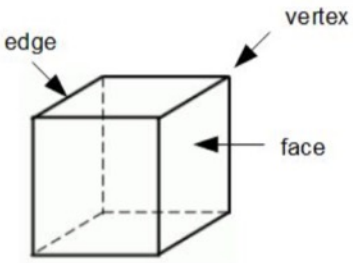
equal - with the same value



equation - expressions with an equal sign

$$5 = 2 + 3$$

edge - line segment between faces



Glosario de Matemáticas

Término	Definición
adición *(add)*	Juntar o agregar.
algoritmo *(algorithm)*	Es una serie de pasos organizados que describe el proceso que se debe seguir, para dar solución a un problema específico.
alinear *(array)*	Un conjunto de objetos, imágenes o números alineados en columnas y filas.
ancho *(width)*	La medida de un lado de un objeto, generalmente el lado más corto.
ángulo *(angle)*	Dos semirrectas o segmentos de línea recta que comparten un punto final.
ángulo agudo *(acute angle)*	Un ángulo que mide menos de 90°.
ángulo obtuso *(obtuse angle)*	Un ángulo que mide entre 90° y 180°.
ángulo recto *(right angle)*	Un ángulo que mide exactamente 90°.
área *(area)*	La cantidad de unidades cuadradas que cubre una figura geométrica cerrada.
balance presupuestario *(balance the budget)*	Un presupuesto es cuando la cantidad total de dinero gastado, ahorrado y compartido es igual al ingreso total.
báscula *(balance scale)*	Instrumento de medición que se utiliza para medir el peso o la carga.
base (computación) *(base (computation))*	Un número que se multiplica por un exponente.
base (geometría) *(base (geometry))*	La forma inferior de una figura tridimensional.

SPIRAL
Specialized Math Intervention to Reach All Learners

(mathspiral.com)

Use anchor charts

Addition Computation

1
17 ← addend
+ 59 ← addend
76 ← sum

Quadrilaterals

Kite

Rhombus

Parallelogram

Square

Rectangle

Trapezoid

Use graphic organizers

horizontal

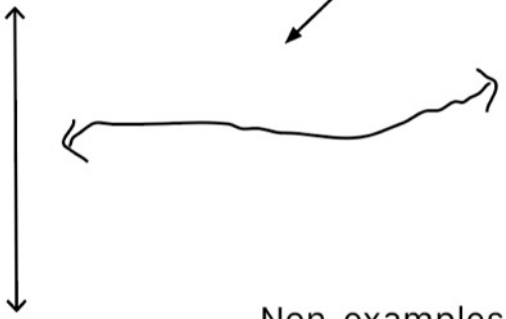
Definition

a straight line that goes left to right or right to left

Picture

Examples

Non-examples



Use games

Name: _____ Date: _____

3RD GRADE MATH

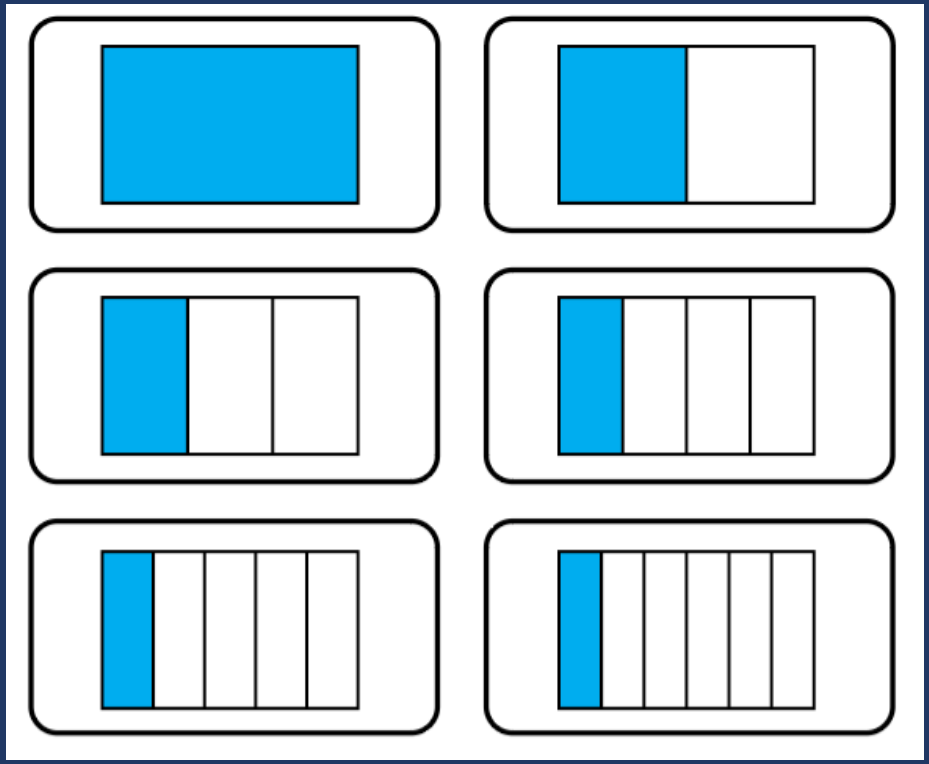
T W D H P T R I Y T S F T O Y T S P X G U C I C
M S G C E N T I M E T E R S Y S C Y Z D R G D J
I M I E U J R L H F O A W Z X U A L F J O Q F T
U P F N Z L R V C N N P Z D D H L P H F A V J Z
Y R U P M I A C O R D P I V J T E I H Y M F M S
E K X H R T T Y U J E S V X S V A T Z W M S E T
W Z Z W G E Q G M P C V C Z E H E L W L T T A A
N U V V B R J F H Y N E O Z H T K U K F A Z S W
Q Y J P S H A C D X A M Y G C U K M Z L J E U P
T Z W B P N R Z Y F L I Y D N A J J Z M W D R S
O W W U W T G I T I A T F F I S Q G X K K I E Z
E W R Q C S U U C U B J L E Z P B H G B C V M Q
F R A C T I O N S K Z O W M D X B M M C O I E C
N Z C A T L I J B J M U F O X X A D O Y L D N K
D E V P T U C M Y S A H K Z S U O W R S C V T G
M X N L N S O L V E R R I Y I B Z N Q O X C P W
M V W Z C D Q Z Z U G X L U K P Y O B S W I M N
K F L J U R J G J R O Q E M X C U O H N M S F D
S U B T R A C T G U L Y Y A K W U N U S O L H H
E G F D D A I O I A I Y R R J I C A P A C I T Y
I L W B A T R D O X K C J G H T D X E G C S M Z
X J Q Y A H L T B J R A I W J B C K I U S S I U
N U O D N R L V T I T L C U P C L W C B F S Z Q
J F N P R R K P Y M R E L U R Q H M A E Y Y Z H

ADD
BALANCED
CAPACITY
CENTIMETERS
CLOCK
DIVIDE
FRACTIONS
GRAM
INCHES
KILO GRAM
LITER
MEASUREMENT
MULTIPLY
NOON
RULER
SCALE
SOLVE
SUBTRACT
TIME

square	circle	decagon	triangular prism	cone
cylinder	cube	sphere	line	pyramid
parallelogram	octagon	FREE	trapezoid	oval
kite	pentagon	cylinder	rectangle	line segment
hexagon	rhombus	triangle	quadrilateral	rectangular prism

https://wordmint.com/public_puzzles/13055

Use games

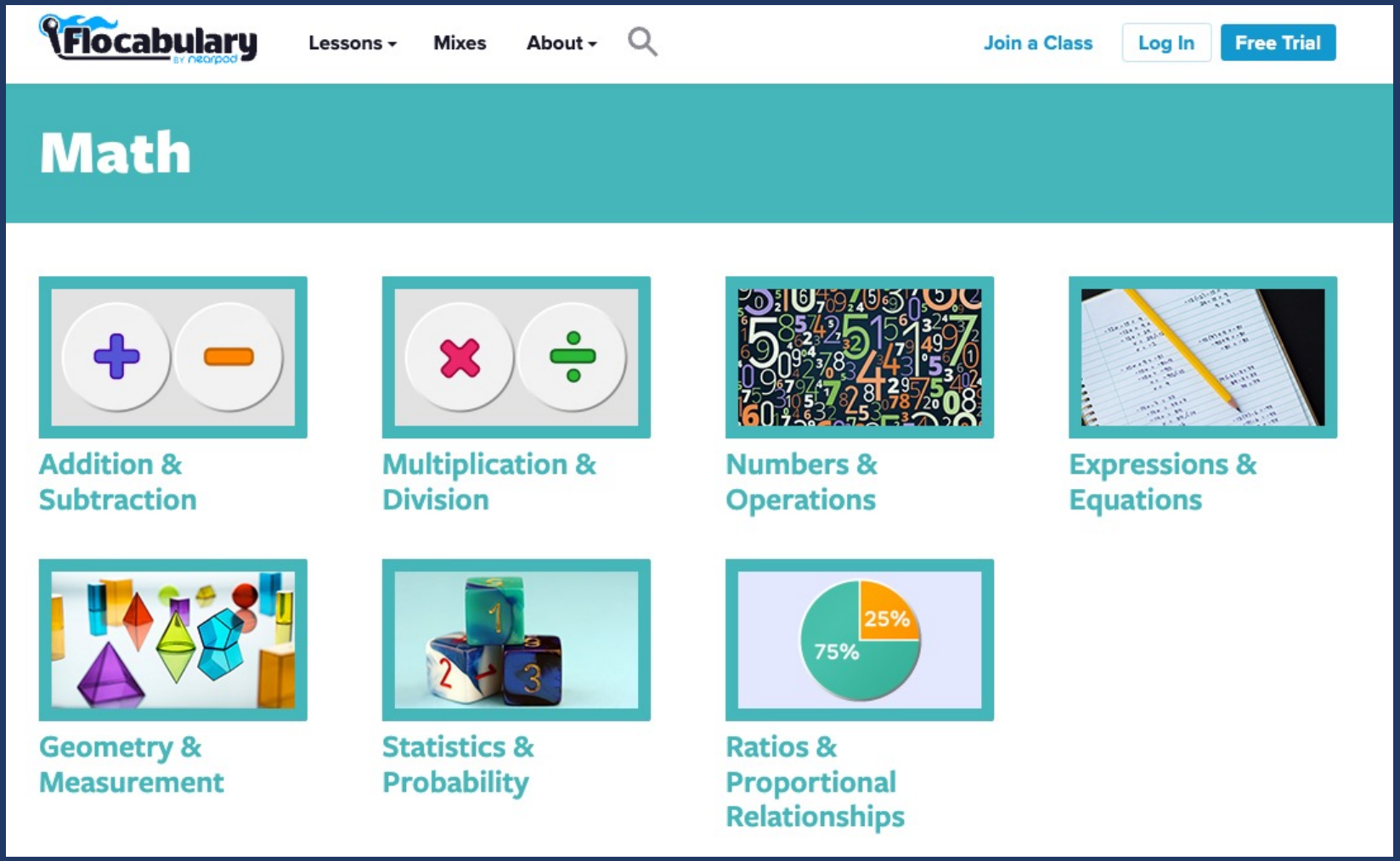


one	one-half
one-third	one-fourth
one-fifth	one-sixth

Use discussion

add
change
compare
decrease
difference
increase
part
put together
subtract
total

Use technology



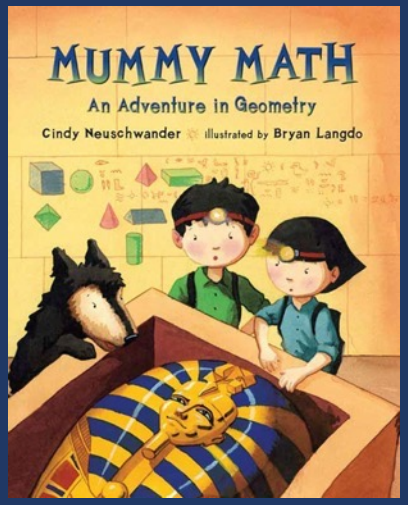
Use math writing

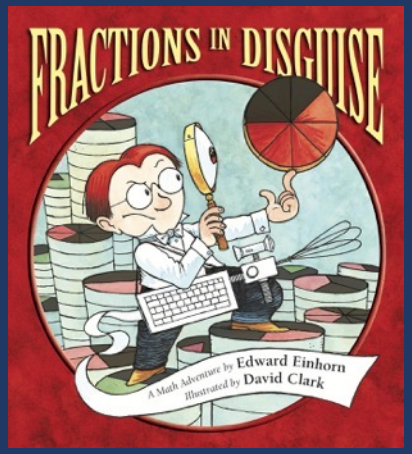
I would help Cole out by not putting them into thirds because he put 1 and 2/3 but the answers should be 3/5. So next I would draw the lines the make them into 5 pieces but put next to eachother. Then you shade in 3 of the squares And keep the others alone. then that would be the correct way to do 3/5.

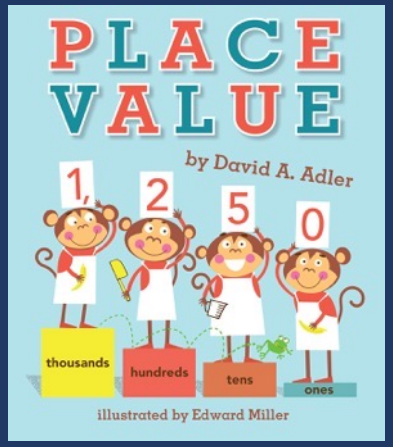
What Alex did wrong was that it was that he has five rectangles but they are different sizes so first what I would do is draw 5 rectangles side by side then what you would do is shade 3 of them and that would equal 3/5 so that would be the correct way to solve it.

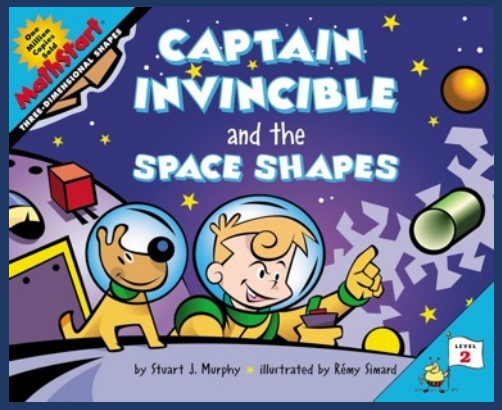
I would help Bo out by he shaded three rectangls which is corect but the he left 5 shaded and there should Only be 2 blank rectangles so if you close 3 rectangles then then answer would be 3/5 and that is the correct way to solve it.

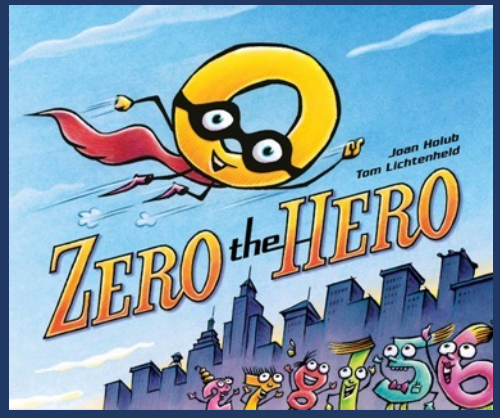
Use read-alouds

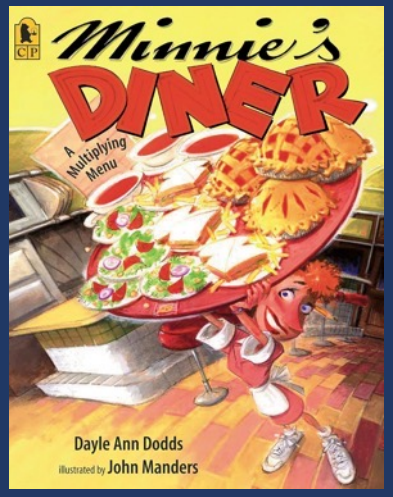












(Purpura et al., 2017)

Model and practice

Teacher	**Let's work on addition. Today, let's think about addition as combining. What does it mean to combine?**
Students	Put together.
Teacher	**When we combine, we put things together. When you cook, you put ingredients together. For example, to make macaroni and cheese, you combine what?**
Students	Macaroni noodles and cheese!
Teacher	**That's right. You combine macaroni and cheese! Now, let's think about combining numbers. Look at this problem.** (Show problem.)

(Powell & Driver, 2015; Stevens et al., 2022)

Make sure students use mathematics vocabulary as much as adults do!

Common Language on the STAAR: Grades 4 and 5 Math Terms

Below are two tables of math terms commonly found on the Mathematics STAAR for Grades 4 and 5.
*Note: *indicates terms found at Grades 4 and 5.*

Grade 4 Terms

acute angle	angle	area	array	comparison
cups	degrees	digit	*equation	equivalent
estimate	exactly	*expression	feet	*fraction
*frequency table	*greater	*hundredths	inches	*intersect
*length	*less than	*measure	millions	mixed number
*more than	obtuse angle	parallel	*perimeter	perpendicular
protractor	quart	*stem and leaf plot	strip diagram	sum
thousands	twice	*value	variable expenses	

Grade 5 Terms

additive relationship	balance the budget	centimeters	coordinate grid	cubic units
data	diagram	difference	earn	*equation
equilateral triangle	*expression	*fraction	*frequency table	graph
*greater than	greatest	horizontal line	*hundredths	*intersecting
isosceles triangle	least	*length	*less than	*measure
miles	model	*more	multiplicative relationship	ordered pairs
pentagon	*perimeter	polygon	quadrilateral	rectangular prism
relationship	represent	rounding	scalene triangle	scatterplot
simplify	square inches	*stem and leaf plot	total	triangle
true	two-dimensional figure	*value	volume	*x*-coordinate
y-coordinate				

mathspiral.com

The STAAR features commonly used vocabulary terms.

Look at the STAAR. What are the 20 terms your students need to know?

VOCABULARY

Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

What are your strengths with vocabulary?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

Students require modeling and practice on how to use the language of mathematics.

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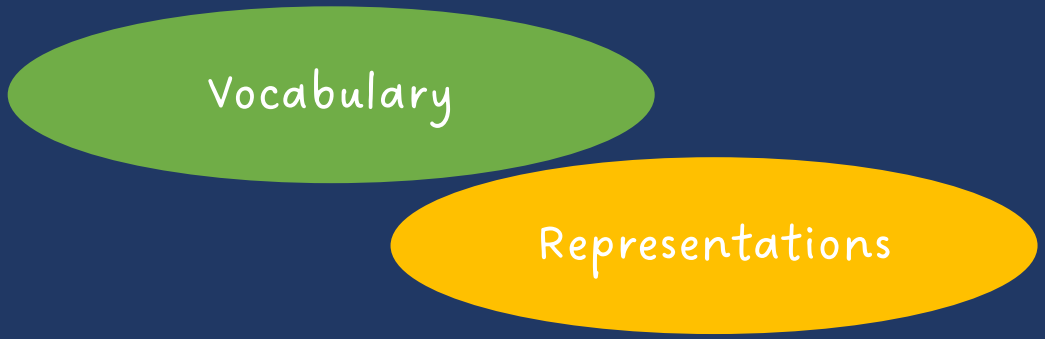
Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Representations

Instructional Platform

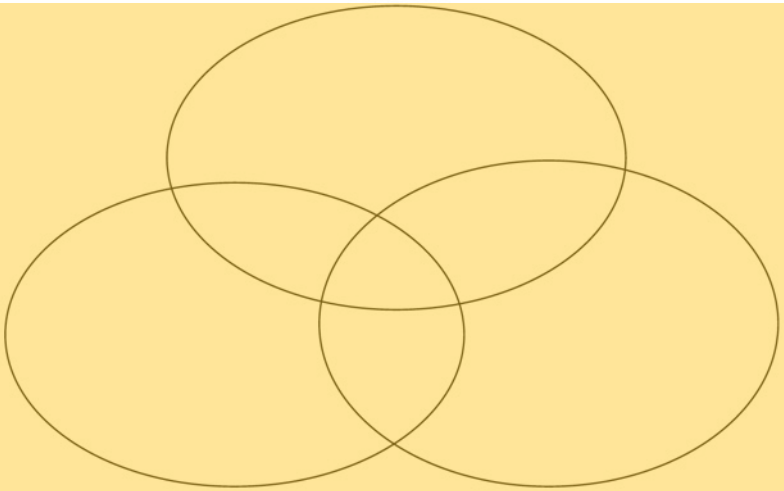
INSTRUCTIONAL DELIVERY



INSTRUCTIONAL STRATEGIES

REPRESENTATIONS

Research and Information



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MATH

MATH

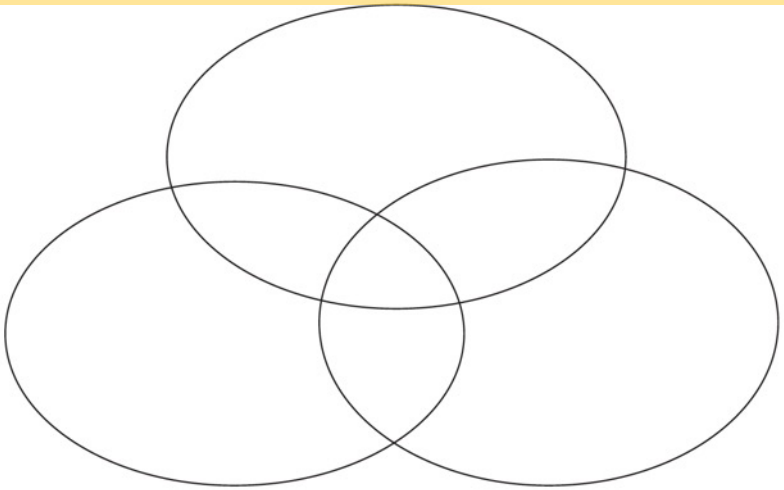
Hands-on manipulatives contribute to increases in mathematics performance.
(Bouck & Park, 2018; Carbonneau et al., 2013; Namkung & Bricko, 2021; Sherman & Bisanz, 2009; Strickland & Maccini, 2012)

Virtual manipulatives contribute to increases in mathematics performance.
(Bouck et al., 2020; Satsangi et al., 2016)

Other visuals (e.g., graphic organizers) contribute to increases in mathematics performance.
(Jitendra et al., 2009; Sharp & Dennis, 2017; van Garderen, 2007; Xin, 2008)

