Foundations For College Mathematics 3e

Prospectus & Philosophy

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Introduction:

Foundations for College Mathematics 3*e* is a remedial algebra textbook using a function approach with the introduction to new content based in real-world contexts. Guided discovery learning is implemented with an expectation of student understanding (as opposed to student memorization), and the use of the graphing calculator is required. The over-all structure of *Foundations 3e* is in concert with the way the brain learns, understands, remembers, and recalls. At the same time, the table of contents of the traditional textbooks has been relatively preserved. It's just that the traditional topics are placed after a discovery study of function and function behaviors (Chapters Two & Three) – so that the concept of function and function behaviors can be used to teach these more traditional topics – more on this later.

Foundations was originally written for a developmental intermediate algebra course at the college level, and was published in preliminary versions in 1992 and 1994 by national-based publishers. Currently, it is self-published by Red Bank Publishing, first in 2000 and then in 2003 and 2008. It has now been revised again as a third edition with a 2013 copyright and targets the high school senior remedial Algebra II course where it is has been adopted in 17 states.

About the Author:

Ed Laughbaum is an emeritus professor of mathematics from Columbus State Community College, and recently retired as the director of the Ohio Early Mathematics Placement Testing Program and the College Short Course Program in the mathematics department at The Ohio State University. He is presently interested in the implications of basic brain processes on understanding and memory/recall as related to the teaching of remedial algebra with handheld technology. Ed has authored over 60 publications in professional journals and by commercial publishers. He has given over 270 presentations at state, national, and international conferences in over 12 countries. Ed has taught mathematics for over 40 years, and has won numerous teaching awards including the Presidential Award and the Mathematics Excellence Award from the American Mathematical Association of Two-Year Colleges.

Ed's professional work at the state level included numerous committees and task forces for the Ohio Board of Regents. He represented Ohio on the Achieve (achieve.org) National Algebra II End-of-Course Exam and the Ohio Model Curriculum Program for the Ohio Department of Education.

A Focus on Remedial/Developmental Algebra:

Ed was on the remedial/developmental math writing team for the 1995 AMATYC *Standards for introductory mathematics before calculus*. He was the founder and presenter of the AMATYC Outer Banks Summer Institute "Developmental Algebra Using a Function Approach" that ran each summer from 1999 to 2008. Ed has taught the 2 or 3-day version of the same course at over 20 colleges in the US, and at over 45 high schools. All remedial/developmental algebra materials require a graphing calculator to facilitate the function concepts and neural response teaching methods.

Recent Remedial/Developmental Algebra Publications:

2012 "Learning - Changing the Connections in the Brain," *The New Jersey Mathematics Teacher Journal*. In press
2012 "Neural Associations," *PCTM Magazine*. In press
2011 "Capitalizing on Basic Brain Processes in Developmental Algebra – Part Three," *MathAMATYC Educator 3 (1)*, 10-13.
2011 Capitalizing on basic brain processes in developmental algebra – Part two," *MathAMATYC Educator 2 (3)*, 28-31.
2011 The neuroscience of connections visualizations & generalizations. In *Enhancing Mathematics with Digital Technologies*. London, UK. Continuum Press. http://www.math.osu.edu/~laughbaum.6/papers/2009_PREPRINT_The_Neuroscience_of_Connections_Visualizations_Generalizations.pdf
2011 Capitalizing on basic brain processes in developmental algebra – Part one," *MathAMATYC Educator 2 (2)*, 4-7.

2009 Generalizing patterns in algebra for long-term memory and understanding," California Math Council, *ComMuniCator*, *34*, 2. http://www.math.osu.edu/~laughbaum.6/papers/CMC_DEC_2009_Laughbaum_GeneralizePatterns.pdf

2008 Implications of neuroscientific research on teaching algebra. In the *International Journal on Continuing Engineering Education and Life-Long Learning*, *Vol. 18, Nos. 5/6, 2008* 585. Inderscience Publishers, Switzerland. http://www.math.osu.edu/~laughbaum.6/papers/Implications_of_neuroscientific_research_on_teaching_algebra.pdf

2007 Breaking tradition in teaching developmental algebra, *Community College Times*, March, 2007.

2003 Hand-held graphing technology in the developmental algebra curriculum, *Mathematics and Computer Education, 37(3)*, 301-314.

