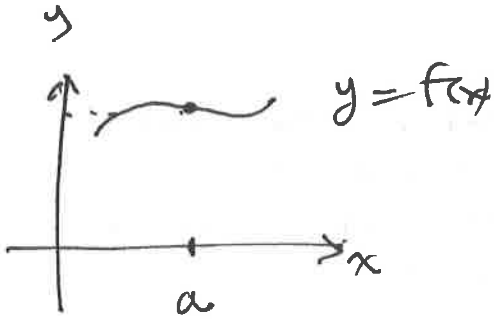
Math 1497 - Calc II

Calc I

limits



$$\lim_{x\to a} f(x) = L$$

Ex1 $\lim_{x\to 1} 2x+3 = 2(1)+3 = 5$

Ex2 $\lim_{x\to 1} \frac{x^2-1}{x-1} =$ "$\frac{0}{0}$" ⇐ means nothing!

Ways to investigate

(1) graphically

(2) numerically

.9	.99	.999	1.001	1.01	1.1
1.9	1.99	1.999	2.001	2.01	2.1

(3) analytically

$$\lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = \lim_{x\to 1} x+1 = 2$$ Factoring

(ii) rationalization

$$\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}$$

$$\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \lim_{x\to 0} \frac{\cancel{x}+\cancel{1}-\cancel{1}}{\cancel{x}(\sqrt{x+1}+1)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x+1}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

(iii) <u>squeeze Th^{m}</u>

if $\cos x \le \frac{\sin x}{x} \le \frac{1}{\cos x}$

then $\lim_{x\to 0} \cos x \le \lim_{x\to 0} \frac{\sin x}{x} \le \lim_{x\to 0} \frac{1}{\cos x}$

$$1 \le \lim_{x\to 0} \frac{\sin x}{x} \le 1$$

$$\Rightarrow \lim_{x\to 0} \frac{\sin x}{x} = 1$$

Also $\lim\limits_{x \to 0} \dfrac{1-\cos x}{x} = 0$

$$\lim_{x \to 0} \frac{1-\cos x}{x} \cdot \frac{1+\cos x}{1+\cos x} = \lim_{x \to 0} \frac{1-\cos^2 x}{x(1+\cos x)}$$

$$= \lim_{x \to 0} \frac{\sin^2 x}{x(1+\cos x)} = \lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \frac{\sin x}{1+\cos x}$$

$$= 1 \cdot \frac{0}{2} = 0$$

Another Limit

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

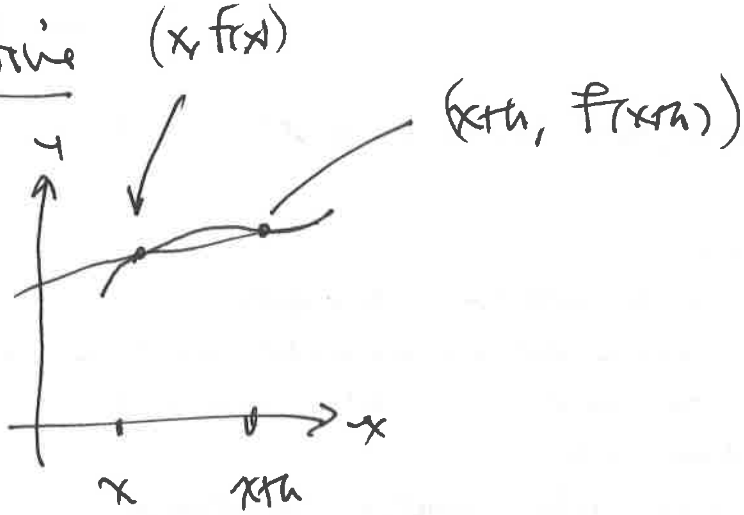
Other special limits

$$\lim_{x \to 0} \left(1+\frac{1}{x}\right)^x = 1 \qquad \lim_{x \to \infty} \left(1+\frac{1}{x}\right)^x = e$$

↑ limits at infinity

$\lim\limits_{x \to 0} \dfrac{1}{x^2} = \infty$ ← infinite limits

Derivative



$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h} \quad \text{secant}$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad \text{tangent}$$

Called derivative

$= f'(x)$, $\frac{dy}{dx}$, y', $\frac{df}{dx}$ ← all notation for a derivative

then we derived the rules & standard deriv

(1) power $f(x) = x^n$, $f'(x) = nx^{n-1}$

(2) sum diff $(f \pm g)' = f' \pm g'$

(3) const mult

$$(cf)' = cf'$$

(4) product $(fg)' = f'g + fg'$

(5) quot $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

chain $(f(g))' = f'(g) \cdot g'$

or $y = f(g(x)) =$

let $u = g(x)$ $y = f(u)$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Ex $y = \sqrt{x^2+1}$ $u = x^2+1,\ y = u^{1/2}$

$u' = 2x$ $y' = \frac{1}{2} u^{-1/2}$

$$\frac{dy}{dx} = \frac{1}{2} u^{-1/2} \cdot 2x = (x^2+1)^{-1/2} \cdot x = \frac{x}{\sqrt{x^2+1}}$$

b) Implicit

$x^2+y^2=1 \qquad 2x+2yy'=0$

$\Rightarrow \quad y' = -\dfrac{x}{y}$ ← involves x & y

Applications

Graphing

optimization problems

Mean Value Thm

L'Hôpital's Rule ← we will do tomorrow

Newton's Method

Related Rates

Anti derivative

F anti derivative of f

if $\qquad F'(x) = f(x)$

Ex $f(x) = 3x^2$ then $F(x) = x^3 + c$

see $\frac{d}{dx}(x^3 + c) = 3x^2 + 0$

Indefinite Integrals $\int f(x)dx = F(x) + c$

ex

$$\int 3x^2 dx = x^3 + c$$

$$\int x^n dx = \begin{cases} \frac{x^{n+1}}{n+1} + c & \text{if } n \neq -1 \\ \ln|x| + c & \text{if } n = -1 \end{cases}$$

and we have all the special integrals

Also

$$\int (f \pm g)dx = \int f dx \pm \int g(x)dx + c$$

$$\int c f(x)dx = c \int f(x)dx + c$$

$$\int \left(2x + \frac{4}{1+x^2} - 3e^x\right)dx = x^2 + 4\tan^{-1}x - 3e^x + c$$