REPRESENTATIONS

Research and Information



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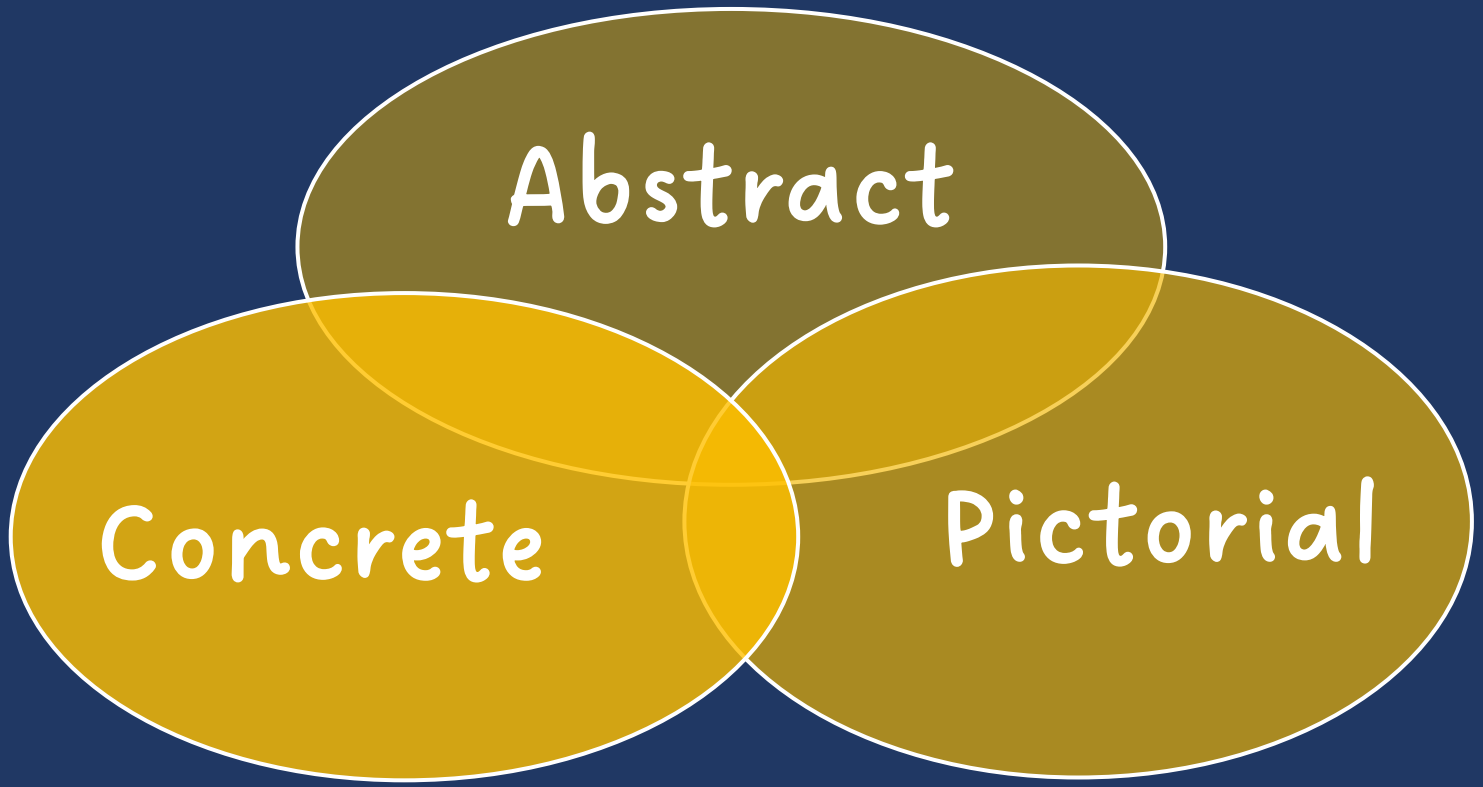
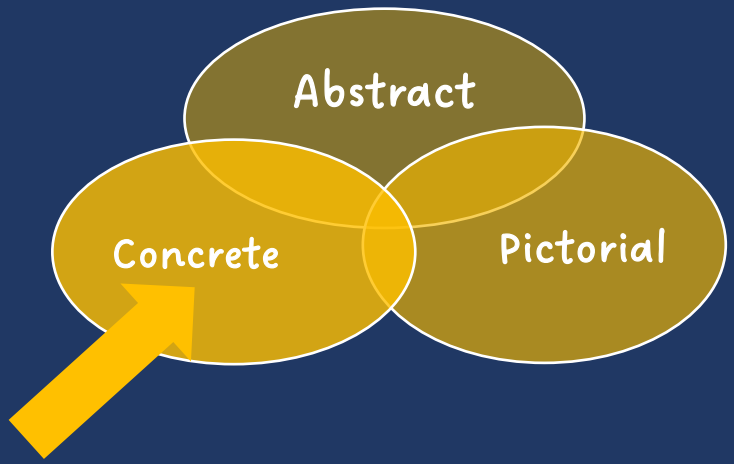
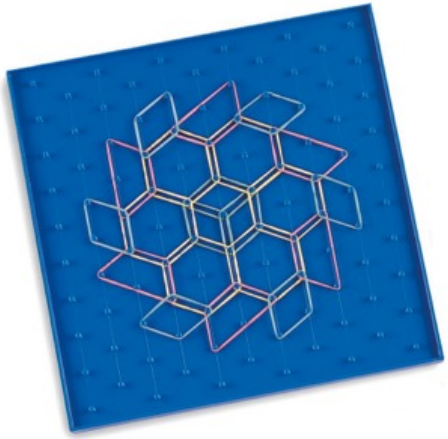
MATH

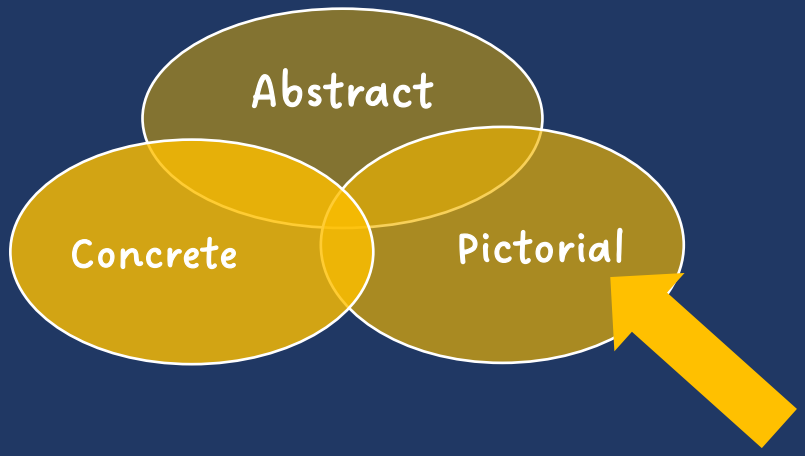
MATH

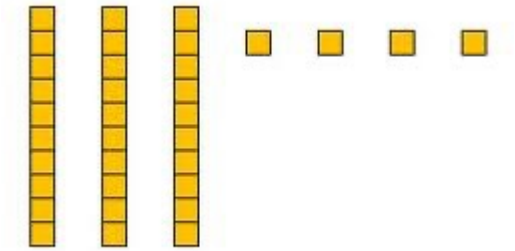


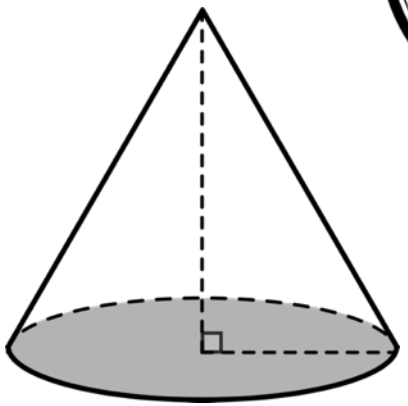


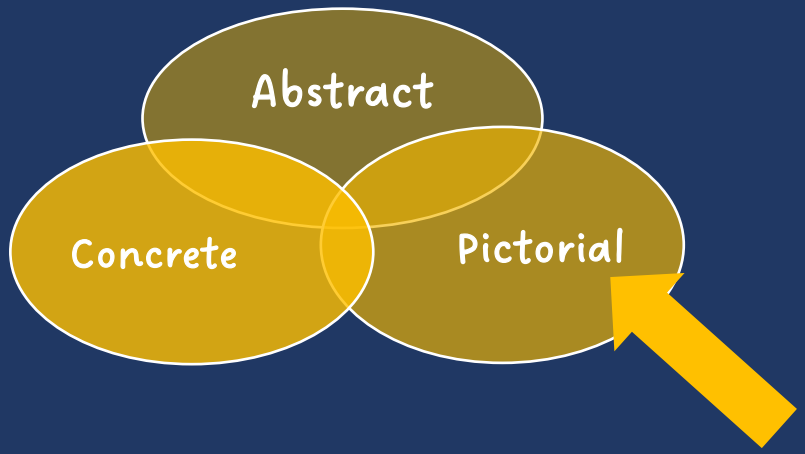


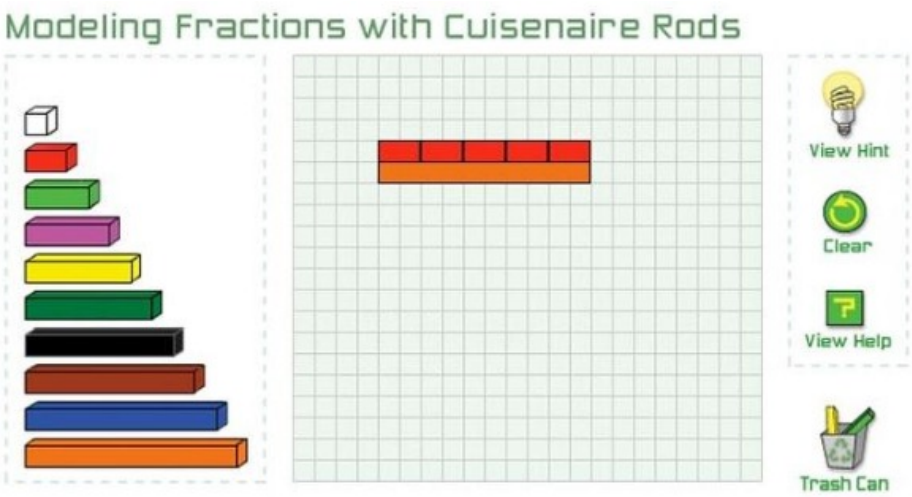
Describe your favorite hands-on materials.

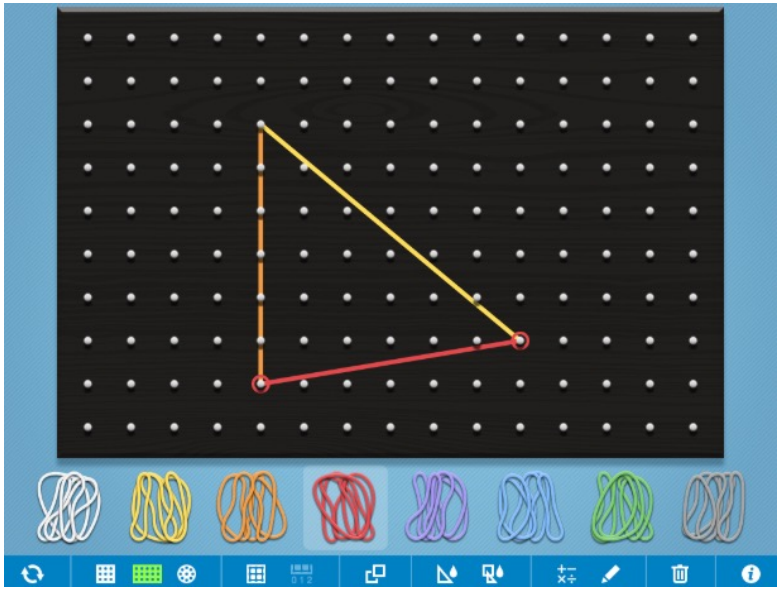


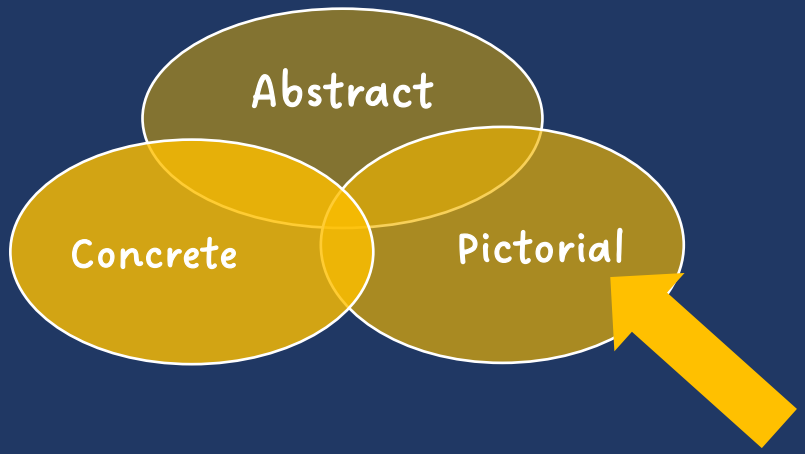


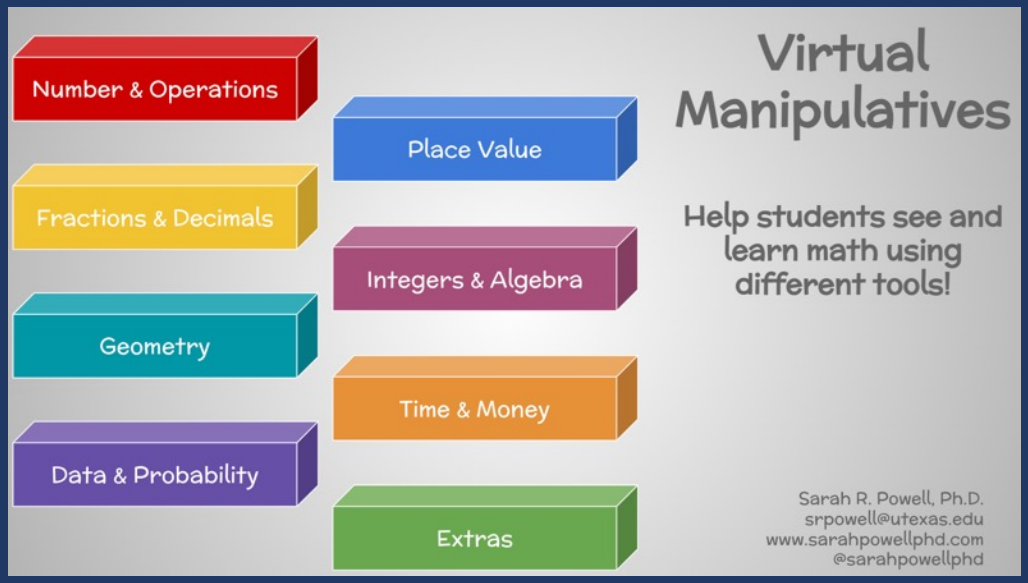












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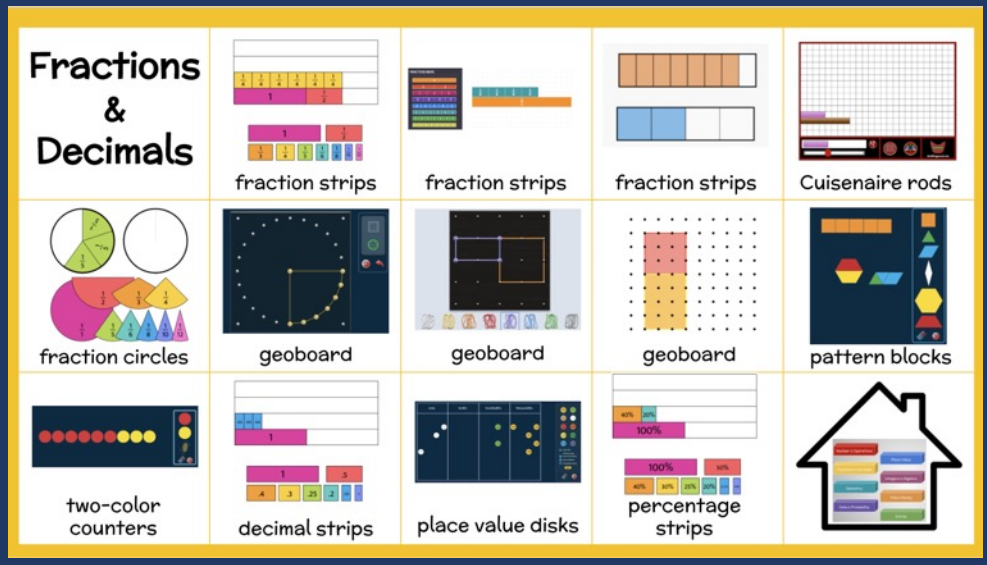
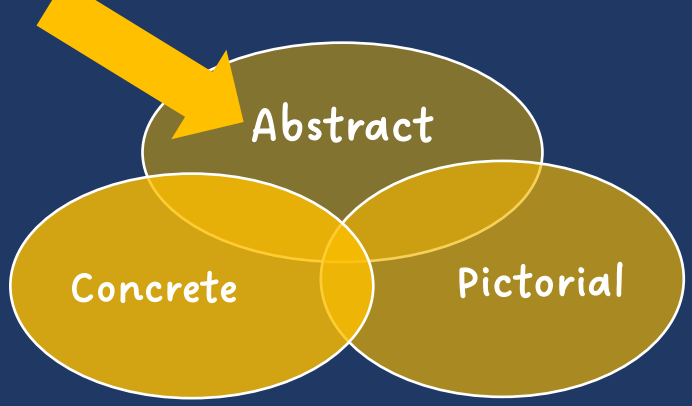


Explore 3 virtual manipulatives.

Share with a partner.



$2 + 8 = 10$

34 = 3 tens and 4 ones

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

$$\begin{array}{r} 4{,}179 \\ +\ \ 569 \\ \hline \end{array}$$

REPRESENTATIONS

STAAR Representations

STAAR Item	Representations

What are your strengths?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

The STAAR features common representations.

Identify STAAR items with representations and describe the representation.

REPRESENTATIONS

STAAR Representations

STAAR Item	Representations

What are your strengths?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

What are your strengths with multiple representations?

What are your opportunities for growth?

What are your plans for next Monday?

Next month?

Next year?

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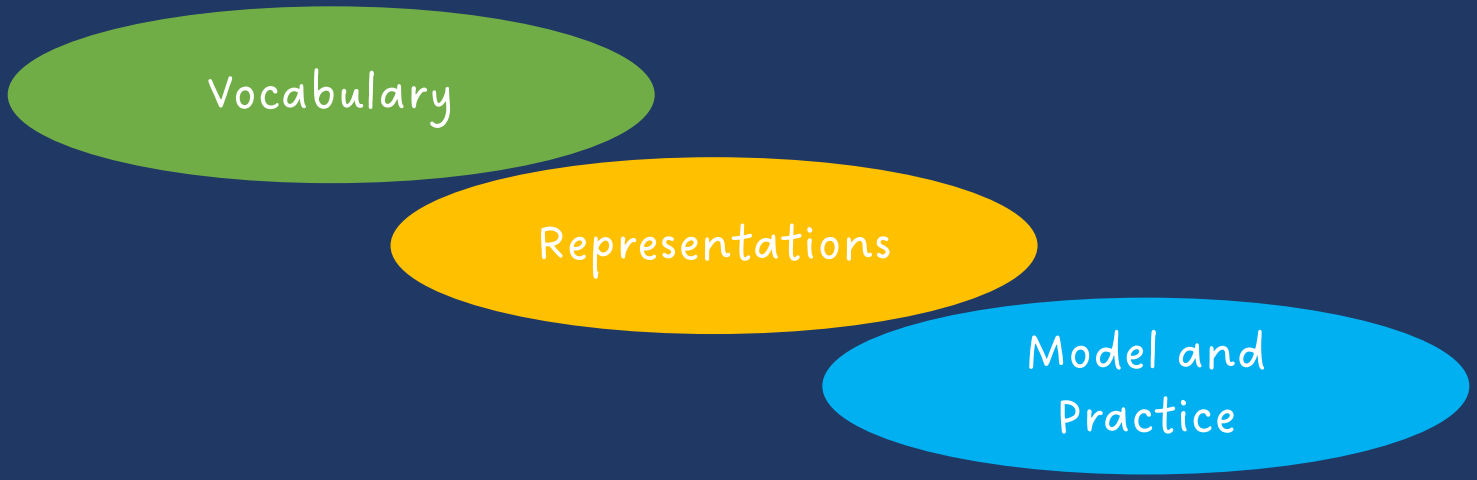
Model and Practice

Instructional Platform

INSTRUCTIONAL DELIVERY



INSTRUCTIONAL STRATEGIES

MODEL AND PRACTICE

Research and Information

MODELING

PRACTICE

SUPPORTS

Over a half century of research supports explicit (i.e., direct, systematic) instruction.
(Stockard et al., 2018)

When compared to discovery approaches, explicit instruction demonstrates higher outcomes.
(Alfieri et al., 2011; Kroesbergen et al., 2004; Poncy et al., 2010)

Numerous meta-analyses and large-scale studies have identified explicit instruction as essential for the teaching and learning of mathematics.
(Chodura et al., 2015; Ennis & Losinski, 2019; Jitendra et al., 2018; Kong et al., 2021; Morgan et al., 2015; Nelson & McMaster, 2019; Powell et al., 2021).

MODEL AND PRACTICE

Research and Information

MODELING

PRACTICE

SUPPORTS

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

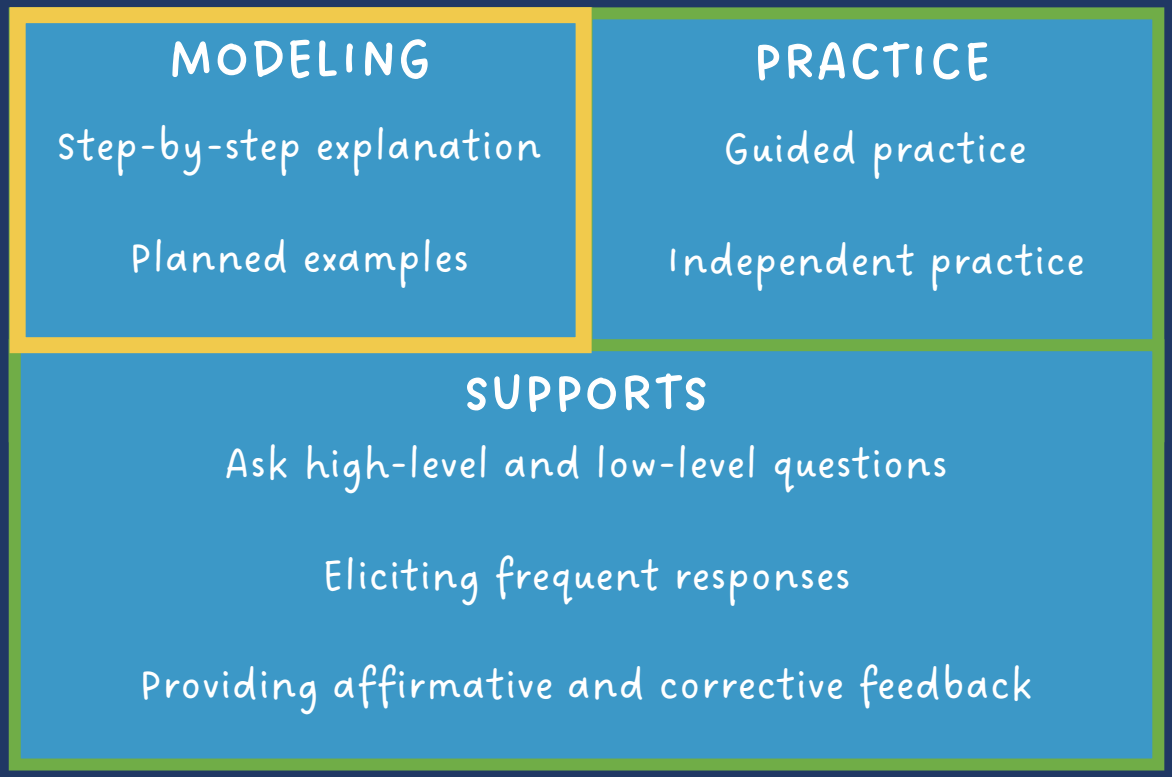
SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

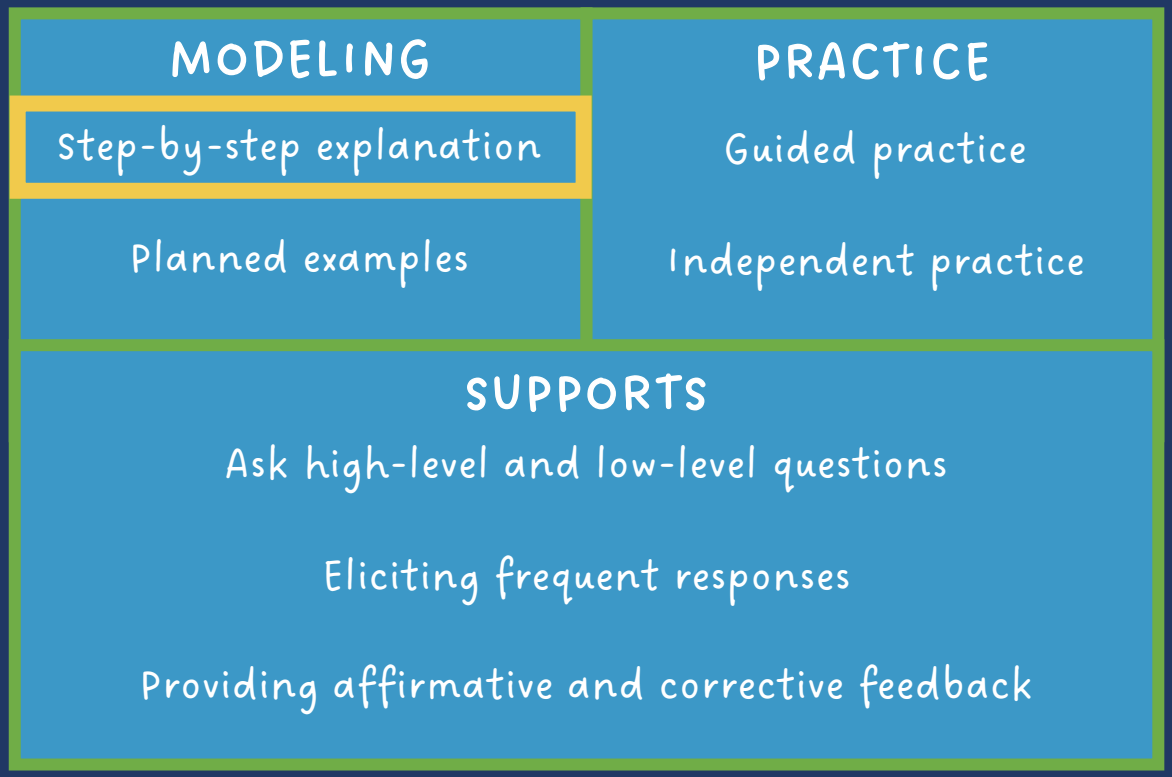
Providing affirmative and corrective feedback

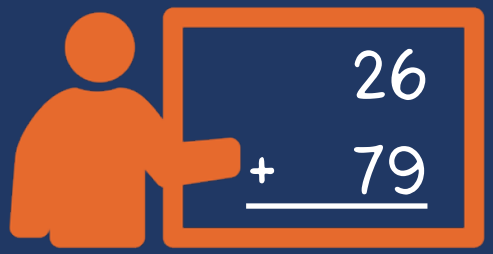
Modeling is a dialogue between the teacher and students.