2003 Developmental algebra with function as the underlying theme, *Mathematics and Computer Education*, *37 (1)*, 63-71.

Target Audience:

At the school level, *Foundations 3e* is typically used for a course in mathematics for juniors and/or seniors not prepared for pre-calculus after having difficulty in Algebra I and/or II – yet still needing another math credit for graduation.

At the college level, *Foundations 3e* is appropriate for intermediate algebra courses or as a beginning/intermediate algebra combined course. College developmental programs may choose to use any of the popular traditional texts for beginning algebra, but then use *Foundations 3e* as the intermediate algebra text so that students are given a chance to really understand algebra and see its relevance to the real world – things that are often missing in traditional textbooks.

Rationale for the Concept of a Function Approach

To neuroscientists, learning is not necessarily complicated. Learning is simply the process of creating a memory. The brain does it automatically and it is an on-going process. Forgetting is another normal process. In teaching, the issue becomes how to create a long-term memory of, for example, how to solve any equation. But long-term memory isn't worth much unless students understand what it means to solve an equation. We want students to learn (create memories), understand what they learned, keep the memory long term, and be able to recall it in a month or five years. The brain does all of these things on a molecular/cellular level in response to external events/actions.

In education, the thinking has been that if you practice you will learn and remember. This is likely the case for physical learning like riding a bicycle, playing a sport, etc. As for the learning (creating a memory) of an abstract concept, or a mathematical procedure, practice will only take students so far. Practicing a procedure (like factoring, equation solving, etc.) for a day and then moving on to practicing another procedure the day after and a different one the day after that is not enough of, or the correct type of practice required to create a recallable long-term memory with understanding. When practice is followed by an absence of practice the memory will soon fade, and students will have intermittent recall and/or false memory reconstruction.

Basic brain function includes an electro/chemical value system. Brain structures assign a value to the things we think about such as: $2x - 5$, faces, $f(x)$, where I parked my car yesterday, $3x + 7 = 12$, etc. The value systems help determine if we will store a particular memory long term. That is, the brain structure called the hippocampus processes the storing and reconstruction of our memories. Based on the value of a particular memory (something learned), it will process the long-term storage of the memory or not. Memories assigned "of no value," like what you had for dinner 3 days ago, will likely not be processed into long-term memory and will fade within minutes, hours, or days. Recall of a stored memory depends on what it is connected to, it's meaning, time passed, and assigned value. As taught through "drill-and-skill", algebra may not have appropriate connections, meaning, understanding, or value as assessed by a student's brain.

Neuroscience tells us we can improve understanding/memory/recall using the following external events.

- ✓ Connect the mathematics in each lesson to previously learned math and to a contextual situation. Connections are required to store and recall a long-term memory. When teachers don't provide connections, each brain automatically creates their own. These become unusable in future teaching.

- ✓ Use dynamic visualizations – like found on a graphing calculator – near the beginning of a lesson. Understanding is improved through visualizations. Neuroscience research shows that using a purely symbolic procedure can cause the brain to assign it a low value (neurons do not release dopamine).
- ✓ Use a real-world contextual situation at the beginning of a lesson that models the algebra to be taught. This provides an emotional tag that improves understanding/recall/reconstruction of the concept/skill being taught. This method also adds meaning which has the same cognitive benefits. Further, students can then function at a higher cognitive level and learn more quickly.
- ✓ Use a guided-discovery pattern-building activity designed to have students generalize the mathematics in the lesson. This creates a memory of the generalization and an understanding of the mathematics.

Neural response teaching methods are implemented through **a function approach** in the **senior remedial Algebra II** textbook ***Foundations for College Mathematics 3e*** written by Ed Laughbaum. *Foundations* has been adopted in over 170 high schools (colleges) in the USA to teach differentiated Algebra II (**Intermediate Algebra**) to students who had difficulty in traditional main-stream algebra, but still have aspirations of college success.

Comments from Current Users:

"I keep hearing from students and their parents that for kids who traditionally always struggled in math; this was a positive course for them! We use it for kids who have scraped by but still need a 4th year of high school math (as is now required in Texas). I think this is a GREAT course. The methodology of the text works. Period." (From a teacher in the 2012 *Newsweek* top 1000 high schools in the country. This school was 47th.)

Dallas, Texas

"I did a little within school comparisons using the EMPT scores and found that the students in *Foundations of College Math* did significantly better than other students in our Algebra II classes. Our *Foundations of College Math* students came from Algebra II classes where they failed, dropped the class or didn't have the confidence to take pre-calculus."

Ohio

"The book is wonderful. The kids keep asking me where you get all those wonderful facts ... (the most recent favorite: 'there is enough TNT in the world for each person to have 10,000 pounds')."

Ohio

"I didn't realize that my attendance at your summer institute would have such far reaching effects. At first, co-workers thought I was crazy, but I stuck with your recommendations for teaching Algebra through function. My co-workers, especially at the Algebra II level, are now sold on the idea as well. Our Algebra II SOL scores were outstanding last year."

Virginia

"We are using your series *Foundations for College Mathematics* as a senior course for those students going to college but not in a math related field. At this time, I am very pleased with the format of the textbook and the exercises at the end of each section. I like the function approach and believe it has helped my students."

Ohio

"Your book is a big adjustment for everyone – it's just a different approach. But, so far, everyone loves the way it's done."

Texas

"The students are doing amazingly well, and I credit part of that to a well-written text."

Ohio

"I visited classrooms and talked with both teachers and students. They are still pleased with the *Foundations* curriculum."

Ohio

"For awhile now, the struggling math students in our school have found great success using the function approach."

Texas

Current Table of Contents:
Attached later

Current Users:
There are currently 175 adopters. At two adopter schools, the teachers are winners of the USA Presidential Award for Excellence in Mathematics and Science Teaching. Another *Foundations* adopter school ranked 47th out of the 1000 top schools in the country as found in the 2012 *Newsweek* annual survey of public education in the USA.

Ancillaries:
The first is a 350-page student workbook (and a 540-page teacher version) with the following types of activities:

Explorations:
Explorations are of two types. A few are designed for the day before a topic is discussed in class. They are somewhat like guided-discovery exercises. Just like some of the explorations are designed for before you formally teach the related topic, others have been designed for after the formal discussion. If assigned at the wrong time, some will become exceedingly difficult and others may take what was a challenging assignment and turn it into a simpler and low-level thinking task. An exploration that asks students to solve the equation $2[x - 5(3x + 4) - 6] + 6[4 + 7(9x - 1) - 2(2x - 6)] = 0$ is to be assigned right after teaching students to solve linear equations (Section 6.1) – not before the topic is taught. Whereas an exploration something like in Section 2.1 asks students to predict the number of inmates in the US in the year 2000, given a numeric history of the prison population. This is assigned before the topic is taught. The intent is to see if they can use their own ideas to solve this problem. Most explorations require a graphing calculator, a few don't. The author used almost all of the explorations as group assignments; giving them to students as they left a class and collecting them at the beginning of the next class. The title "Explorations" is somewhat descriptive of the kind of work the students will be doing. That is, many times students must explore on the calculator to answer the question.

The explorations are an integral part of the overall assessment tools used with *Foundations for College Mathematics 3e*. Assessment tools like the explorations measure a totally different kind of learning than do skill-based midterms. Explorations typically use pattern building embedded in guided discovery.

Concept Quizzes:
Just like the explorations, some concept quizzes are to be assigned either before a topic is discussed or right after. Many of these are like guided discovery exercises and they would be assigned before the topic is taught. What is unique about the concept quizzes is that many ask students to do something they probably don't do much – be creative. These quizzes contain questions like "If $(f+g)(x) = 2x+7$, develop any two functions $f(x)$ and $g(x)$ whose sum is $(f+g)(x)$." Or "Create any function that has a domain of $[3.6, \infty)$." The concept quizzes usually require a graphing calculator. The quizzes investigate a variety of concepts related to the mathematical topics in *Foundations for College Mathematics 3e*.