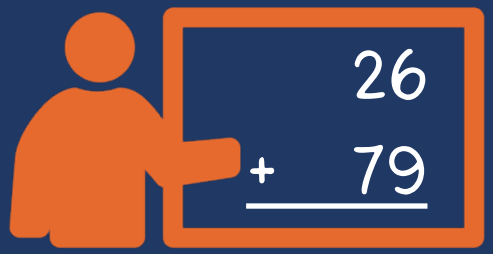
Modeling includes a step-by-step explanation of how to do a math problem.

A teacher may do 1 modeled problem or several.





"Today, we are learning about addition. This is important because sometimes you have different amounts – like money – and you want to know how much money you have altogether."



"Let's solve this problem. What's the problem?"

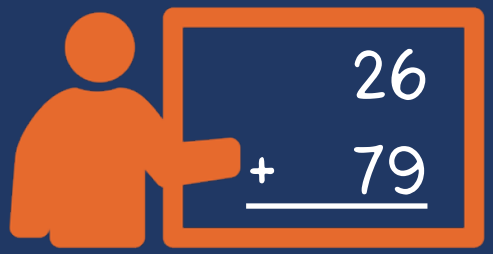
"26 plus 79."

"To solve 26 plus 79, first decide about the operation. Should we add, subtract, multiply, or divide?"

"Add."

"How did you know we want to add?"

"There's a plus sign."



"The plus sign tells us we want to add. To add, let's use the partial sums strategy. What strategy?"

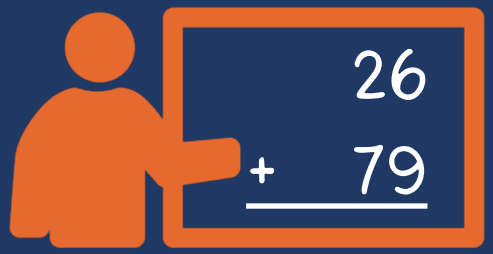
"Partial sums."

"What might partial mean?"

"Part of."

"We'll find parts – or partial sums - then add them together. With the partial sums strategy, we start adding in the greatest place value. What's the greatest place value in this problem?"

"The tens."



"So, let's add the tens. What's 20 plus 70? Use your base-10 blocks or other tools."

"90."

"20 plus 70 equals 90. Let's write 90 right here below the equal line. What will we write?"

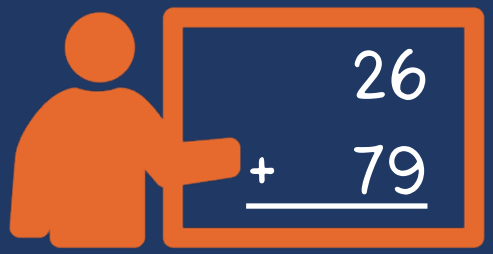
"90."

"90 is the partial sum when you add the tens. What does 90 represent?"

"It's the partial sum of adding 20 plus 70."

"Now, let's add the ones. What should we add?"

"6 plus 9."



"6 plus 9 equals what? Use your base-10 blocks or other tools."

"15."

"How did you get 15?"

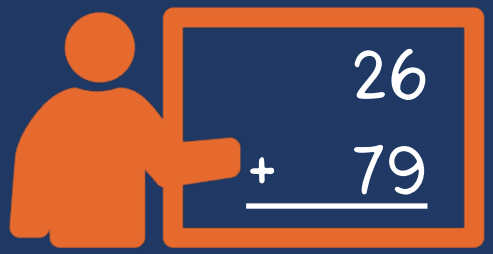
"We knew we had 9, then we added on 6."

"Let's write 15 below the 90. Where do we write the 15?"

"Below the 90."

"15 is the partial sum when you add the ones. Now, let's add the partial sums together. What will we add?"

"90 plus 15."



"What's 90 plus 15? Use your go-to strategy."

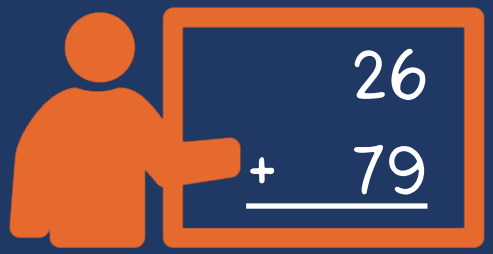
"105."

"How did you add those addends?"

"I added 90 plus 10 then added 5 more."

"So, when you add 26 plus 79, the sum is 105. Who can share how we solved this problem?"

"We used the partial sums strategy. We added the tens then added the ones. Then we added the partial sums."

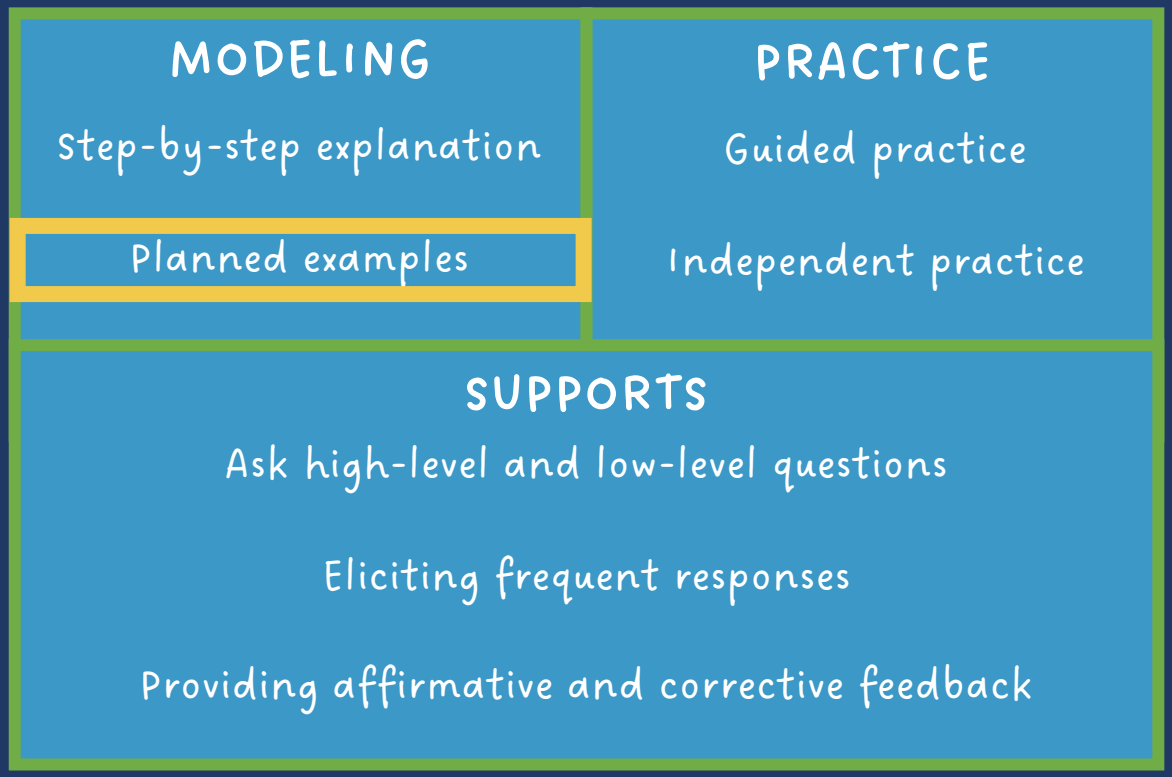


What did you observe?

How would you improve this example?

Modeling needs to include planned examples.

These examples should be sequenced so easier skills lead to more difficult skills.



MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

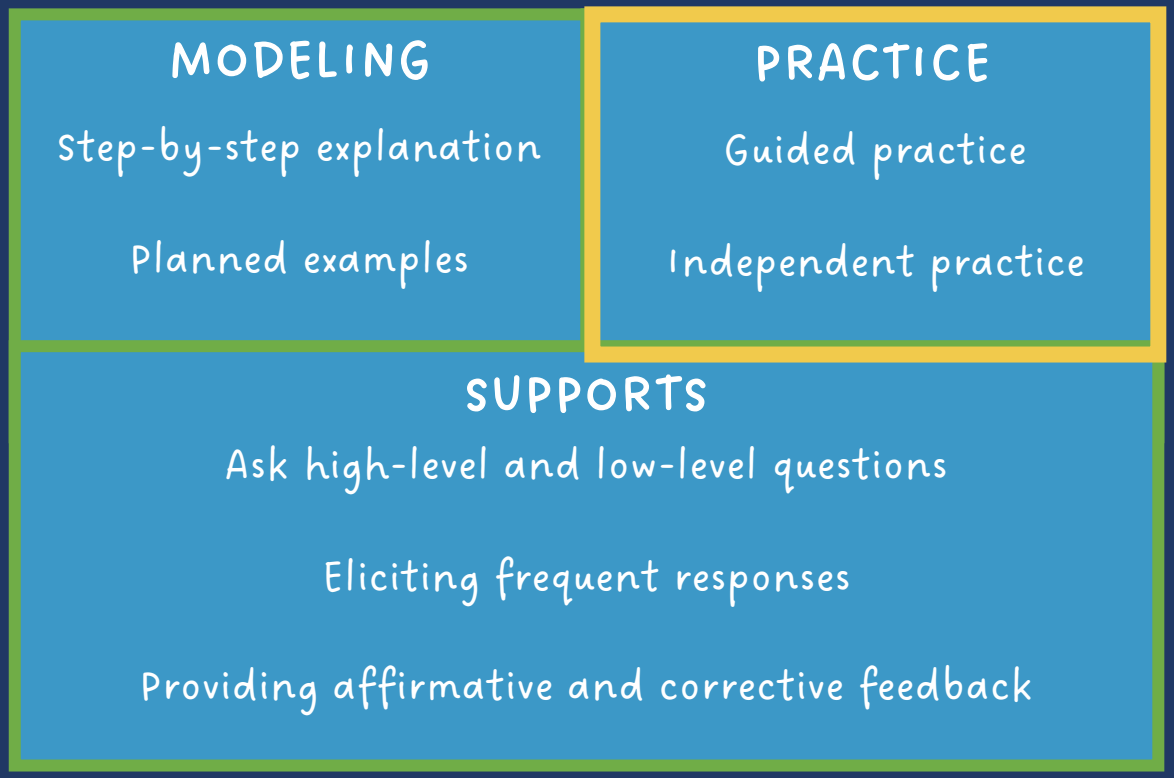
Providing affirmative and corrective feedback

MODEL AND PRACTICE

Problem	Step-by-Step Explanation

Select a STAAR problem.

Work with a partner to outline a step-by-step explanation.



Practice continues as a dialogue between the teacher and students.

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Guided practice is practice in which the teacher and students practice problems together.

"Let's work on a problem together."

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Independent practice is practice in which the students practice independently with teacher support.

"Now, you'll practice a problem on your own. Use your attack strategy!"

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

MODEL AND PRACTICE

Problem

Practice Opportunities

High-Level Questions

Low-Level Questions

Affirmative Feedback

Corrective Feedback

Describe how you would engage students in practice.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

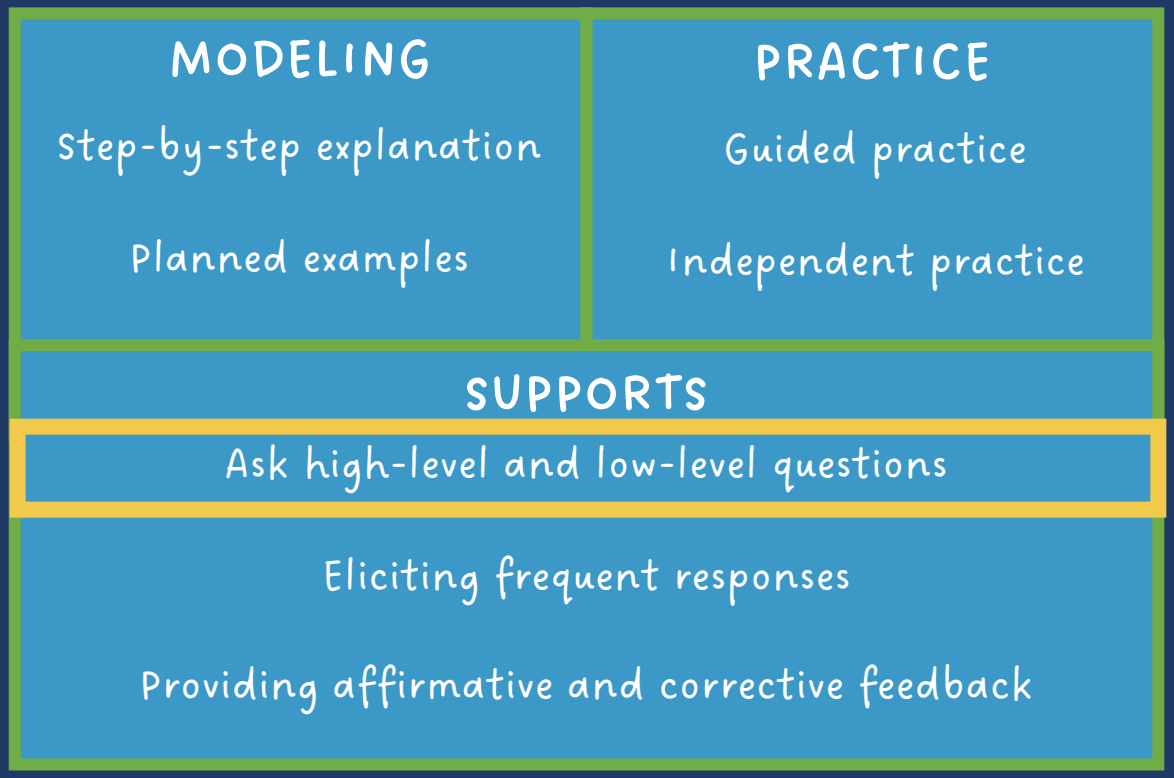
SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

These **Supports** should be used in both **Modeling** and **Practice.**



During **Modeling** and **Practice**, it is essential to engage students and check for understanding.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"What is 7 times 9?"

"63."

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

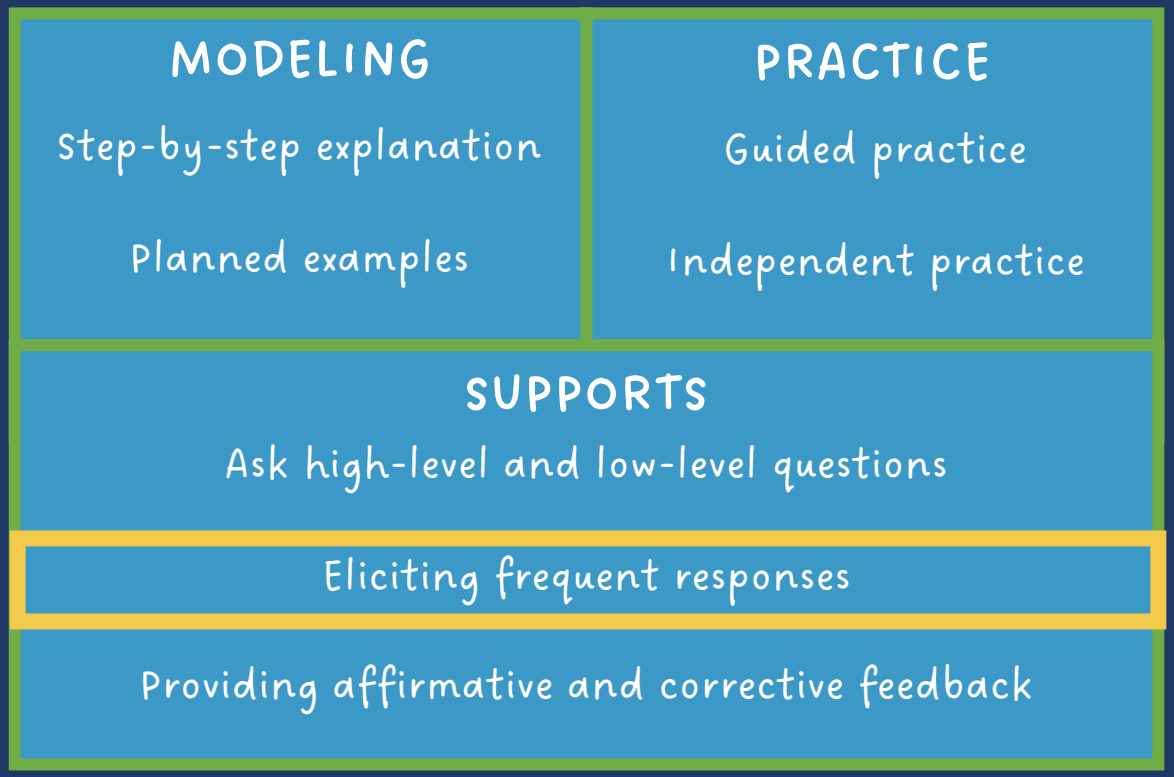
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Why do you use zero pairs?"

"Because a positive 1 and a negative 1 equal 0. I use the zero pair to help me subtract."



During **Modeling** and **Practice**, students should frequently respond. The frequent responses keeps student attention and keeps student learning active.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

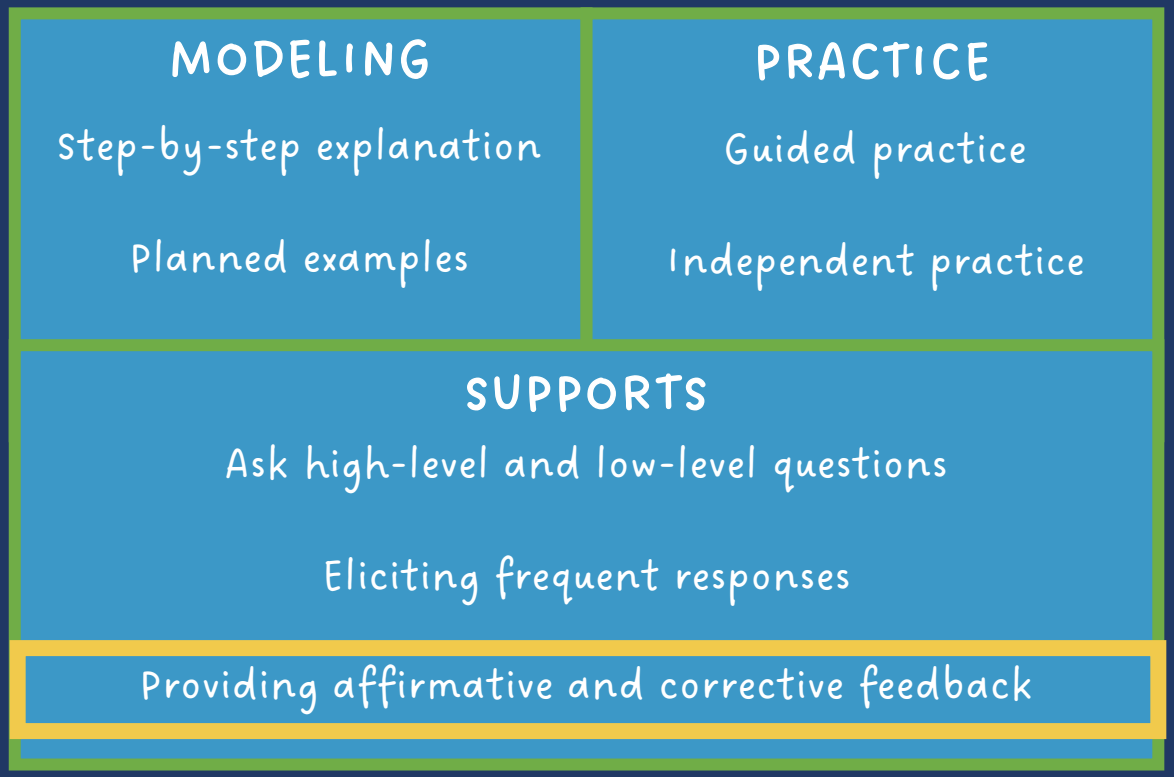
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback







During **Modeling** and **Practice**, students should receive immediate feedback on their responses.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Nice work using your word problem attack strategy."

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Let's look at that again. Tell me how you added in the hundreds column."

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

MODEL AND PRACTICE

Problem

Practice Opportunities

High-Level Questions

Low-Level Questions

Affirmative Feedback

Corrective Feedback

Provide several of your questions.

Provide examples of your feedback.

MODEL AND PRACTICE

What are your strengths?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

What are your strengths with modeling and practice?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

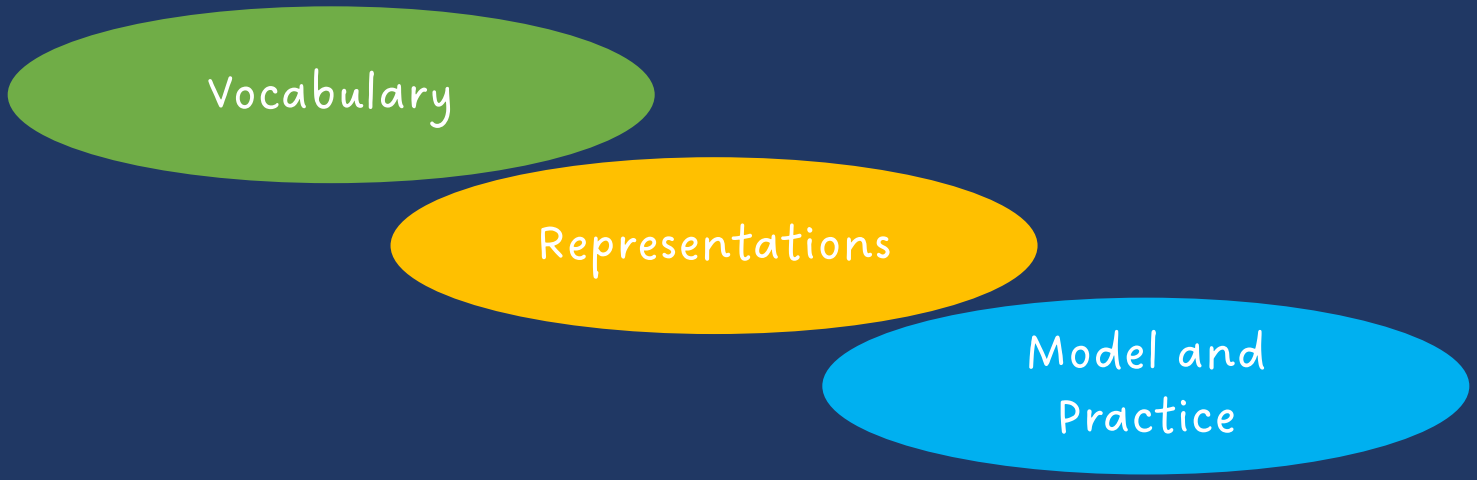
Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Fluency

Instructional Platform

INSTRUCTIONAL DELIVERY



INSTRUCTIONAL STRATEGIES

FLUENCY

Research and Information

Types of Fluency

Type	Memorization?	
	Yes	No

Mathematics fluency, particularly fluency with facts and computation, is related to overall mathematics performance.

(Bailey et al., 2012; Cirino et al., 2019; Koponen et al., 2007, 2017; Vukovic et al., 2014

Fact fluency practice improves mathematics fact performance.

(Burns et al., 2010; Codding et al., 2011; McCallum et al., 2004; Nelson et al., 2013; Poncy et al., 2010; Schutte et al., 2015; Stocker & Kubina, 2017)

Fluency is doing mathematics easily and accurately.

Fluency in mathematics makes mathematics easier.

Fluency provides less stress on working memory.

Fluency helps students build confidence with mathematics.

With fluency, it is important to emphasize both conceptual learning and procedural learning.

Addition	Subtraction
Multiplication	Division

Counting

Comparison

Money

Time

Fractions

Geometry

Measurement

Algebra

Ease and accuracy

Memorization or automaticity

FLUENCY

Research and Information

Types of Fluency

Type	Memorization?	
	Yes	No

List different types of fluency. Discuss whether they require memorization.

FLUENCY

Addition

Subtraction

Multiplication

Division

Addition

100 addition facts

Single-digit addends sum to a single- or double-digit number

```
   5    (addend)
+  4    (addend)
-----
   9    (sum)
```

Total

Addition

Count one set, count another set, put sets together, count sum

$$2 + 3 = 5$$

Change

Addition

Start with a set, add the other set, count sum

$$2 + 3 = 5$$

Total

Addition

Parts put together into a **total**

Karly saw **4** cardinals and **5** blue jays. How many birds did Karly see?

Change

Addition

An amount that **increases** or decreases

Premila had $4. Then they earned $5 for cleaning their room. How much money does Premila have now?

3 + 9 = __

Addition

If you teach elementary:

What's a Total story to show addition?

If you teach secondary:

What's a Change/Join story to show addition?

100 subtraction facts

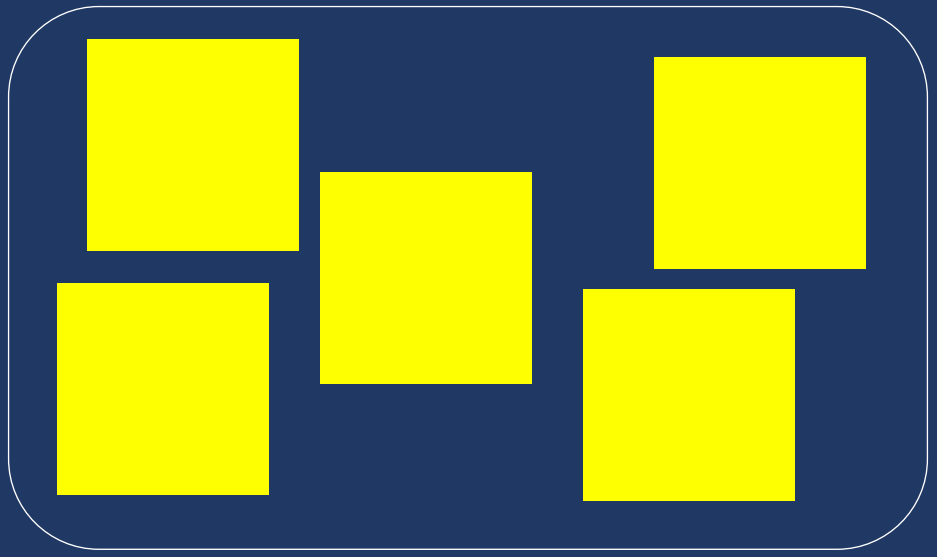
Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

16	(minuend)
− 8	(subtrahend)
8	(difference)

Change

Subtraction

Start with a set, take away from that set, count difference

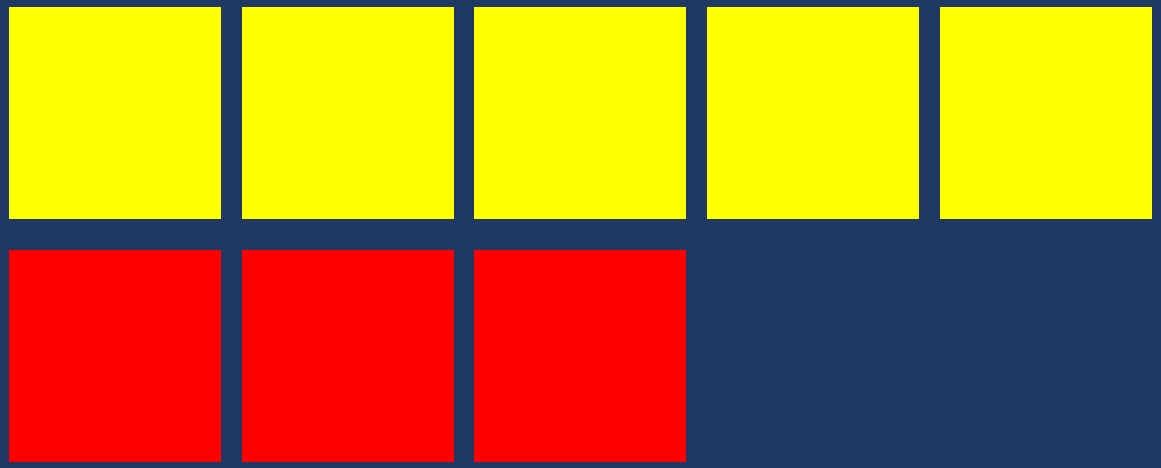


$$5 - 3 = 2$$

Difference

Subtraction

Compare two sets, count difference



$$5 - 3 = 2$$

Change

Subtraction

An amount that increases or **decreases**

Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?

Difference

Subtraction

Greater and lesser amounts compared for a difference

Rachel has 9 apples. Jodie has 2 apples. How many more apples does Rachel have? (How many fewer does Jodie have?)

9 - 5 = __

Subtraction

If you would chose beaches:

What's a Change/Separate story to show subtraction?

If you would chose mountains:

What's a Difference story to show subtraction?

Multiplication

100 multiplication facts

Multiplication of single-digit factors results in a single- or double-digit product

$$\begin{array}{rl} 2 & \text{(factor)} \\ \times\ 3 & \text{(factor)} \\ \hline 6 & \text{(product)} \end{array}$$

Equal Groups

Multiplication

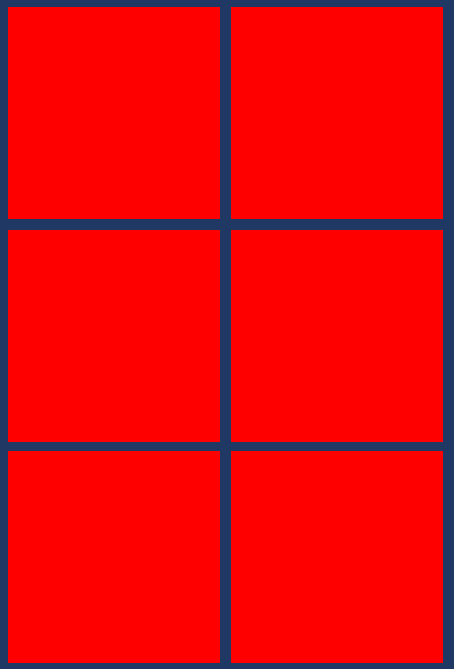
Show the groups, show the amount for each group, count product

$$3 \times 2 = 6$$

Equal Groups

Multiplication

Show the groups, show the amount for each group, count product

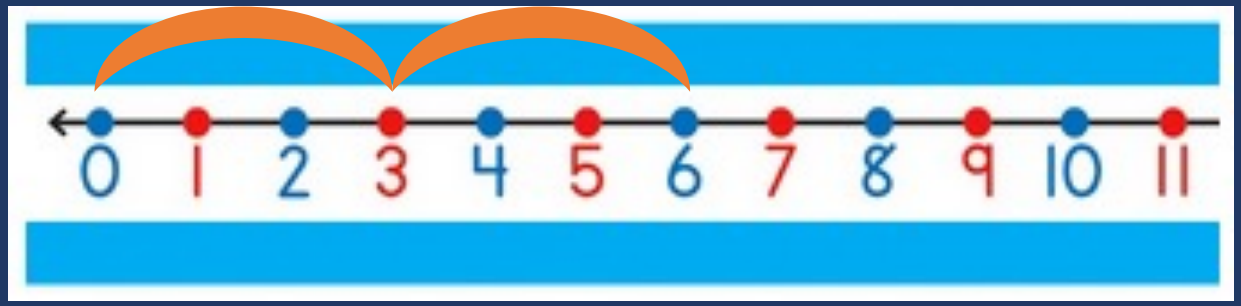


$3 \times 2 = 6$

Comparison

Multiplication

Show a set, then multiply the set



$$3 \times 2 = 6$$

Equal Groups

Multiplication

Groups multiplied by number in each group for a product

Rhiannon has 3 boxes of crayons. There are 8 crayons in each box. How many crayons does Rhiannon have altogether?

Comparison

Multiplication

Set multiplied by a number of times for a product

Vivienne had 8 stickers. Jessica had 3 times as many stickers as Vivienne. How many stickers did Jessica have?

$2 \times 5 = \_\_$

Multiplication

If you wear glasses:

What's an Equal Groups story to show multiplication?

If you don't wear glasses:

What's a Comparison story to show multiplication?

Division

90 division facts

Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

8 ÷ 4 = 2

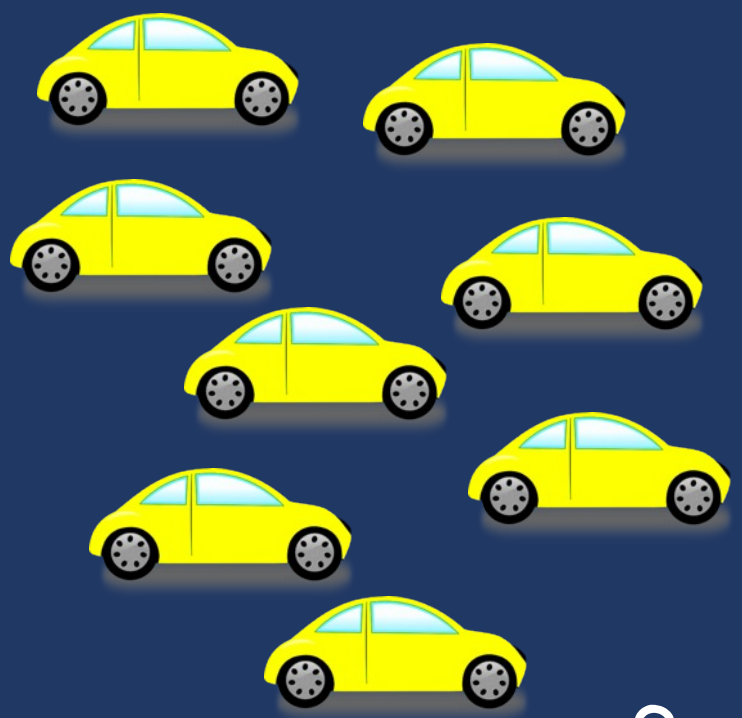
(dividend) (divisor) (quotient)

Equal Groups

(Partitive Division)

Division

Show the dividend, divide equally among divisor, count quotient



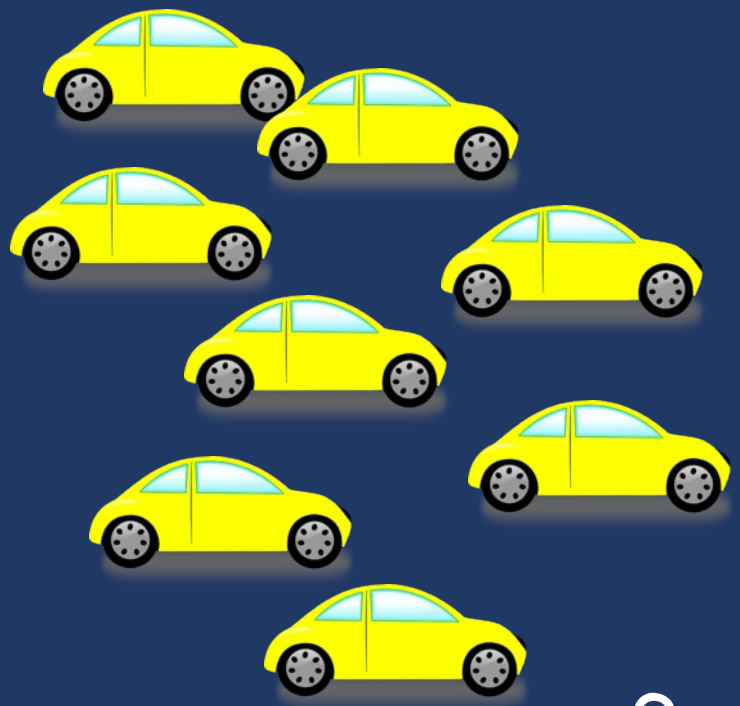
$8 \div 2 = 4$

Equal Groups

(Quotative Division)

Division

Show the dividend, make groups of the divisor, count groups



$$8 \div 2 = 4$$

Equal Groups

Division

Groups multiplied by number in each group for a product

Stefanie has 12 pencils. She wants to share them equally among her 2 friends. How many pencils will each friend receive?

Nicole has 12 pencils. She put them into pencil pockets with 6 pencils each. How many pencil pockets did Nicole use?

$12 \div 4 = \_\_$

Division

If you watch comedies:

What's a Partitive story to show division?

If you watch dramas:

What's a Quotative story to show division?

Addition	Subtraction
Multiplication	Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

$$\begin{array}{r} 5 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ -\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ \times\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ \div\ 8 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

Build fluency with whole-number computation

$$\begin{array}{r} 15 \\ +\ 28 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 1009 \\ -\ 724 \\ \hline \end{array}$$

$$\begin{array}{r} 7250 \\ \div\ 15 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

Build fluency with rational-number computation

$$\begin{array}{r} 1.4 \\ +\ 3.9 \\ \hline \end{array}$$

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{9}{4} - \frac{3}{8}$$

$$\begin{array}{r} 7.892 \\ \div\ 0.14 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

Build fluency with integer computation

$$\begin{array}{r} 1.4 \\ +\ -3.9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times\ -12 \\ \hline \end{array}$$

$-14 - (-7) =$

$-135 \div 2 =$

Partial Sums

A.

$$\begin{array}{r} 74 \\ +\ 18 \\ \hline 80 \\ +12 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \\ +\ 365 \\ \hline 1{,}000 \\ 80 \\ +\ 10 \\ \hline 1{,}090 \end{array}$$

$$\begin{array}{r} 227 \\ +\ 185 \\ \hline \end{array}$$

Opposite Change

A.

$$\begin{array}{r} 74 \xrightarrow{-4} 70 \\ +\ 18 \xrightarrow{+4} +22 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \xrightarrow{+5} 730 \\ +\ 365 \xrightarrow{-5} +360 \\ \hline 1{,}090 \end{array}$$

$$\begin{array}{r} 227 \\ +\ 185 \\ \hline \end{array}$$

Partial Differences

A.

$$\begin{array}{r} 62 \\ -\ 17 \\ \hline +50 \\ -\ 5 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ -\ 96 \\ \hline +300 \\ -90 \\ -1 \\ \hline 209 \end{array}$$

$$\begin{array}{r} 232 \\ -\ 164 \\ \hline \end{array}$$

Same Change

A.

$$\begin{array}{r} 62 \xrightarrow{+3} 65 \\ -\ 17 \xrightarrow{+3} -20 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \xrightarrow{+4} 309 \\ -\ 96 \xrightarrow{+4} -100 \\ \hline 209 \end{array}$$

$$\begin{array}{r} 232 \\ -\ 164 \\ \hline \end{array}$$

Add Up

A.

$$\begin{array}{r} 62 \\ -\ 17 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ 20 \quad 3 \\ 60 \quad 40 \\ 62 \quad +\ 2 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ -\ 96 \\ \hline \end{array}$$

$$\begin{array}{r} 96 \\ 100 \quad 4 \\ 300 \quad 200 \\ 305 \quad +\ 5 \\ \hline 209 \end{array}$$

$$\begin{array}{r} 232 \\ -\ 164 \\ \hline \end{array}$$

Partial Products

A.

$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline 800 \\ 160 \\ 60 \\ +\ 12 \\ \hline 1{,}032 \end{array}$$

B.

$$\begin{array}{r} 132 \\ \times\ 53 \\ \hline 5000 \\ 1500 \\ 100 \\ 300 \\ 90 \\ +\ 6 \\ \hline 6996 \end{array}$$

$$\begin{array}{r} 13 \\ \times\ 47 \\ \hline \end{array}$$

Area (Array)

A.

$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline \end{array}$$

20	4	
800	160	40
60	12	3

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 800 \\ 160 \\ 60 \\ +\ 12 \\ \hline 1{,}032 \end{array}$$

B.

$$\begin{array}{r} 132 \\ \times\ 53 \\ \hline \end{array}$$

100	30	2	
5000	1500	100	50
300	90	6	3

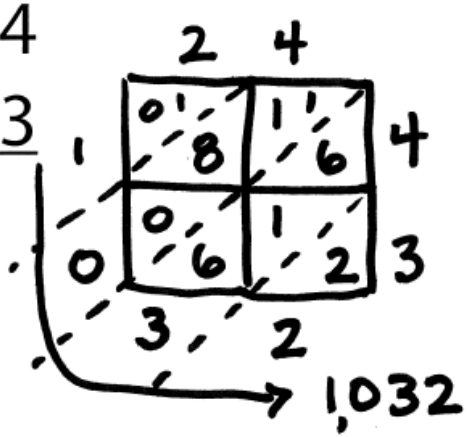
$$\begin{array}{r} 5000 \\ 1500 \\ 100 \\ 300 \\ 90 \\ +\ 6 \\ \hline 6996 \end{array}$$

$$\begin{array}{r} 13 \\ \times\ 47 \\ \hline \end{array}$$

Lattice

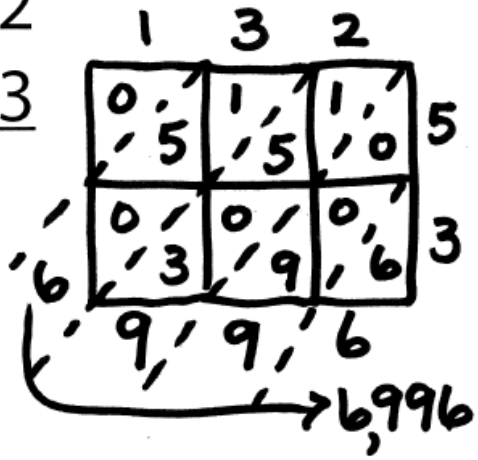
A.

$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline \end{array}$$



B.

$$\begin{array}{r} 132 \\ \times\ 53 \\ \hline \end{array}$$



$$\begin{array}{r} 13 \\ \times\ 47 \\ \hline \end{array}$$

Partial Quotients

A.

$$
\begin{array}{r|l}
12\overline{)158} & \\
-120 & 10 \\
\hline
38 & \\
-36 & +3 \\
\hline
2 & 13\ \text{R}2
\end{array}
$$

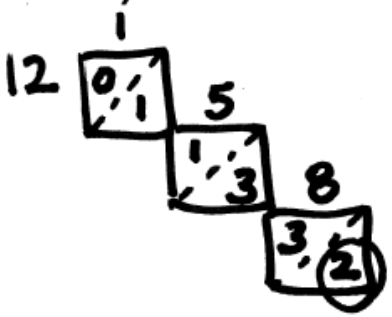
B.

$$
\begin{array}{r|l}
34\overline{)970} & \\
-680 & 20 \\
\hline
290 & \\
-170 & 5 \\
\hline
120 & \\
102 & +3 \\
\hline
18 & 28\ \text{R}18
\end{array}
$$

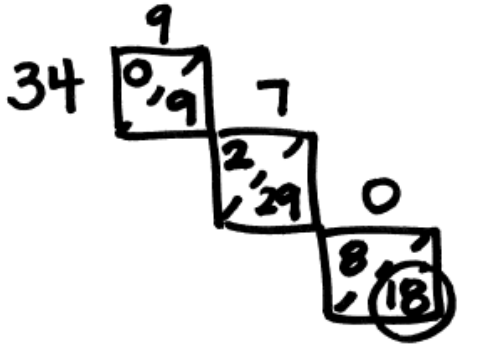
$$
\begin{array}{r}
804 \\
\div\ 12 \\
\hline
\end{array}
$$

Lattice

A. $12\overline{)158}$ = 13 R2



B. $34\overline{)970}$ = 28 R 18



$$804 \div 12$$

FLUENCY

Strategies for Building Fluency

What are your strengths?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

DAILY

BRIEF

Work on small sets of facts

Work on unknown facts
(in combination with known facts)

Dice

Roll the Dice

8 + 7 = 15

____ + ____ = ____

____ + ____ = ____

____ - ____ = ____

____ - ____ = ____

____ + ____ = ____

____ + ____ = ____

____ - ____ = ____

____ - ____ = ____

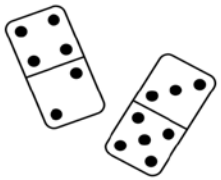
Beach Ball

4 plus 6 equals 10.