The concept quizzes are an integral part of the overall assessment tools used with *Foundations for College Mathematics 3e*. Assessment tools like the concept quizzes measure a totally different kind of learning than do skill-based midterms and the explorations that require "exploration" and a little tenacity. Concept quizzes typically use pattern building embedded in guided discovery.

Investigations:
The investigations usually take a situation or single idea and ask a multitude of questions about the situation or idea. They are assigned after a topic has been developed in class. They usually require a graphing calculator. The intent of the investigations is to require a thorough analysis of a topic or idea. For example, Section 2.1's investigation asks 14 questions about the electricity charge data from the North Carolina Public Utilities Commission. Or the Section 4.4 investigation has 26 questions about gasoline usage.

The investigations are an integral part of the overall assessment tools used with *Foundations for College Mathematics 3e*. Assessment tools like the investigations measure a totally different kind of learning than do skill-based midterms and the explorations that require "exploration" and a little tenacity, or the concept quizzes that ask students to be creative. Investigations typically use pattern building embedded in guided discovery.

Writing Mathematics:
These assignments are typical writing assignments like found in many reform textbooks. Just like all of the above tools, the writing materials are intended as another method for assessing other facets of student understanding. While all of these tools are used as assessments, many are used to enhance understanding and to promote learning. Many of the other assessment/teaching tools listed above also require some writing. As do the modeling projects described below.

Modeling Projects:
The modeling projects require exploration, conceptual understanding, tenacity, and a graphing calculator. They assume some knowledge of the connection between function behavior and function parameters and/or they require the use of geometric transformations to create the mathematical model of real-world data. A study of statistics is not assumed. Students typically may require 2 to 10 hours to finish a modeling project.

The modeling projects provide an opportunity to ask students to apply the mathematics that is in the text. Mathematics such as using the relationships between the parameters in a function and the behavior of the function, geometric transformations, arithmetic operations of functions and the resulting change in the domain, behavior near the zeros of functions, etc. Students must recognize the shapes of the basic elementary functions. They must know how arithmetic operations on functions change the geometry of the graphical representation of the functions. The main goal of the modeling projects is to find a symbolic representation of any function that models the data.

Just as an engineer, physicist, business person, or astronomer may solve a problem by following a prescribed procedure, your students are directed through this problem solving process by being asked a series of questions about the data. Once they have solved the main problem and have developed the symbolic representation of the function (mathematical model), they are asked to defend their solution by explaining the limitations of their model. This includes giving a situation when their model does not apply. They must explain their thinking on how they developed the model. Students are given the opportunity to use their model when asked questions about data not available in the given information. They are also asked to conjecture on what type of professional person might use their model. And finally, they must identify the references and resource person(s) used to help them in the problem solving process.

Class time may be used to work on the projects or they provide a good assignment outside of class. Depending on the level of mathematical sophistication of students, the projects may take from one to four hours. The modeling projects provide another tool for assessment that measures still other traits of the math student.

<u>The remaining ancillaries are:</u>
Data Sets:
All data used in *Foundations* is available in TI-84 or TI-83 Plus calculator programs, grouped by chapters as TI calculator group files. Data sets are also available for the TI-Nspire CX calculator. The TI-83/84 programs and the Nspire documents are named by data description followed by the page number where the data is used in the textbook. The reason for the programs/documents is that they make the data interactive which improves memory and understanding of the related mathematics.

e-Activities:
A series of 48 StudyCard stacks on algebra topics that can be used as a “Power Point” like presentation, or assigned to students to be executed right before (or at the beginning) class. They are to be processed in a group of two students. A subset of the 48 activities is to be used as a summative assessment tool that can be used in the place of a quiz. See the full description in the supporting documents below.