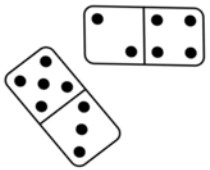
7 plus 6 equals 13.

2 plus 2 equals 4.

Dominoes



Dominoes



4 + 6 = 10

____ + ____ = ____

____ − ____ = ____

____ − ____ = ____

____ × ____ = ____

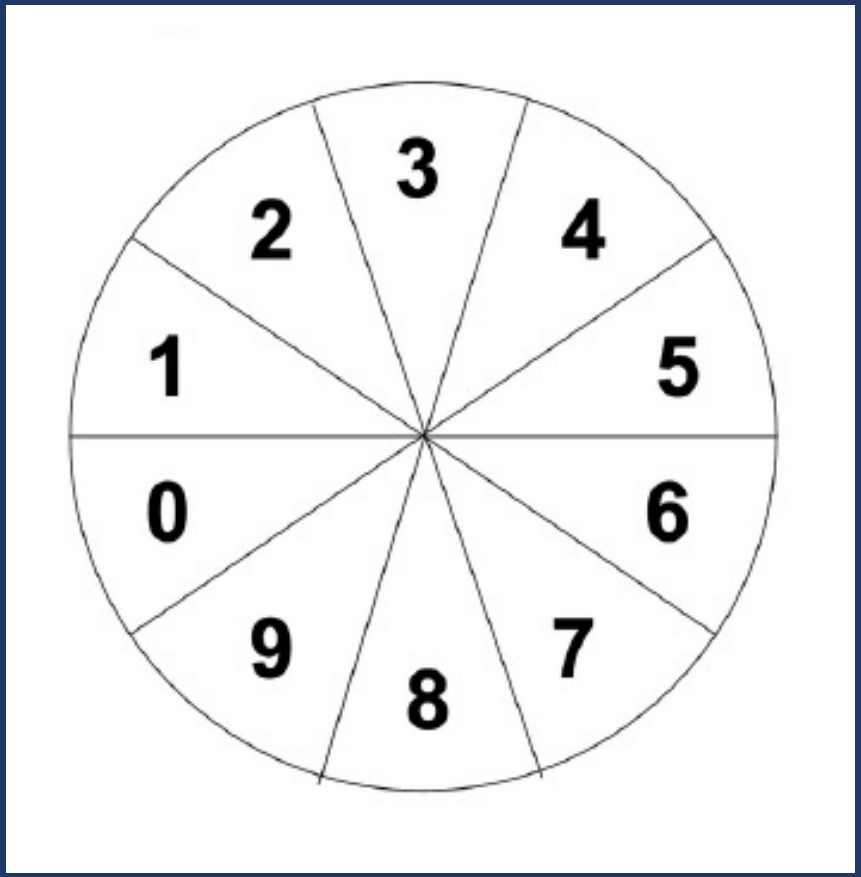
____ × ____ = ____

Spinner

2 times 4 equals 8.

6 times 9 equals 54.

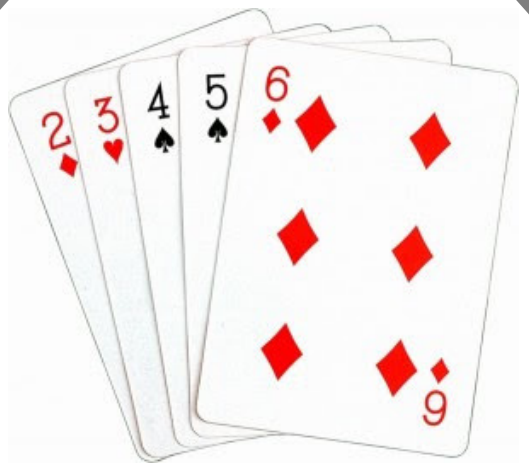
7 times 1 equals 7.



Playing Cards

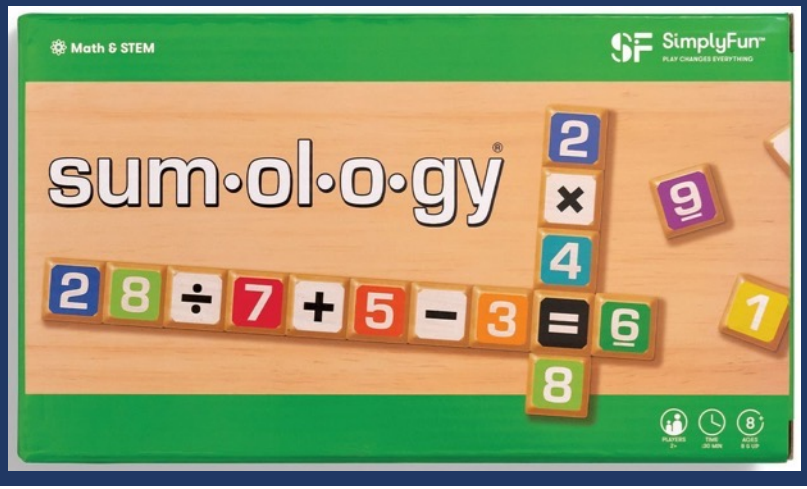
Cards 

____ + ____ = ____	____ + ____ = ____
____ + ____ = ____	____ + ____ = ____
6 − 2 = 4	____ − ____ = ____
____ − ____ = ____	____ − ____ = ____

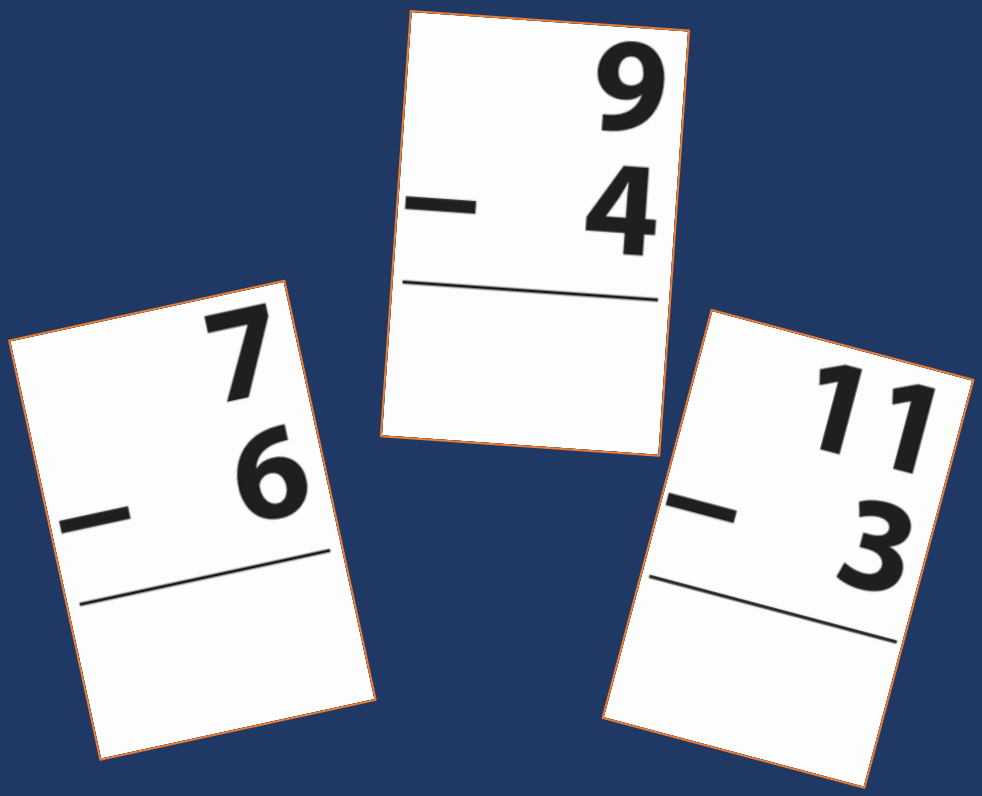


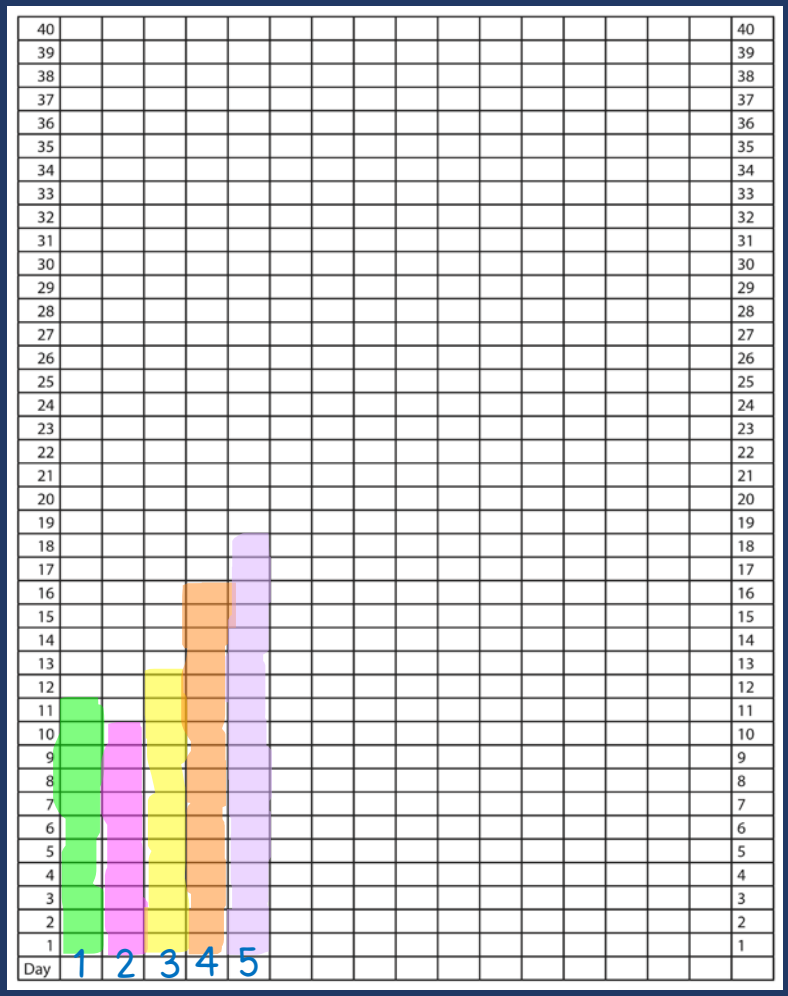
Wrap-Ups

Mobi Math



Flash Cards

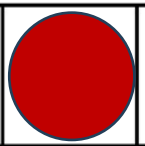
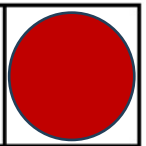
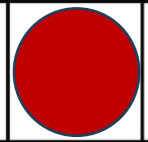
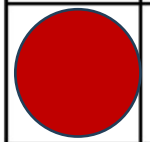




Bingo

Math Bingo

Your teacher will call out a math problem. Quickly solve the problem. If you have the answer on your bingo card, cover it with a chip. The middle space is free!
The first person to finish the pattern your teacher decides wins!
(5 in a row, 4 corners, etc.)

12		24	100	
15	0	42	16	20
8	35		6	4
	2	40	27	7
50	10	30	48	14

8 times 10 equals...

3 times 1 equals...

2 plus 3 equals...

Magic Squares

Magic Squares Board

1. Place the sum or product in the bottom right corner.
2. In the bottom row, create a fact with a sum or product of the bottom right corner.
3. In the right column, create a fact with a sum or product of the bottom right corner.
4. Create two columns with a sum or product of the bottom number.
5. Create two rows with a sum or product of the right column number.
6. Write the created facts below.

0	2	2
5	4	9
5	6	11

$0 + 2 = 2$

$5 + 4 = 9$

$2 - 0 = 2$

$9 - 5 = 4$

4	5	9
2	0	2
6	5	11

7	3	10
1	0	1
8	3	11

6	1	7
3	2	5
9	3	12

4	4	8
2	2	4
6	6	12

5	1	6
4	3	7
9	4	13

5	1	6
3	4	7
8	5	13

6	3	9
2	3	5
8	6	14

1	5	6
6	2	8
7	7	14

6	2	8
3	4	7
9	6	15

4 5 2 0

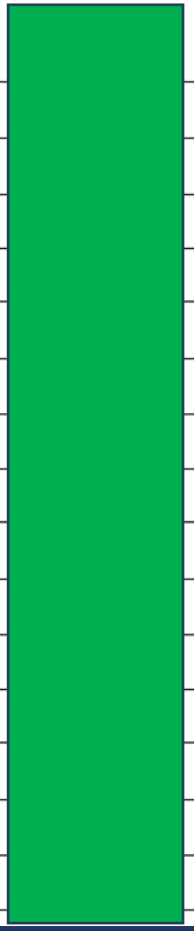
5 6 9 11 2

Cover, Copy, Compare

Cover, Copy, Compare

	9 x 6 54	8 × 6 48	
7 × 8 56		6 × 5 30	
9 × 9 81		7 × 9 63	
6 × 7 42		8 × 5 40	
8 × 8 64		7 × 7 49	

File Folder



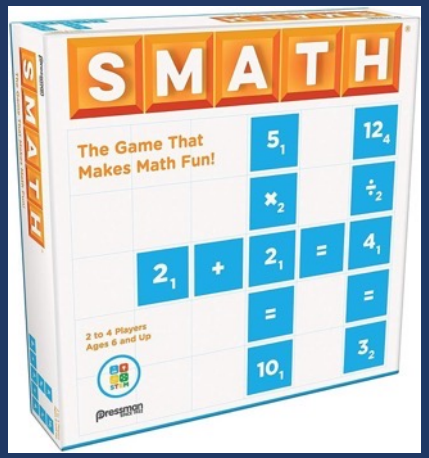
6 + 3 = 9
1 + 7 = 8
6 + 4 = 10
7 + 3 =
2 + 7 =
5 + 6 =
4 + 7 =
7 + 8 =
6 + 7 =
7 + 9 =
7 + 6 =
8 + 7 =
7 + 0 =
9 + 6 =
6 + 0 =
6 + 8 =

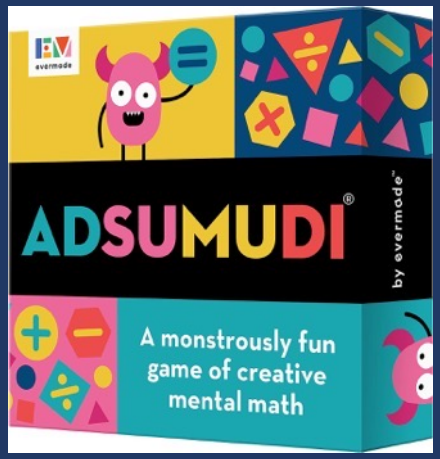
Taped Problems

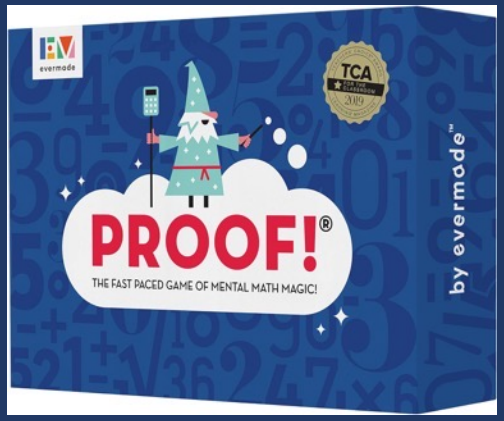
Taped Problems

$8 \times 8 = 64$	$7 \times 7 = 49$	$8 \times 7 =$
$6 \times 5 =$	$6 \times 7 =$	$6 \times 8 =$
$5 \times 7 =$	$5 \times 5 =$	$5 \times 6 =$
$6 \times 6 =$	$8 \times 6 =$	$7 \times 6 =$
$7 \times 8 =$	$8 \times 5 =$	$7 \times 5 =$

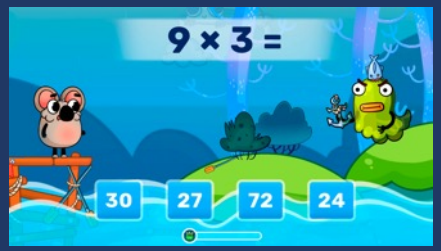
Games

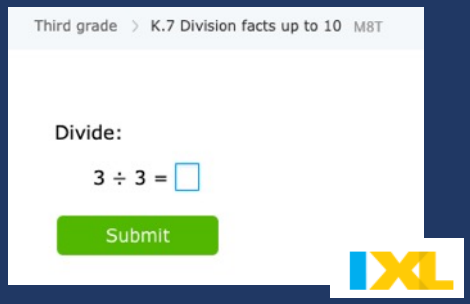


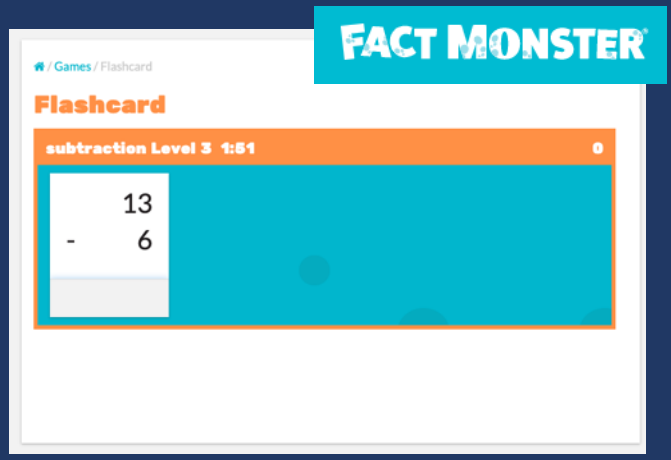




Technology







DAILY

BRIEF

Work on small sets of facts

Work on unknown facts
(in combination with known facts)

FLUENCY

Strategies for Building Fluency

What are your strengths?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

What are your strengths with fluency?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Word Problems

Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and Practice

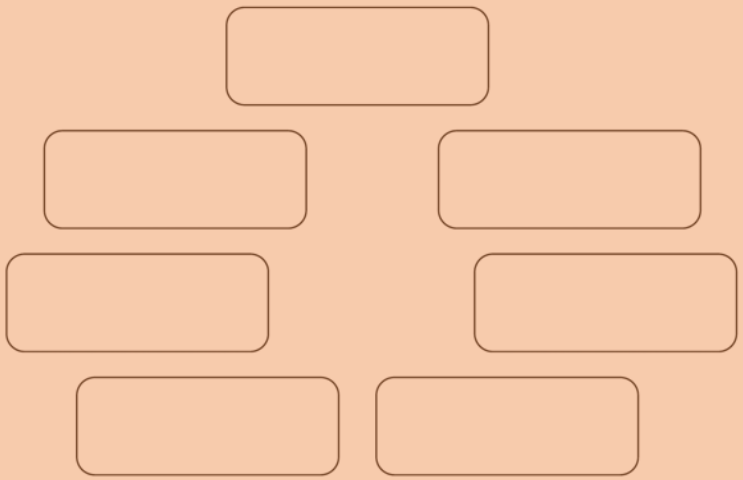
INSTRUCTIONAL STRATEGIES

Fluency

Word Problems

WORD PROBLEMS

Research and Information



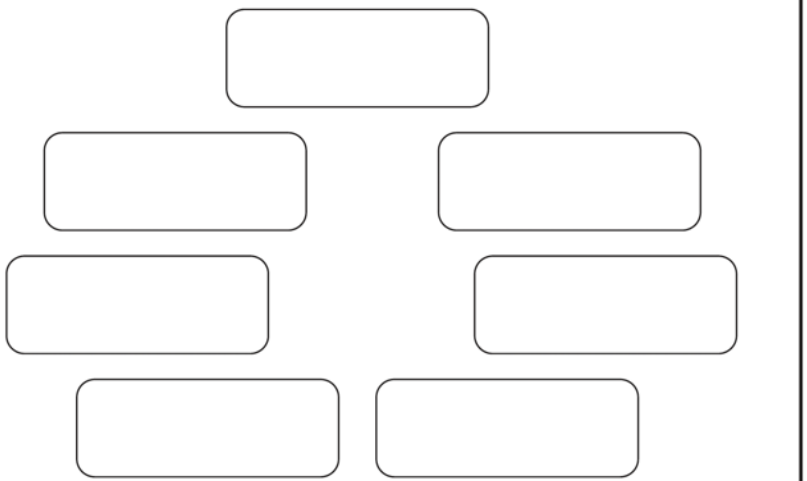
Key words tied to operations is an ineffective word-problem strategy.
(Karp et al., 2019; Powell et al., 2022)

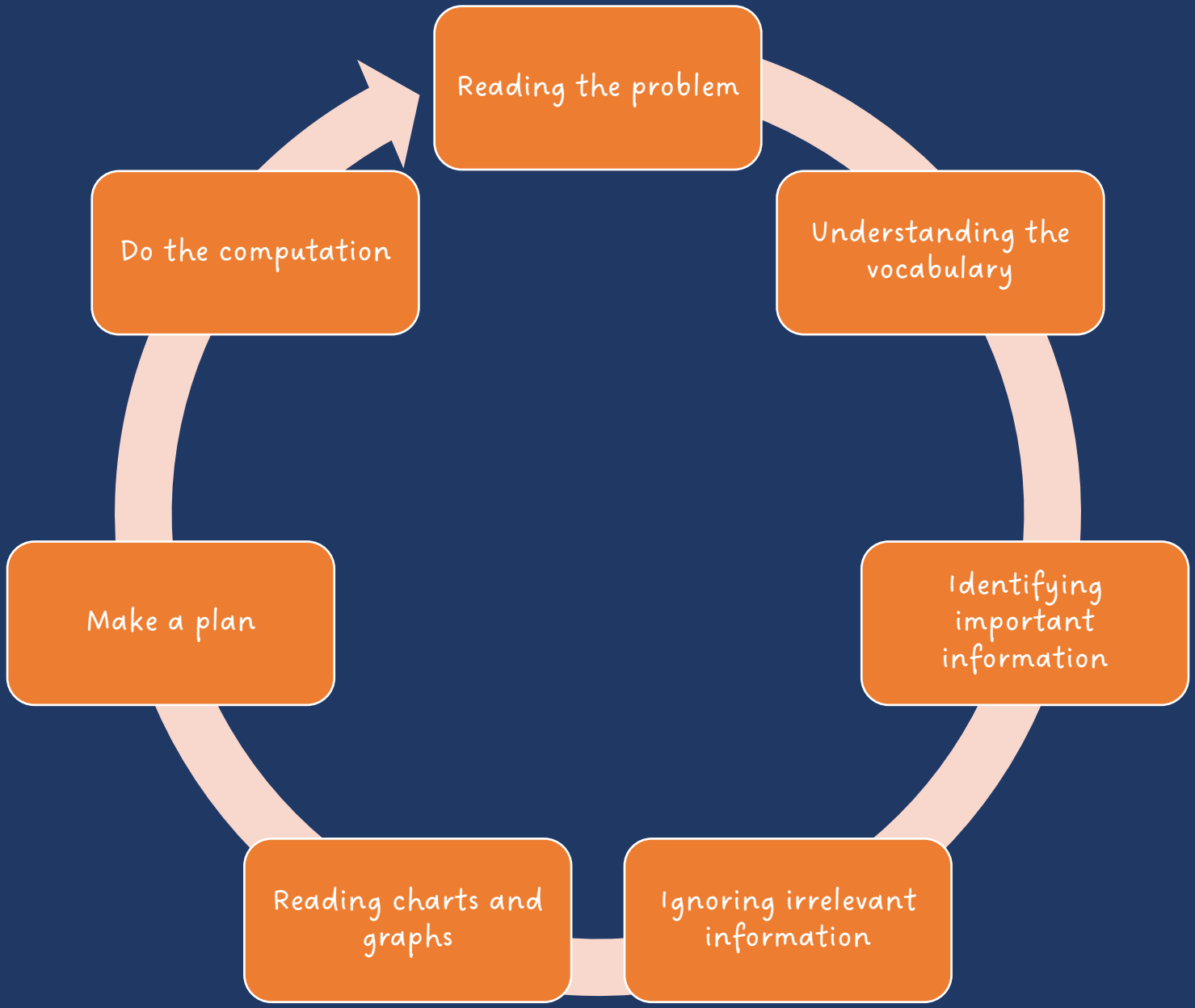
Using a meta-cognitive strategy improves word-problem performance.
(Freeman-Green et al., 2015; Krawec et al., 2012; Montague et al., 2011; Swanson et al., 2014)