Apps:
The StudyCard activities will be available as apps for the iPad, Kindle, Nook, iPhone, and Android phones as of November 2012. The apps are:
Behaviors of the Core Functions of Algebra (ISBN: 978-0-9858683-0-7) with 6 lessons
Solving Equations Containing the Core Functions of Algebra (ISBN: 978-0-9858683-1-4) with 8 lessons
Summative Review of the Core Functions of Algebra (ISBN: 978-0-9858683-2-1) with 6 lessons
Foundations of Algebra (ISBN: 978-0-9858683-3-8) with 9 lessons
Symbol Manipulations in Algebra (ISBN: 978-0-9858683-4-5) with 12 lessons

In the spring of 2013, the following apps will be available with content taken from the ancillary activity book:
Explore Linear Functions (ISBN: 978-0-9858683-5-2) with 13 lessons
Explore Quadratic Functions (ISBN: 978-0-9858683-6-9) with 10 lessons
Explore Exponential Functions (ISBN: 978-0-9858683-7-6) with 8 lessons
Explore Solving Equations through Functions (ISBN: 978-0-9858683-8-3) with 8 lessons

Miscellaneous:
A 100-page document that addresses how selected homework exercises can be solved (a teacher manual). Even Answer Key/Chapter Test Key and a two semester suggested course outline.

Competition:
At the high school level: No major publisher publishes a remedial textbook for seniors who cannot learn from the mainstream books. If they did, it would imply that their regular textbooks do not work for everyone. Typically, mainstream textbooks will fail in about 40% of the students required to take the courses.

At the college level: Developmental algebra texts have yet to use a true function approach. Several others claim to use the function approach, but it simply means moving the chapter on functions from eight to two (for example). Of the five or six claiming to use a function approach they never use function concepts and behaviors of functions to teach mathematics. Further, functions rarely appear anywhere else in the text except the function chapter. That is, no one is using function and function behaviors to teach mathematics. No one is using function and function behaviors to do mathematics such as polynomial addition & subtraction, reducing rational expressions (function), laws of exponents, factoring, equation solving, systems of equations, inequalities, properties of inequalities, definitions, etc.

Supporting Documents:
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CHAPTER THREE
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CHAPTER FOUR
FUNCTIONS: NOTATION AND OPERATIONS

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CHAPTER TEN
ADVANCED ANALYSIS OF THE QUADRATIC FUNCTION

CHAPTER ELEVEN
BASIC GEOMETRY

CHAPTER TWELVE
BASIC TRIGONOMETRY

CHAPTER THIRTEEN
SYSTEMS OF EQUATIONS AND INEQUALITIES

CHAPTER FOURTEEN
INTRODUCTION TO THE ANALYSIS OF THE LOGARITHMIC FUNCTION

Preface to Students and Teachers

This text contains terminology, content, and algorithms that may not be found in a traditional textbook because it is the author's intention to break from tradition and prepare students for the mathematics needed in a modern society. Further, as learning progresses, terminology may change to reflect new understandings – just as in scientific discovery. The author also recognizes that mathematics is learned by understanding, not by rote memorization. Developing mathematical ideas in the context of a real-world situation helps students understand mathematics; at the same time, students learn that mathematics relates directly to the world outside the classroom.

The use of this text requires a graphing calculator with function notation; the calculator will be used as a tool to help us understand mathematical concepts and perform mathematical algorithms. Technology, which offers students and teachers a variety of methods for solving problems, is used in this text to explore mathematical ideas, and it changes what is considered important. Many traditional mathematical topics have diminished importance. This text offers content that is important to students directly entering the work force and to students continuing the study of mathematics and science in college. Many of the exercise sets offer questions that are engaging and demand higher level thinking.

Students have the responsibility to read and study the text in order to learn. Students have the responsibility to view the exercise sets as questions that are to stimulate thinking, not questions that are designed to encourage memorization of facts. Students have the responsibility to recognize that the exercise sets contain questions to help them formulate their own ideas about mathematical relationships.

There are very limited references to calculator keystrokes; when they are included, they are for theTI-83 or TI-84. With slight modification, the keystrokes will also work on other calculators.