A focus on schemas improves word-problem performance.
(Alghamdi et al., 2020; Cook et al., 2020; Flores et al., 2016; Fuchs et al., 2021; Griffin et al., 2019; Jitendra et al., 2013; Lein et al., 2020; Peltier et al., 2020; Powell et al., 2022; Xin & Xhang, 2009; Zheng et al., 2013)

WORD PROBLEMS

Research and Information



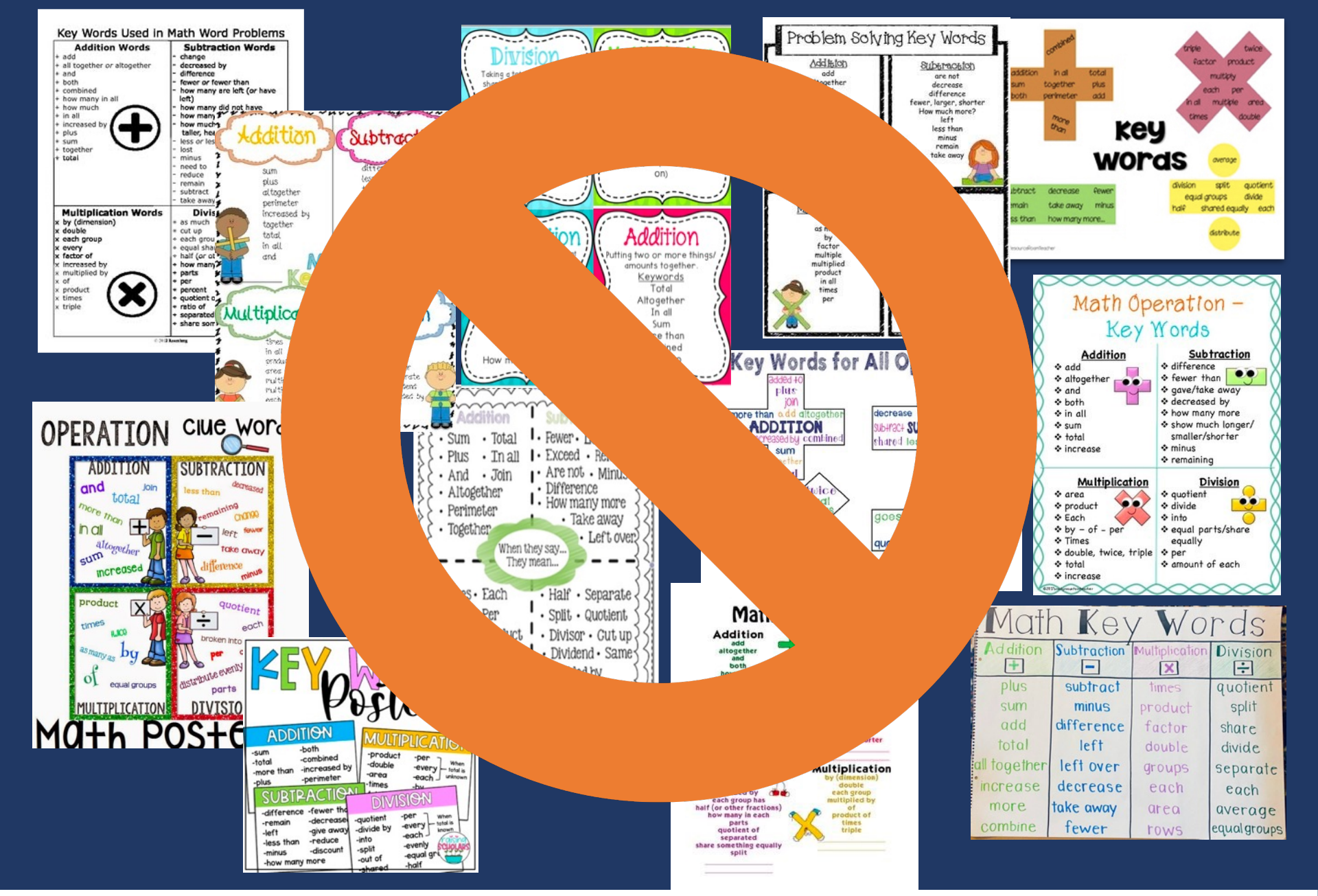


Ineffective Strategies

1. Keywords tied to operations

Carmelita had 8 pencils **fewer** than Jenny. If Jenny had 18 pencils, how many pencils did Carmelita have?

Carmelita had 8 pencils **fewer** than Jenny. If Carmelita had 18 pencils, how many pencils did Jenny have?



Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords[a]		Multiple keywords[a]		Keyword(s) led to correct solution[a]	
	n	%	*n*	%	*n*	%	*n*	%	*n*	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

[a]When a problem featured a keyword.

Description of Multi-Step Word Problems (n = 84)

Schema	Occurrence of schema[a] *n*	Occurrence of schema[a] %	Any keyword *n*	Any keyword %	Keyword(s) led to correct solution[b] *n*	Keyword(s) led to correct solution[b] %
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

[a]Sum across schemas does not equal 100 because each word problem featured more than one schema.

[b]When a problem featured a keyword.

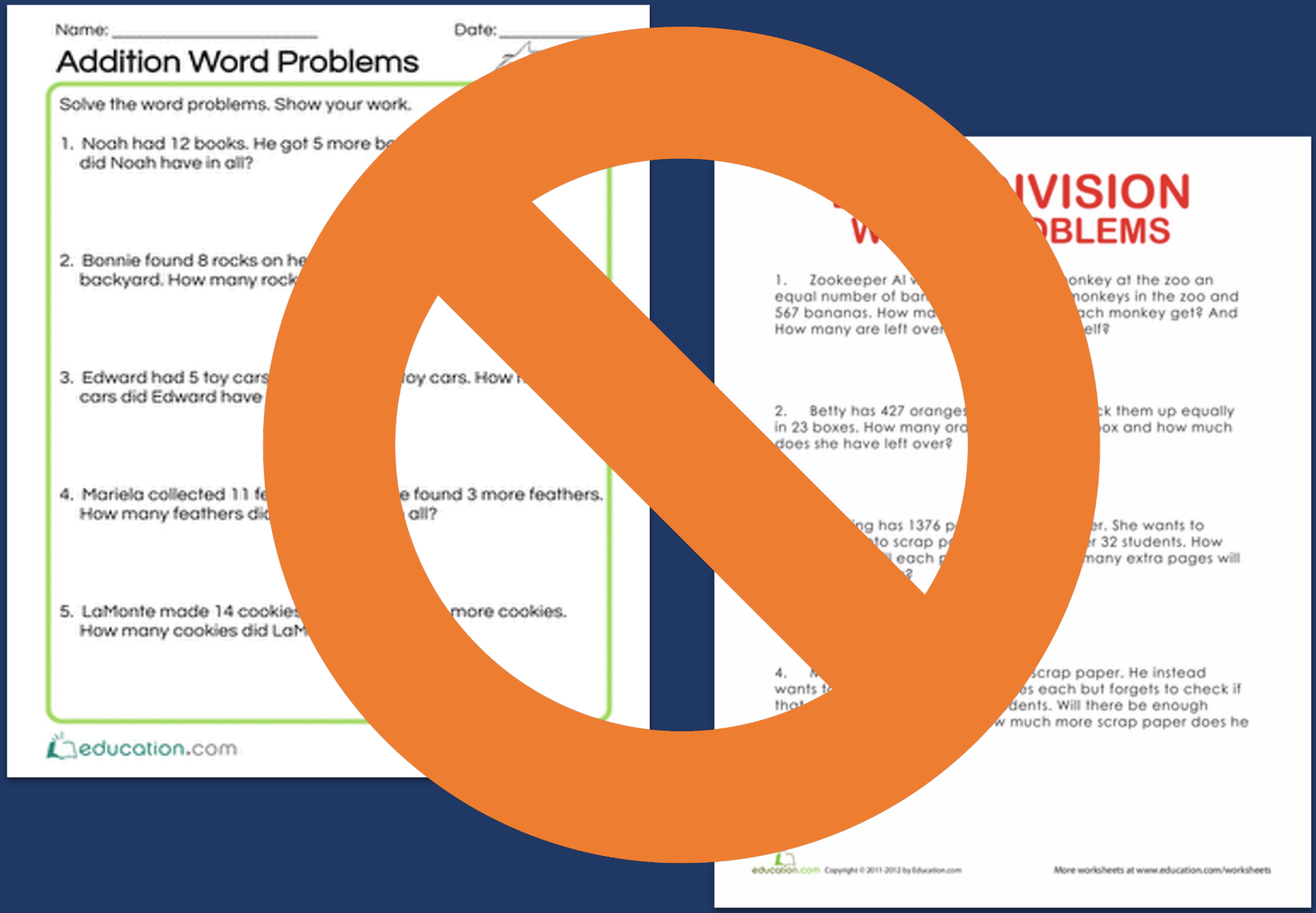
Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords ("What does *more than* tell you about?")

But, *do not* tie a keyword to a specific operation!

2. Presenting problems by operation



WORD PROBLEMS

Attack Strategies

SOLVE

- Study the problem
- Organize the information
- Line up a plan
- Verify the plan
- Examine the answer

R-CUBES

- Read the problem
- Circle key numbers
- Underline the question
- Box action words
- Evaluate steps
- Solve and check

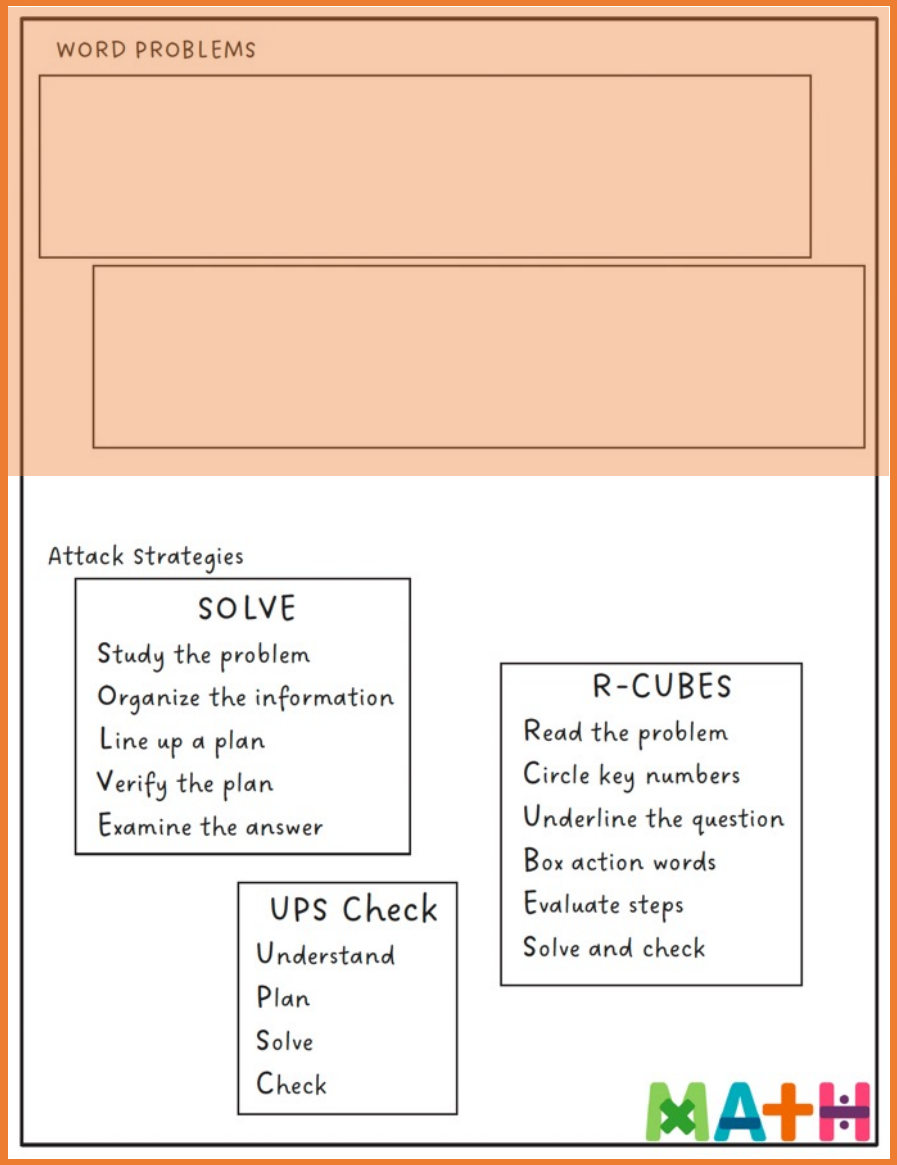
UPS Check

- Understand
- Plan
- Solve
- Check

Effective Strategies

Teach an attack strategy

Teach about schemas



RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RICE

Read and record the problem.
Illustrate your thinking.
Compute.
Explain your thinking.

RIDGES

Read the problem.
I know statement.
Draw a picture.
Goal statement.
Equation development.
Solve the equation.

SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
Picture it. Draw the scenario to show what is happening.
Explain the problem with a number sentence.
Rewrite the answer in a sentence.

SHINES

Slowly and carefully read the problem.
Highlight or underline key information.
Identify the question by drawing a circle around it.
Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity.
Share your answer by writing a sentence.

STAR

Stop and read the problem carefully.
Think about your plan and the strategy you will use.
Act. Follow your plan and solve
the problem.
Review your answer.

SOLVE

Study the problem.

Organize the information.

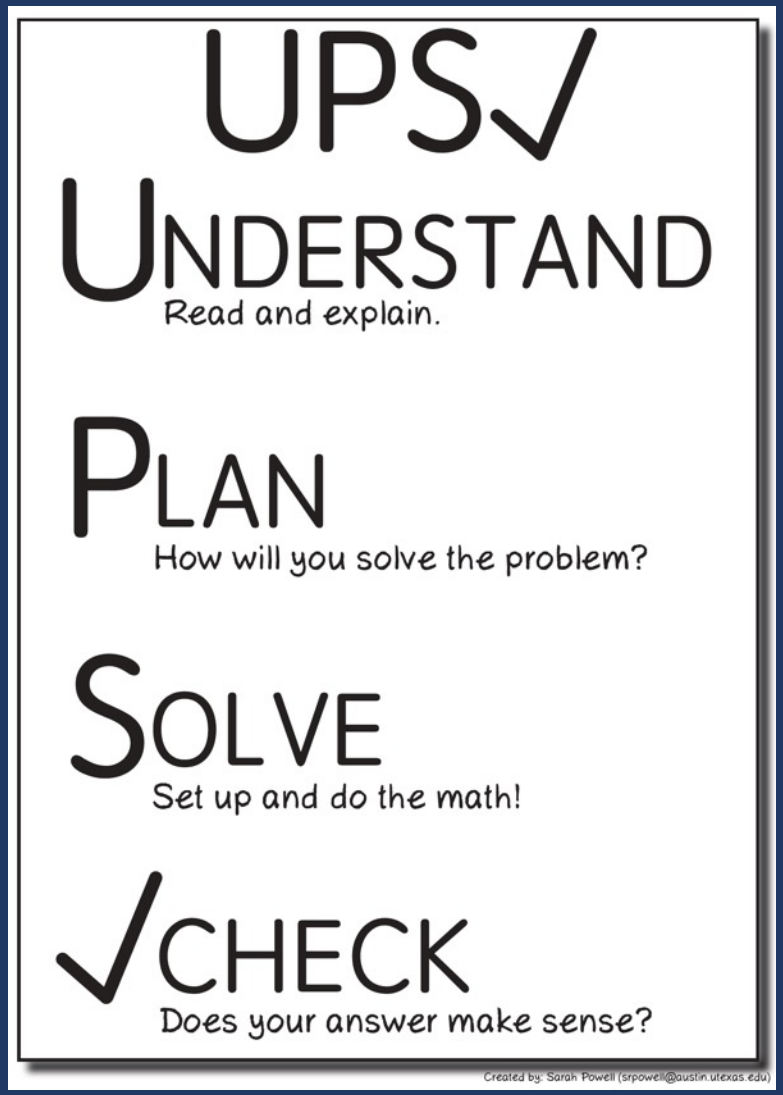
Line up the plan.

Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem.
Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.



Share your favorite attack strategy.

Teach an attack strategy

Teach about schemas

Total

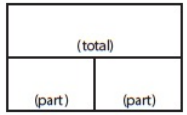
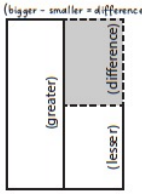
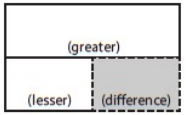
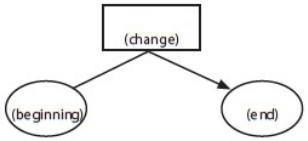
Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	P1 + P2 = T (part + part = total)  	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	B − s = D (bigger − smaller = difference)  G − L = D (greater − less = difference)  	Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	ST +/− C = E (start +/− change = end)   	End (increase) unknown: Jorge had $52. Then, he earned $16 babysitting. How much money does Jorge have now? End (decrease) unknown: Jorge had $52. Then, he spent $29 at the ballpark. How much money does Jorge have now?	Change (increase) unknown: Jorge had $52. Then, he earned some money babysitting. Now, Jorge has $68. How much did Jorge earn babysitting? Change (decrease) unknown: Jorge had $52 but spent some money when he went to the ballpark. Now, Jorge has $23. How much did Jorge spend at the ballpark?	Start (increase) unknown: Jorge has some money, and then he earned $16 for babysitting. Now, Jorge has $68. How much money did he have to start with? Start (decrease) unknown: Jorge had some money. Then, he spent $29 at the ballpark and has $23 left. How much money did Jorge have before going to the ballpark?	Multiple changes: Jorge had $78. He stopped and bought a pair of shoes for $42 and then he spent $12 at the grocery. How much money does Jorge have now?

Powell & Fuchs (2018).
Material collected from: Griffin & Jitendra, 2009; Fuchs et al., 2014; Fuchs, Seethaler, et al., 2008; Fuchs et al., 2010; Jitendra, 2002; Kintsch & Greeno, 1985; Van de Walle, Karp, & Bay-Williams, 2013.

Total

WORD PROBLEMS: TOTAL

A.
An artist poured 6.09 kilograms of orange sand and 14.26 kilograms of blue sand into a mixing container for a project. What was the total amount of sand the artist poured into the container in kilograms?

B.
A movie theater has 710 seats.
- 158 seats are red.
- 247 seats are black.
- 119 seats are yellow.
- The rest of the seats are green.

How many seats are green?

C.
The frequency table shows the number of visitors a park had on three different days.

Park Visitors

Day	Number of Visitors
Friday	𝍸 𝍸 |||
Saturday	𝍸 𝍸 𝍸 𝍸 |
Sunday	𝍸 𝍸 𝍸 ||

What was the total number of visitors the park had over those three days?

Your TOTAL problem:

Total

Part-part-whole
Combine

Parts put together into a **total**

Mandy saw 3 canoes and 8 kayaks. How many boats did Mandy see?

Total

Mandy saw 11 boats. If 3 of the boats were canoes, how many were kayaks?

Part

Mandy saw 11 boats. 8 of the boats were kayaks, how many were canoes?

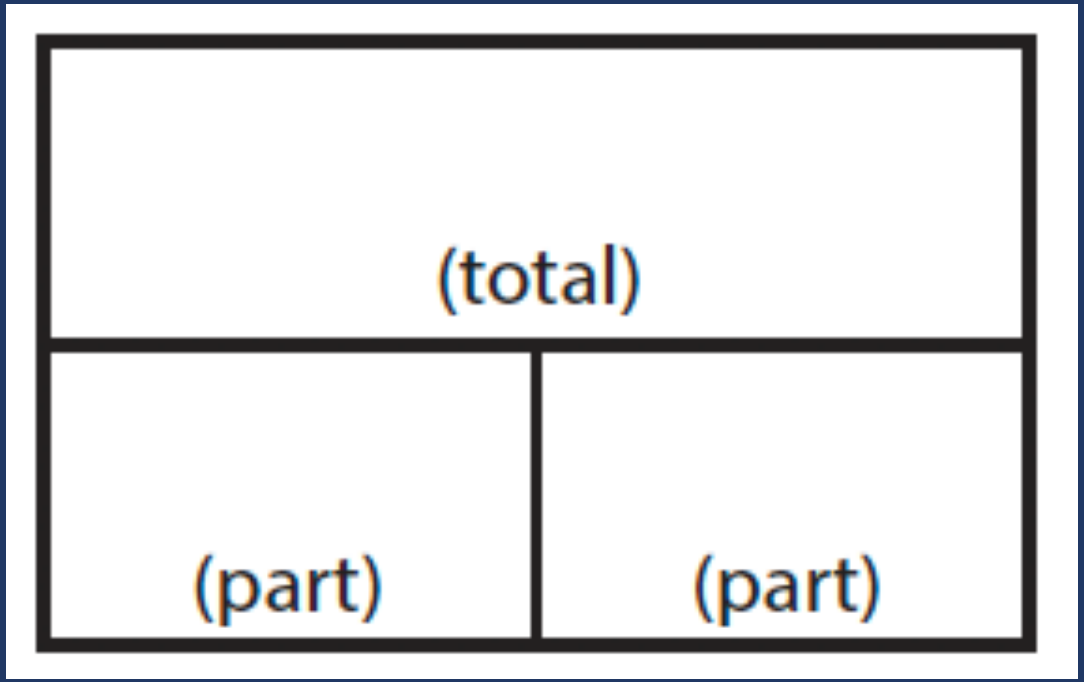
Part

Total

"Are parts put together for a total?"

Total

P1 + P2 = T



Fuchs et al. (2008); Griffin & Jitendra (2009)

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What was the total number of visitors the park had over those three days?

Your TOTAL problem:

Total

Write a Total problem.

Difference

WORD PROBLEMS: DIFFERENCE

D.
There are two lions at a zoo. The weight of the younger lion is 379 pounds. The weight of the older lion is 514 pounds. What is the difference in pounds between these two weights?

E.
The frequency table shows the number of movies watched last month by each student in Mr. Westley's class.

Movies Watched Last Month

Number of Movies	Number of Students
1	𝍸 ||
2	𝍸 𝍸 ||
3	||||
4	||

What is the difference between the number of students who watched 2 movies last month and the number of students who watched one movie last month?

F.
Samantha, Gordon, and Diego each brought an ice chest to a picnic.
- The weight of Samantha's ice chest was 83 pounds.
- The weight of Gordon's ice chest was 28 pounds.
- The weight of Diego's ice chest was 37 pounds.