The teaching features of this text include:

- Technology integrated throughout to enhance long-term memory and promote understanding of mathematical concepts.
- Technology integrated throughout to provide options for performing mathematical algorithms.
- Mathematical concepts introduced in the context of real-world situations.
- Thorough analysis of applications.
- Guided discovery exercises.
- Reduced emphasis on the use of symbol manipulation and increased emphasis on the use of function as a central theme.
- Three methods (numeric, graphic, and algebraic) of representing functions, equations and expressions.
- Distributed learning has been incorporated in this text. For example, the idea of function is introduced in Section 2.1 as a data relationship. The intuitive idea of function is further developed in Chapter Three by looking at algebraic expressions that model data relationships introduced in Chapter Two. Chapter Four continues with formal function notation, and functions are used in equation and inequality solving in Chapter Six. Chapters Seven, Eight, Nine, Ten, Twelve, and Fourteen offer analysis of individual function types.
- A variety of methods for solving problems are encouraged. Students are encouraged to explore on their own.
- Higher level thinking skills are encouraged through projects, open ended questions, and concept questions.
- Sample problems are checked numerically or graphically and students are encouraged to do likewise.
- Exercise sets contain low-level difficulty problems that prepare students for future topics.
- Exercise sets contain high-level difficulty problems that review topics previously taught.

- Exercise sets contain writing questions.
- Exercise sets contain concept questions.
- Exercise sets contain exploration problems. Many of the explorations can be used for group work. Many can be used as portfolio exercises.
- Exercise sets contain open ended questions.
- Mathematical words are defined by bold text.
- Extended laboratory projects on modeling are included with the text.

Embedded Neuroscience

Neuro and cognitive science research provides considerable information about how the brain functions. This textbook capitalizes on this research through the implementation of the cognitive processes of associations, pattern recognition, attention, visualizations, priming, meaning, and the enriched teaching/learning environment.

- We remember algebra longer and have better memory by using associations – made through function permeating the content. That is, we are more likely to remember the mathematics taught because we capitalize on associations made through using a function approach. That is, memory and recall are processed through a series of connected (associated) neural circuits.
- Learning is made simpler, faster, and more understandable by using pattern building as a teaching tool. In the function approach used in this text, almost all of the pencil and paper activities, e-teaching activities, and class discussions use pattern building to reach a generalization about a concept or skill.
- We cannot learn if we are not paying attention. The graphing calculator is used to draw attention to the mathematics through its basic functionalities including, various app software.
- Without visualizations, we do not understand or remember the mathematics as well. In the function approach visualizations are used first before any symbolic development. This greatly increases the likelihood that we will remember the mathematical concept being taught.
- Considerable information processing takes place in the unconscious brain, including a learning module. To make this processing possible for our purposes, the brain must be primed. The function implementation module (Chapters Two and Three) and early e-learning activities prime the brain for all the algebra that follows.
- The enriched teaching/learning environment promotes correct memory of math content. The wide variety of teaching activities facilitated by the function approach provides the enriched environment.
- Contextual situations (often represented as relationships) provide meaning to the algebra learned. Algebra taught without meaning creates memories without meaning that are quickly forgotten.

Spread throughout *Foundations 3/e* are short articles called *Basic Brain Function*. The idea behind them is to acquaint you with information about the brain and how it learns. If you know more about basic brain function, you will know more about how you learn and what is needed to learn algebra – the goal of this textbook. The articles are not in any particular order.

End-of-Section Exercises

You may not recognize the headings of the “exercises” at the end of each section. Here is a brief explanation.

You learn new ideas (concepts) and skills from reading the text and/or participating in class, or listening to the teacher’s presentation. But if this is the end of your learning, you will soon forget the algebra taught. So, each “exercise” set is entitled **Strengthening Neural Circuits** because when you do the exercises, you will make all of the related neural circuits stronger. Stronger implies less likely to forget and more likely to be able to use the algebra taught. Stronger implies the circuits are more likely to “fire” when needed.

Each section of **Strengthening Neural Circuits** starts with four items designed to get the brain thinking about the up-coming content in the next three sections of the text. Well over 90% of the brain’s thoughts are done on an unconscious level. The method of getting the brain to start processing the ideas found in the first four items is called priming. Therefore, these items are entitled **Priming the Brain**.

Each section of **Strengthening Neural Circuits** continues with six items that are review items of the previous sections. Reviewing causes the related neural circuits to fire. This strengthens the neural circuits. So these items are referred to as **Neural Circuits Refreshed**.

Each section of **Strengthening Neural Circuits** continues with what are typically referred to as “homework exercises.” The “exercises” are called **Myelinating Neural Circuits** because when you practice a skill, or revisit/use concepts; glial cells called oligodendrocytes wrap myelin around the neuronal axons in the related circuit. This process means that the circuits that contain these neurons will be more likely to fire when needed. The thousands of neurons that make up the circuit must fire to bring the stored memory to consciousness, or to be used in unconscious processing of the related algebraic skill or concept. (The brain is made up of 15% neurons and 85% glial cells.)

Finally, in each section of **Strengthening Neural Circuits** you will find items called **Developing the Pre-Frontal Lobes**. These items often require reasoning and deeper thinking. As it turns out, the pre-frontal lobes of your brain process reasoning and deeper thinking. This area of the brain is the last to develop and may not be complete until you are around 21 – 23 or so, years of age. If you do not fully develop your pre-frontal lobes in high school and college, you will not make good decisions or be able to reason well as an adult.

Algebra approached through function, as in this textbook, has reordered the content and capitalized on function concepts to <u>develop understanding, long-term memory, and skills</u>.

This text has been published as a preliminary edition, as a revised preliminary edition, a first edition text, second edition, and now as a third edition. It has been class tested over a twelve year period and has been revised and edited no less than fourteen times. It has been reviewed by over twenty-five reviewers.

Philosophy and Use

***Foundation for College Mathematics* 3*e* -** Laughbaum

Chapters Two and Three contain material that is intended as an introduction to several basic functions. The other chapters continue the distributed learning spiral with more advanced material on these and other functions. Although Chapters Two and Three introduce students to various functions in an intuitive fashion, you will also recognize that the included topic of “behavior of functions” is an extremely important part of the curriculum and is used throughout the text *Foundations for College Algebra 3e*. It is wise to use as many as three weeks on the content in Chapters Two and Three. Time spent here will save time two to three-fold in later material. Further, not spending three weeks on Chapters Two and Three will cause difficulties for students later in the book. For the full learning effect, materials from *Leading Discussions, Explorations, Concept Quizzes, Writing Mathematics, Investigations, and Modeling Projects for Foundations for College Mathematics* should be utilized, starting in Chapter One and fully implemented in Chapter Two and beyond.

The terminology in this companion work to *Foundations for College Algebra 3e* may be slightly different from that in other textbooks in four ways: (1) In order to apply it to real-world situations, the topic of *domain* has been split into two parts. If mathematics is used without reference to functions representing anything except real numbers, the domain is called the *normal domain*. However, if a function is used to represent a relationship in the real world, the domain will quite often become a subset of the normal domain and is called the problem domain. For example, the function $S(c) = \dfrac{300}{6c + 300} \cdot 100$ has a normal domain of all real numbers except –50. But, if $S(c)$ is used to model the strength of a coffee-cream mixture where c represents the number of 6-*ml* cream containers added to 300-*ml* of coffee, the normal domain is of little use. The domain that makes sense here is not the normal domain but a problem domain such as the positive integers. (2) This manual uses the notation $S(c) = \dfrac{300}{6c + 300} \cdot 100$ with the notation $\dfrac{300}{6c + 300} \cdot 100$ interchangeably as the symbolic representation of a function. The first notation is the symbolic form of the ordered pairs of numbers $(c,\ S(c))$, and the second is the symbolic form of the set of ordered pairs of numbers $\left(c, \dfrac{300}{6c + 300} \cdot 100\right)$. Both $S(c)$ and $\dfrac{300}{6c + 300} \cdot 100$ represent the strength of the coffee and c represents the number of cream containers added to the coffee. (3) Emphasis is placed on rate of change; therefore, use of the word *slope* has been minimized because it is used little in the real world. (4) The different representations of a function relationship are called symbolic representation, numeric representation, and graphic representation. This is used because the word “representation” ties all three together. This connects all three to the concept of function; unlike the words rule, table, and graph.