What was the difference in pounds between the weight of Samantha's ice chest and the combined weight of Gordon's and Diego's ice chests?

Your DIFFERENCE problem:

Difference

Compare

Greater and **lesser** amounts compared for a **difference**

L'Tanya has 10 pencils. Vickie has 4 pencils. How many more pencils does L'Tanya have?

L'Tanya has 6 more pencils than Vickie. If Vickie has 4 pencils, how many does L'Tanya have?

Vickie has 6 fewer pencils than L'Tanya. L'Tanya has 10 pencils. How many pencils does Vickie have?

Difference

Greater amount

Lesser amount

Total

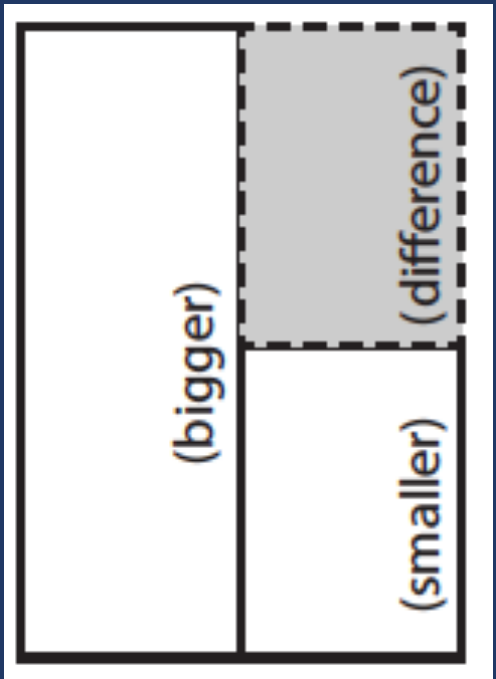
"Are parts put together for a total?"

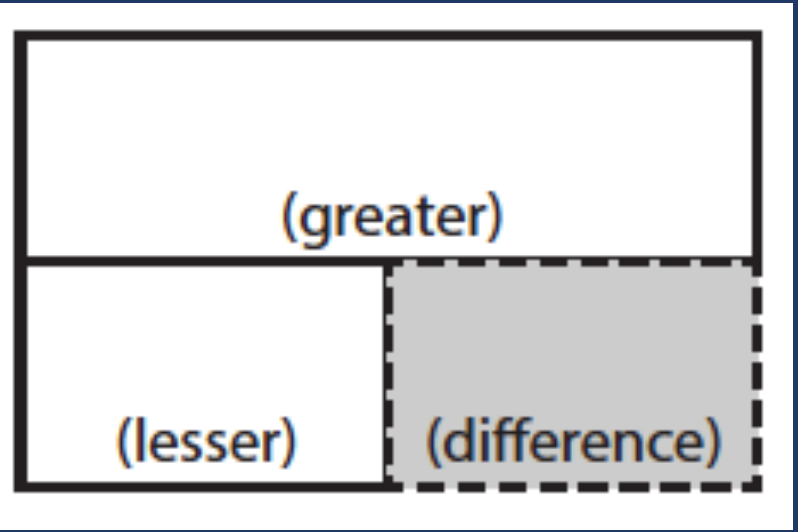
Difference

"Are amounts compared for a difference?"

Difference

G – L = D





Fuchs et al. (2008); Griffin & Jitendra (2009)

Difference

WORD PROBLEMS: DIFFERENCE

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What was the difference in pounds between the weight of Samantha's ice chest and the combined weight of Gordon's and Diego's ice chests?

Your DIFFERENCE problem:

Difference

Write a Difference problem.

Change

WORD PROBLEMS: CHANGE

G. Landon had one string that was 10 meters long. He used 6.275 meters of this string for a project. What was the length of string in meters that Landon had left?	H. A parade began at 11:30 a.m. and ended at 2:18 p.m. How long did the parade last?
I. Ms. Fitzgerald had gallons of fruit punch. She served gallon of the fruit punch to her family at lunch. How many gallons of fruit punch did Ms. Fitzgerald have left after lunch?	Your CHANGE problem:

Change

Join

An amount that **increases** or decreases

Annette had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Annette have now?

End amount

Annette had 6 notebooks. Then, she bought a few more notebooks. Now, Annette has 9 notebooks. How many notebooks did she buy?

Change amount

Annette had some notebooks. Then, she bought 3 notebooks. Now, Annette has 9 notebooks. How many notebooks did she have to start with?

Start amount

Change

Separate

An amount that increases or **decreases**

Jenny baked 9 cookies. Then, she ate 3 of the cookies. How many cookies does Jenny have now?

End amount

Jenny baked 9 cookies. Then, she ate some of the cookies. Now, she has 6 cookies. How many cookies did Jenny eat?

Change amount

Jenny baked some cookies. She ate 3 of the cookies and has 6 cookies left. How many cookies did Jenny bake?

Start amount

Total

"Are parts put together for a total?"

Difference

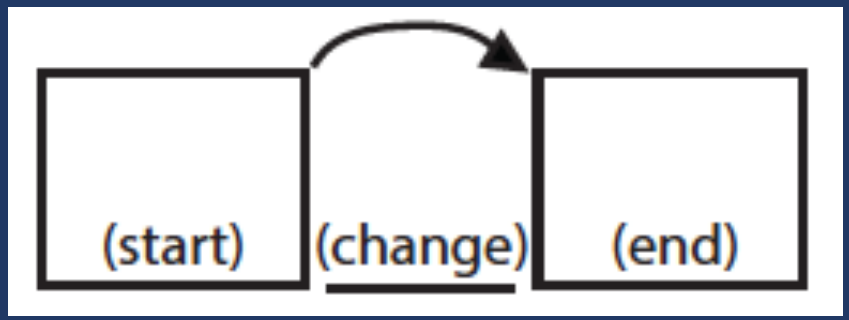
"Are amounts compared for a difference?"

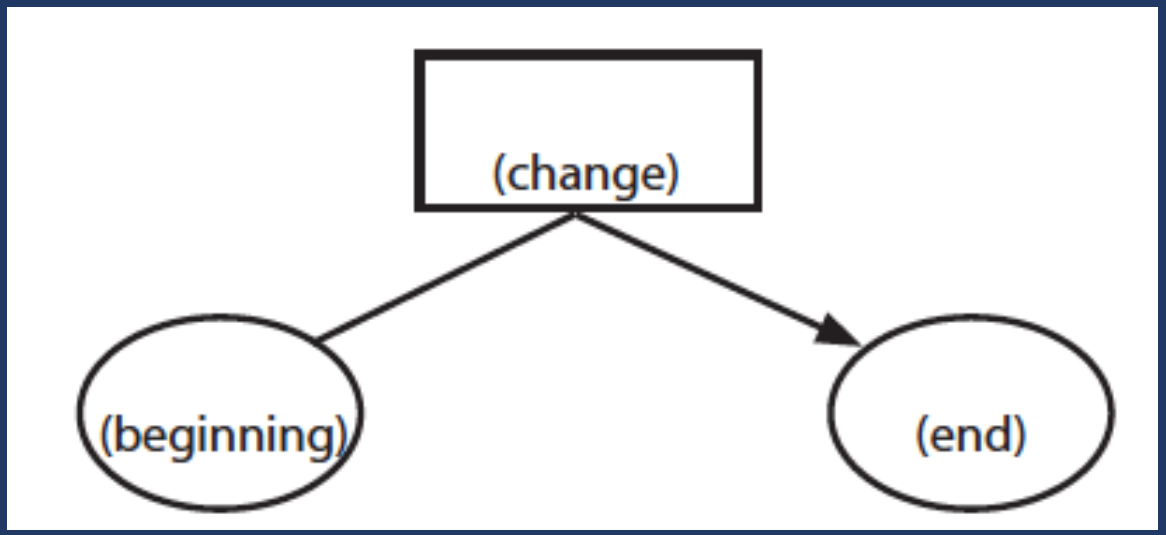
Change

"Does an amount increase or decrease?"

Change

ST +/− C = E





Fuchs et al. (2008); Griffin & Jitendra (2009)

Change

WORD PROBLEMS: CHANGE

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I. Ms. Fitzgerald had gallons of fruit punch. She served gallon of the fruit punch to her family at lunch. How many gallons of fruit punch did Ms. Fitzgerald have left after lunch?	Your CHANGE problem:

Change

Write a Change problem.

Schema Check!

Change

Pablo goes to a stamp show where he can share, buy, and sell stamps.

26. Part A

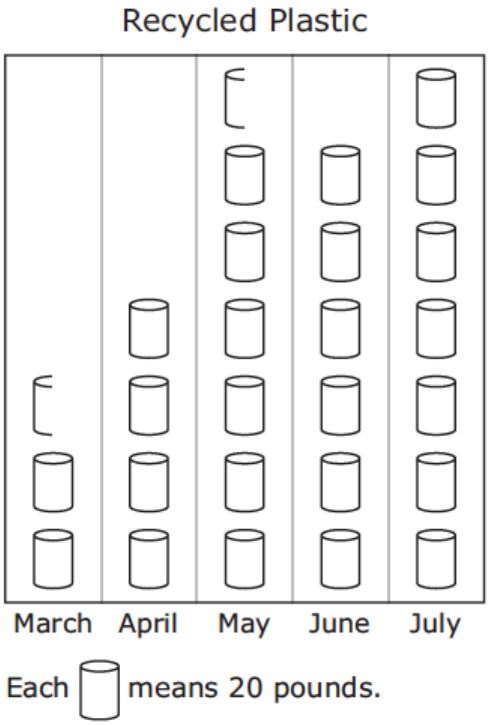
The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.

What is the total number of stamps that Pablo has after the first day of the stamp show?

Grade 3 PARCC

Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.



Based on the graph, how many more pounds of plastic did the family recycle in July than in April?

Grade 3 STAAR

Total

Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.

10. Part A

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?

Ⓐ 300

Ⓑ 340

Ⓒ 350

Ⓓ 360

Grade 3 PARCC

Total

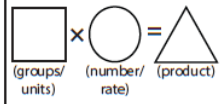
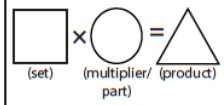
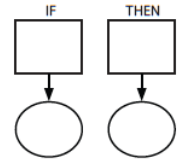
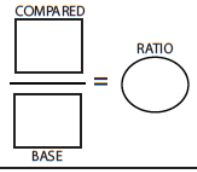
Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units	GR x N = P  	Product unknown: Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	Groups unknown: Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	Number unknown: Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	With rate: Maria bought 5 cartons of eggs. Each carton cost $2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set	S x T = P  	Product unknown: Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	Set unknown: Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	Times unknown: Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	With fraction: Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Ratios/ Proportions (Percentages; Unit Rate) Relationships among quantities	 	Subject unknown: Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	Object unknown: Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?		With percentage: Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?
Ratio	 	Base unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?	Compared unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?	Ratio unknown: Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	With unit rate: Paula bought 5 boxes of markers. She spent $9.75. What is the price of one box of markers?

Material collected from: Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra & Star, 2011; Jitendra et al., 2009; Van de Walle et al., 2013; Xin, Jitendra, & Deatline-Buchman, 2005; Xin & Zhang, 2009.

Equal Groups

WORD PROBLEMS: EQUAL GROUPS

J. There are 4 erasers on each table in a classroom. There are 5 tables in the classroom. What is the total number of erasers on all of the tables in this classroom?	K. Carmine has 291 balloons. She put the same number of balloons into 3 groups. What is the best estimate of the number of balloons in each group? A. 90 B. 100 C. 75 D. 85
L. Kelsi spends $6.75 every Saturday for breakfast. What is the total amount of money Kelsi spends on breakfast for 14 Saturdays?	Your EQUAL GROUPS problem:

Equal Groups

Array
Vary

Groups multiplied by **number in each group** for a **product**

Evan has 2 boxes. There are 6 muffins in each box. How many muffins does Evan have?

Product

Evan has 12 muffins. They want to place them equally into 2 boxes. How many muffins will Evan place in each box?

Number in each group

Evan has 12 muffins. They put them into boxes with 6 muffins each. How many boxes did Evan use?

Groups

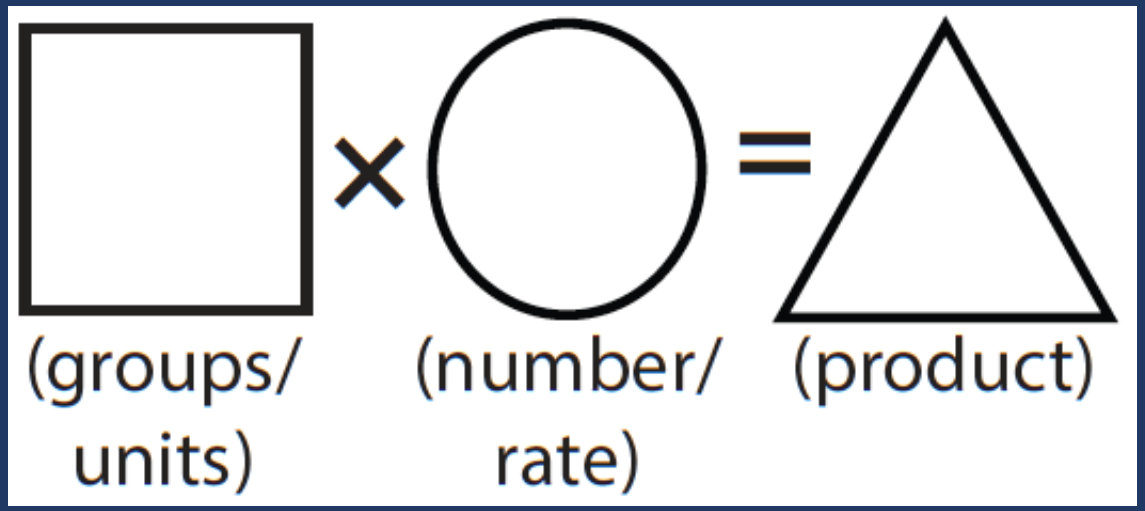
Equal Groups

"Are there groups with an equal number in each group?"

Equal Groups

GR × N = P

GR × E = P



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Equal Groups

Write an Equal Groups problem.

Comparison

WORD PROBLEMS: COMPARISON

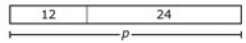
M.
Jonathan and Elizabeth are comparing the masses of their rocks.
- Jonathan's rock has a mass of 0.2 kilogram.
- Elizabeth's rock has a mass 8 times the mass of Jonathan's rock.

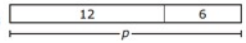
What is the mass of Elizabeth's rock in kilograms??

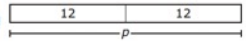
N.
Haruko did 9 sit-ups in P.E. class. The number of sit-ups Tom did can be represented by this expression. 2×9

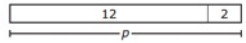
Which statement is true?
F. Tom did 2 times as many sit-ups as Haruko.
G. Haruko did 2 times as many sit-ups as Tom.
H. Tom did 2 more sit-ups than Haruko.
J. Haruko did 2 more sit-ups than Tom.

O.
Erin has 12 pictures from a field trip and some pictures from a vacation. She has twice as many pictures from the vacation as from the field trip. Which strip diagram represents p, the total number of pictures Erin has?

F 

G 

H 

J 

Your COMPARISON problem:

Comparison

Set multiplied by a number of times for a product

Joan ran 6 minutes. L'Tanya ran 4 times longer than Joan. How many minutes did L'Tanya run?

Set

Number of times

Product

Equal Groups

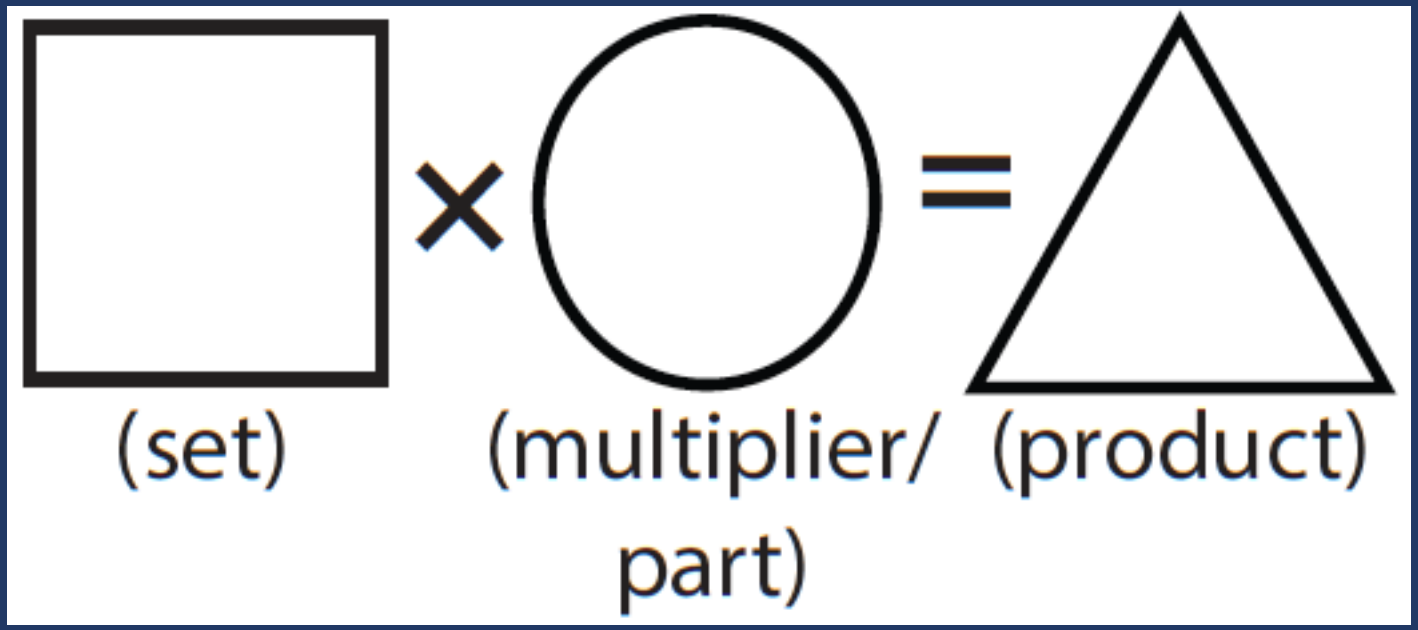
"Are there groups with an equal number in each group?"

Comparison

"Is a set compared a number of times?"

Comparison

S × T = P



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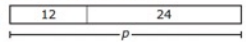
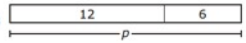
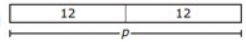
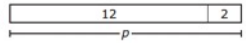
F 

G 

H 

J 

Your COMPARISON problem:

Comparison

Write a Comparison problem.

Ratio/Proportion

WORD PROBLEMS: RATIO or PROPORTION

P. Dennis made an extra $245.00 for selling furniture. The extra $245.00 was 7% of the total value of the furniture he sold. What was the total value of the furniture Dennis sold?	Q. A baseball traveled 330 feet in 5 seconds. Which rate is equivalent to the rate at which the baseball traveled? A. 55 feet per second B. 66 feet per second C. 55 seconds per foot D. 66 seconds per foot
R. During a 90-minute school play, the main character was on stage 80% of the time. What amount of time in minutes was the main character on stage?	Your RATIO or PROPORTION problem

Ratio/Proportion

Description of relationships among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was 3:5. If she baked 25 brownies, how many cookies did she bake?

Emma typed 56 words in 2 minutes. At this rate, how many words could Emma type in 7 minutes?

Equal Groups

"Are there groups with an equal number in each group?"

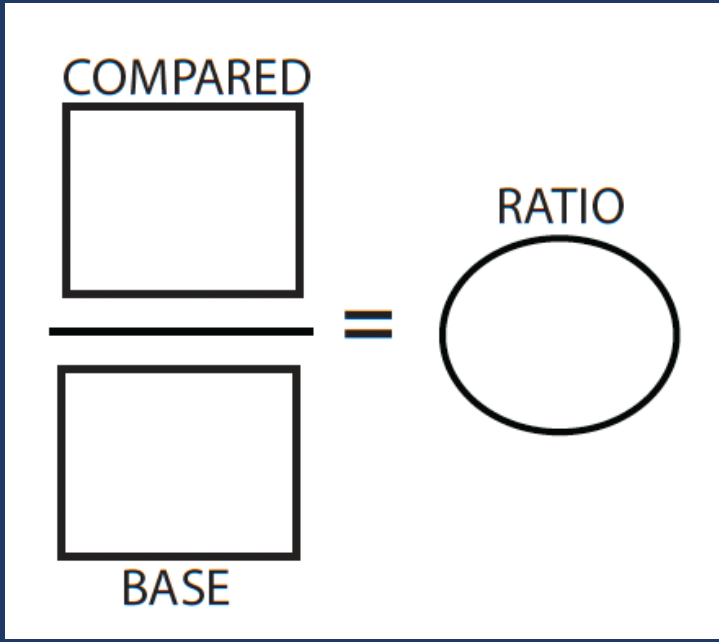
Comparison

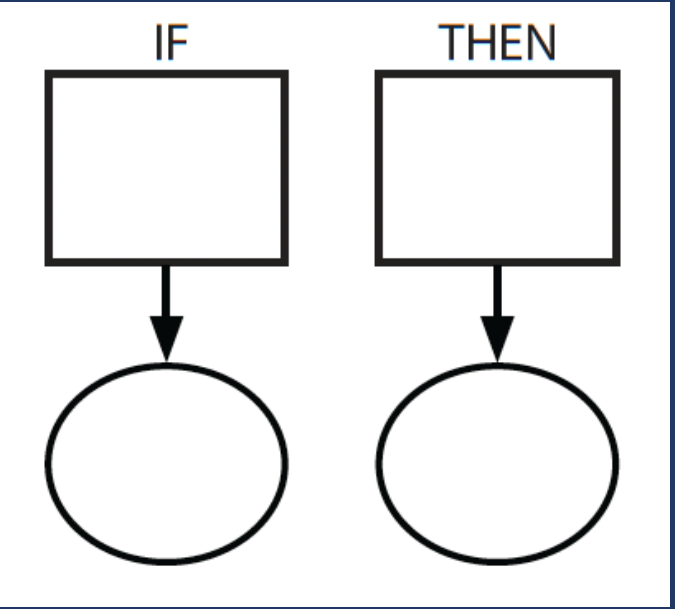
"Is a set compared a number of times?"

Ratio/Proportion

"Are there relationships among quantities - if this, then this?"

Ratio/Proportion





Xin et al. (2005)

Ratio/Proportion

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R. During a 90-minute school play, the main character was on stage 80% of the time. What amount of time in minutes was the main character on stage?	Your RATIO or PROPORTION problem

Ratio/Proportion

Write a Ratios or Proportions problem.

Schema Check!

Equal Groups

Mr. Kowolski ordered 35 boxes of granola bars. Each box contained 24 granola bars.

What is the total number of granola bars Mr. Kowolski ordered?

Grade 4 PARCC

Ratio/Proportion

A company makes 625 cell phone cases each day. How many cell phone cases does the company make in 31 days?

Grade 5 STAAR

Comparison

Danielle's full-grown dog weighs 10 times as much as her puppy. The puppy weighs 9 pounds.

Enter the number of pounds the full-grown dog weighs.

Grade 4 Smarter Balanced

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Total

Difference

Change

Equal Groups

Comparison

Ratio/
Proportion

XXX

Total

Difference

Change

Equal Groups

Comparison

Ratio/
Proportion

XXX

Total

Difference

Change

Equal Groups

Comparison

Ratio/
Proportion

XXX

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Ratio/
Proportion

XXX

Total

Difference

Change

Equal Groups

Comparison

Ratio/
Proportion

XXX

Total

Difference

Change

Equal Groups

Comparison

Ratio/
Proportion

XXX

WORD PROBLEMS: MULTI-STEP

S.
Shay bought 4 packs of markers for $6 each and a box of colored chalk for $11. What was the total cost of the markers and chalk Shay bought?

T.
Rajesh bought 2 salads for $3.65 each and a sandwich for $4.35. He gave the clerk $15.00 to pay for the items. How much change should Rajesh have received in dollars and cents?

U.
Members of the chess club held a bake sale to raise money. Cupcakes and cookies were sold.
- Cupcakes were sold for $1 each.
- Cookies were sold for $0.50 each.
- The members sold a total of 288 items.

23
- Of the items sold, were cupcakes and the remaining items were cookies.

How much money did the chess club members raise from the cookies that were sold?

V.
Alex bought 4 packages of pink golf balls and 2 packages of orange golf balls. There were 12 golf balls in each package. How many golf balls did Alex buy?

WORD PROBLEMS

What are your strengths?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

What are your strengths with problem solving?

What are your opportunities for growth?

What are your plans for next Monday?

Next month?

Next year?

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and Practice

INSTRUCTIONAL STRATEGIES

Fluency

Word Problems

Instructional Platform

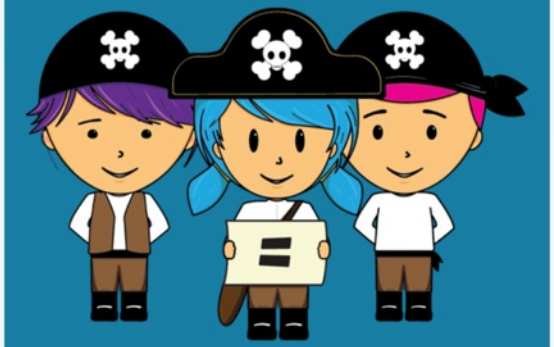
Pirate Math Equation Quest

About | Research | Individual | Small Group | STAAR | Videos

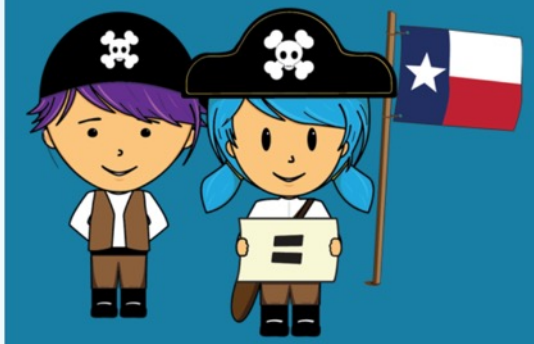
Welcome to Pirate Math Equation Quest!

Individual Word-Problem Intervention

Small-Group Word-Problem Intervention



Small-Group Word-Problem Intervention for STAAR



https://intensiveintervention.org

National Center on
INTENSIVE INTERVENTION
at American Institutes for Research

Search

Intensive Intervention | Tools Charts | Implementation Support | Intervention Materials | Information For...

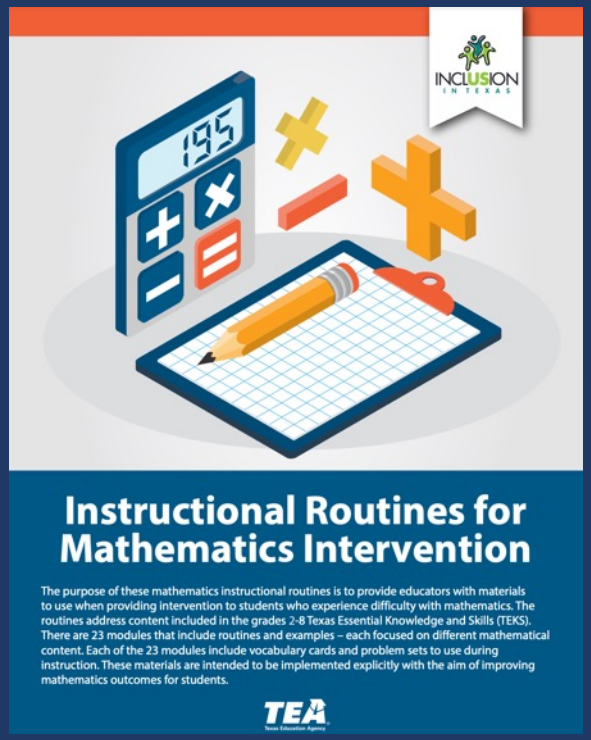
Intensive Intervention in Mathematics Course Content

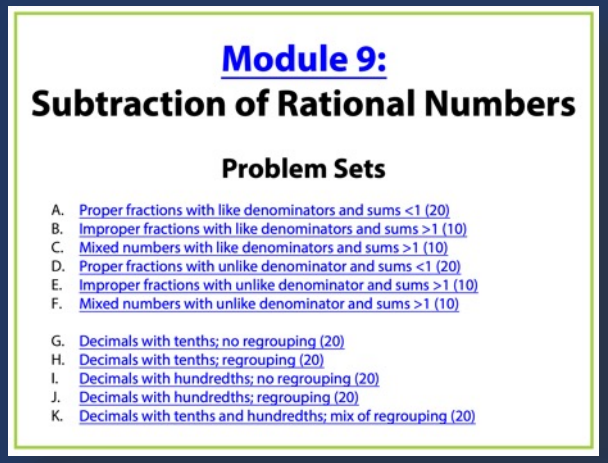
NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

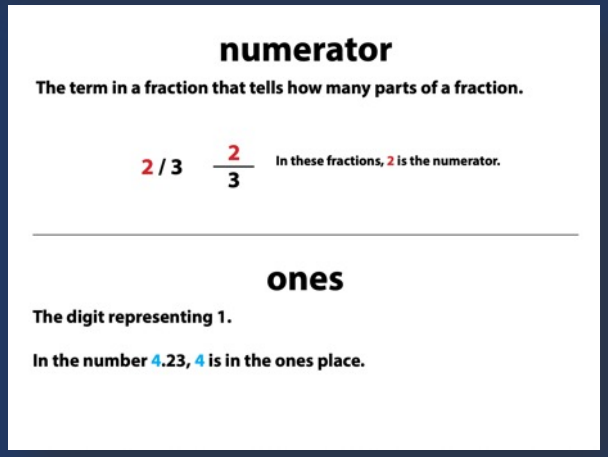
Intensive instruction was recently identified as a high-leverage practice in special education, and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the National Center on Leadership in Intensive Intervention and with support from the CEEDAR Center, developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the Features of Explicit Instruction Course and so we suggest that users complete both courses.

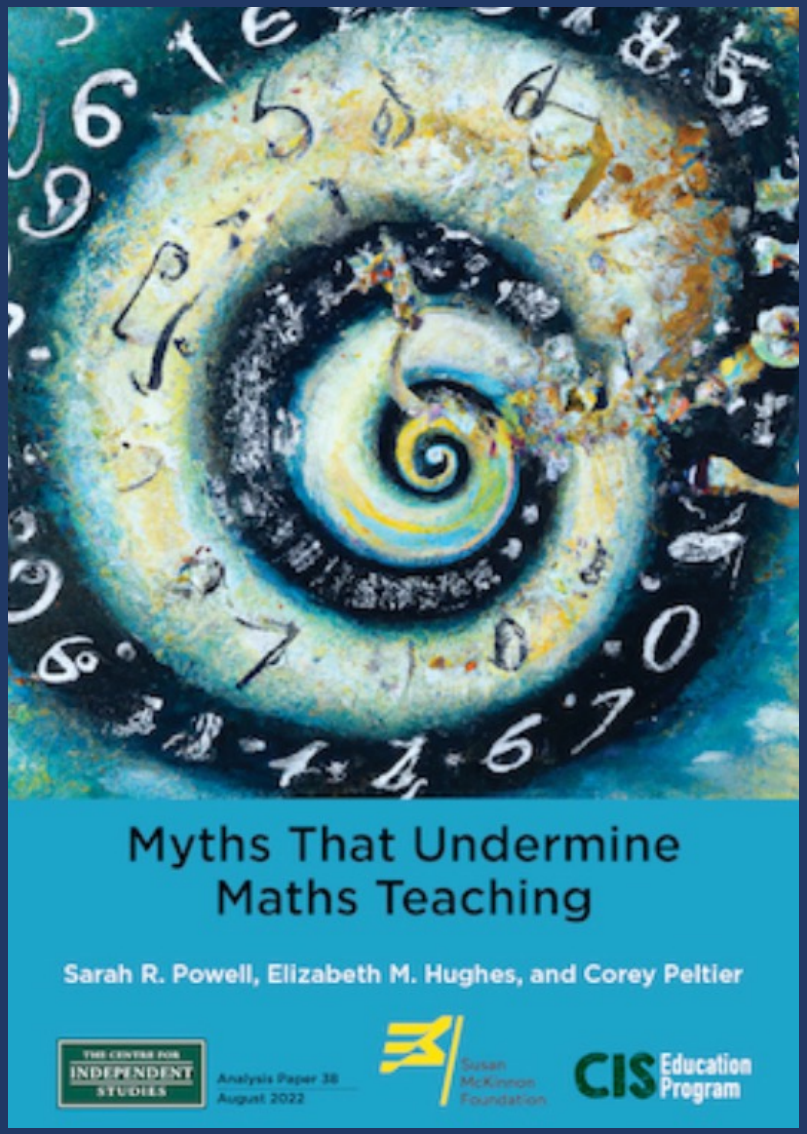
MODULE 5: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL STRATEGIES



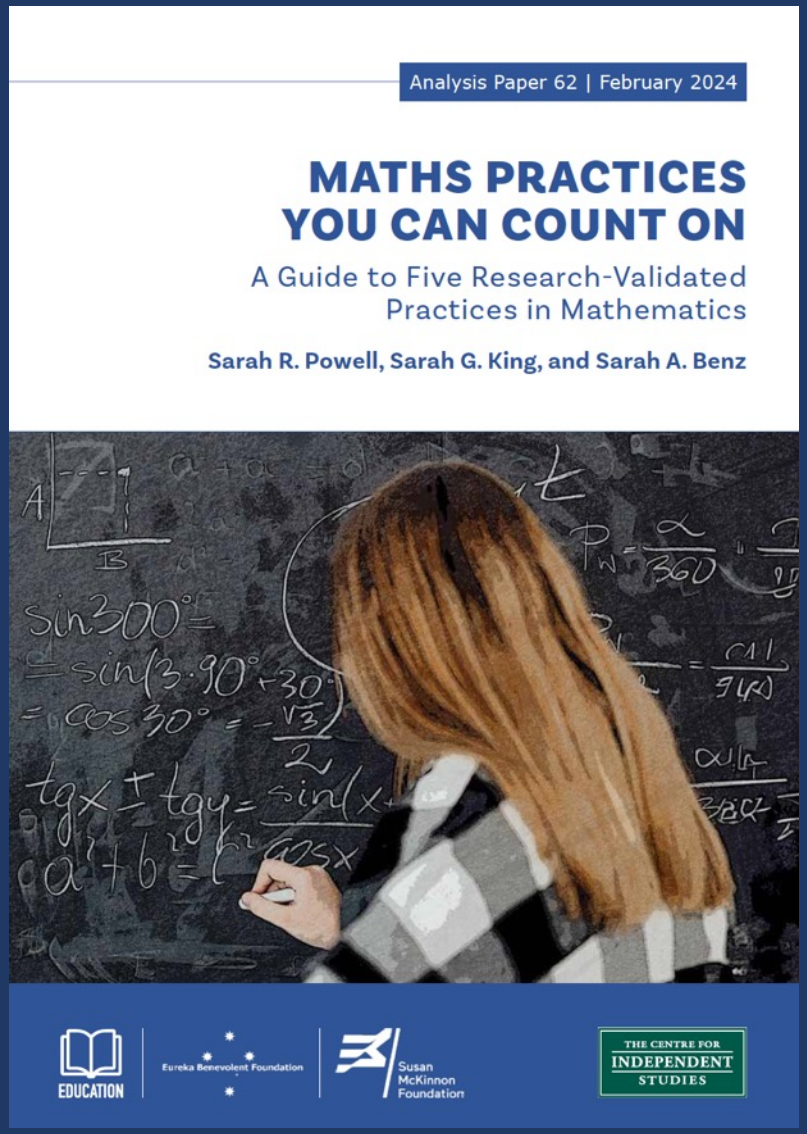




https://spedsupportstage.tea.texas.gov/resource-library/instructional-routines-mathematics-intervention

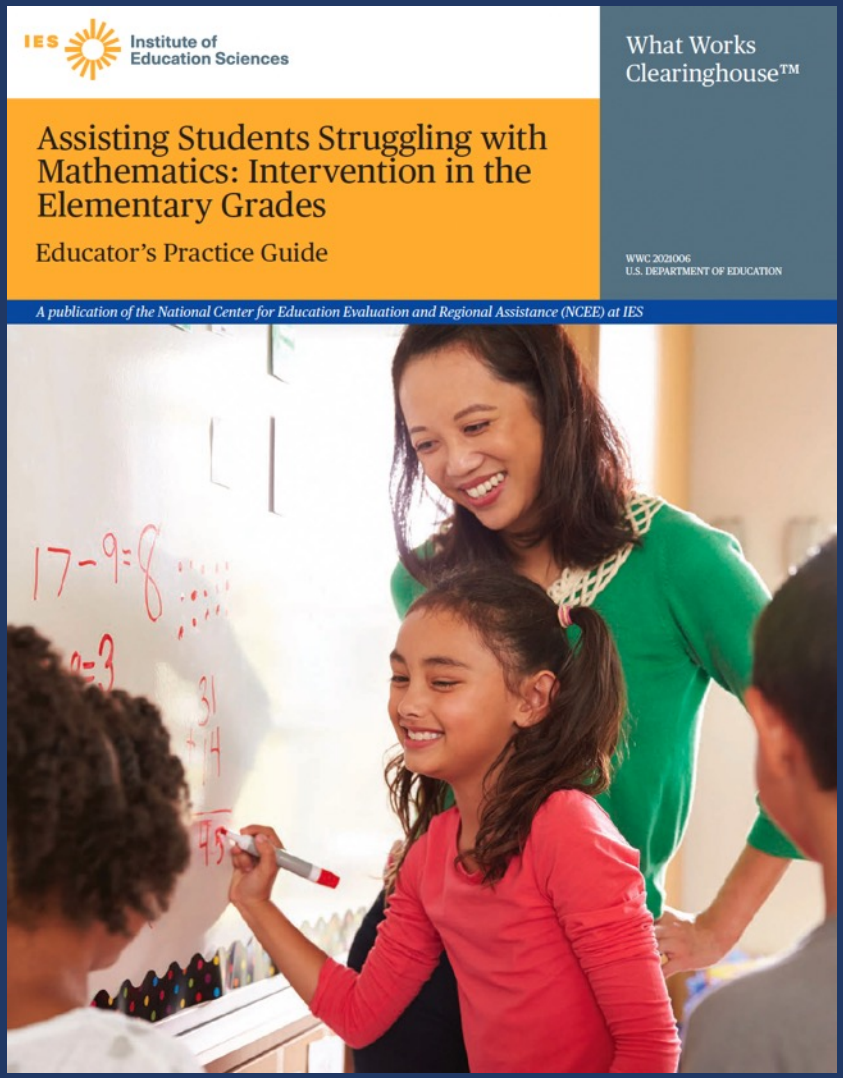


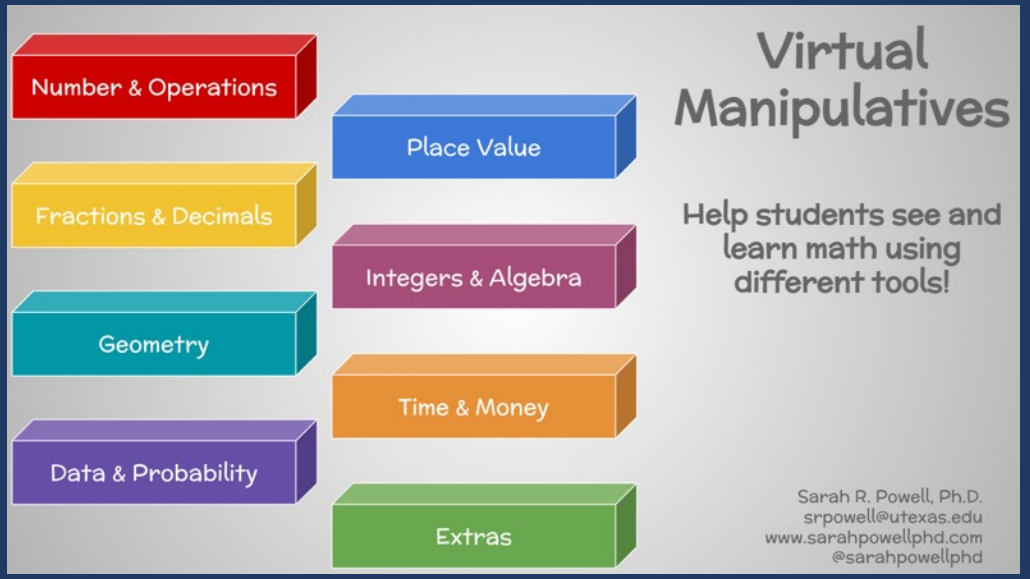
Centre for Independent Studies



Centre for Independent Studies

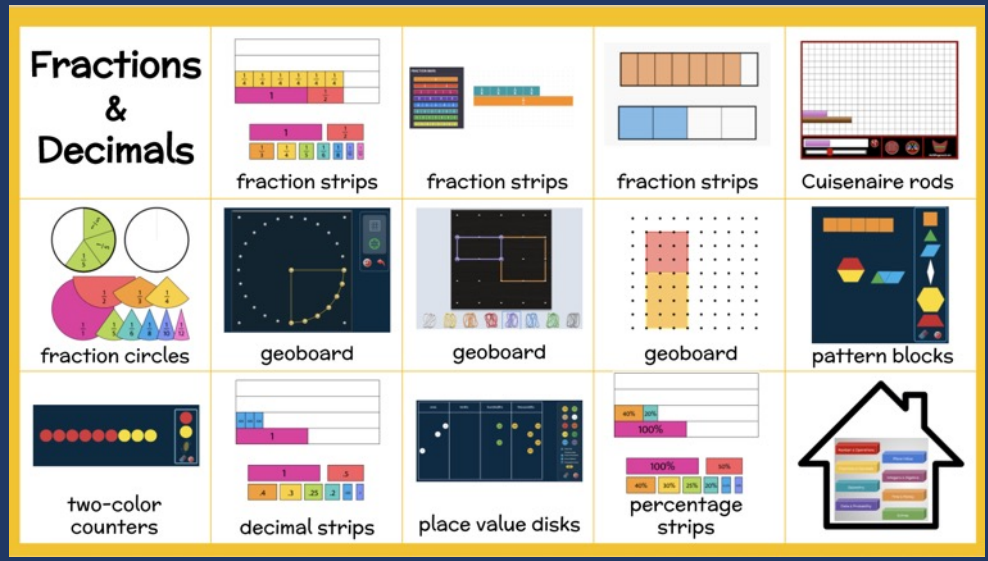
https://ies.ed.gov/ncee/wwc/PracticeGuide/26





FLOWCODE
PRIVACY.FLOWCODE.COM

bit.ly/srpowell



Resources

Instructional Platform

Explicit Instruction

Language

Multiple Representations

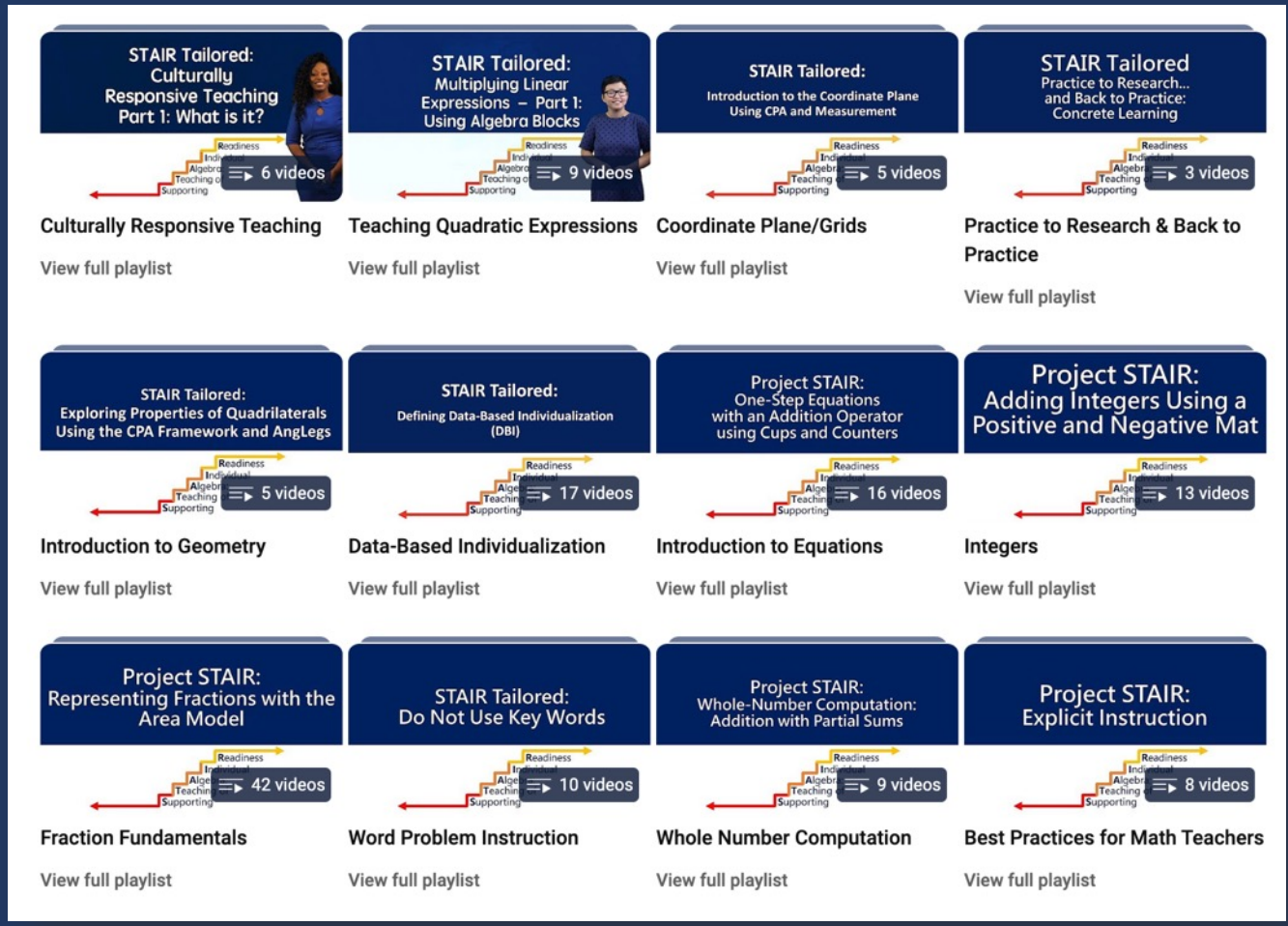
Fluency

Problem Solving

Additional Resources

Coaching

https://www.mathspiral.com



https://www.youtube.com/channel/UCE2puwDtUSNXFON1OhmYmvA

Sarah R. Powell, Ph.D.

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The University of Texas at Austin

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