The materials in *Leading Discussions, Explorations, Concept Quizzes, Investigations, Writing Mathematics, and Modeling Projects for Foundations for College Mathematics* may be duplicated for classroom use only. They may not be reproduced in any format for any other purpose without the written consent of the author.

Below is a listing of features found in *Foundations for College Mathematics 3e*. These features have been recommended by a variety of reform documents. Also included are reasons for the features and how they can be used.

<table>
<tr><th>F E A T U R E S</th><th>R A T I O N A L E   A N D   U S E</th></tr>
<tr><td>

- **Technology integrated throughout to enhance the learning and teaching of mathematical concepts.**

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Every section uses technology to enhance either teaching or learning or both. As early as Section 1.1, the calculator is used to develop an argument for justifying the field properties. Calculator logic statements allow for the demonstration of the use and misuse of the properties of equality and inequality.

Section 1.2 continues the use of technology by "looking" at data pairs through using stat-plot. Students encounter related data in numeric form outside the math classroom; now, they can take this data and discover that it many times has a very definite "shape." Students are therefore immediately introduced to the concept of function using this simple approach.

The purpose of Chapters Two and Three is to set the groundwork for learning about connections between function behavior and function parameters. This is accomplished by using the graphical or numerical representations of primary functions. Guided discovery of the behavior-parameter connection cannot be completed without technology. Chapters Two and Three are also intended as the time for students to become familiar with the calculator. That is, you should not take class time to "teach" the calculator as an end goal. Calculator mechanics must be taught in the context of mathematics. This is a minor, but necessary, goal for Chapters Two and Three.

Of course, since students learn about zeros of functions in Chapters Two and Three, factoring can be developed by studying the connection between zeros and linear factors of polynomial functions in Chapter Four.

There are numerous uses of technology integrated throughout the text. A few uses are to:

- do arithmetic,
- find intercepts and slopes of linear functions,
- study behaviors of functions,
- analyze function operations from a graphical perspective,
- solve all kinds of equations and inequalities,
- model collections of real-world data pairs,
- confirm answers to algebraic simplification,
- develop and confirm algebraic properties,
- discover changes in the domain of algebraically simplified expressions,
- find extraneous roots caused by solving equations with pencil and paper,
- evaluate trig functions,
- solve systems of equations and inequalities,
- study function notation, and
- analyze compositions of functions.

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- **Technology integrated throughout to provide options for performing mathematical algorithms.**

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Students must be given options for performing mathematical algorithms because this is a good model for problem solving in general. Many times, there are multiple methods for solving problems. Further, students must be responsible for selecting an algorithm. This too, models expectations in other arenas. From a pedagogical standpoint, a student may not understand one algorithm, but if they have options, they should understand another and be able to use it.

The primary example is in solving equations. Suppose you assign the following

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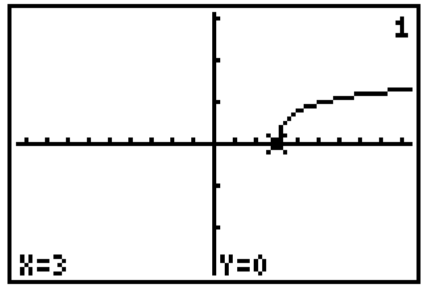
	equation to be solved $(x-2)^2 - 3\lvert x+1\rvert = 4$. This is certainly formidable to the student who only knows how to solve equations with pencil and paper. Chapter Six illustrates four other technology-based algorithms. Three of these: zeros, intersection and numerical work quite well with this equation. On the other hand, asking students to solve the equation $\pi x = 3$ ought to cause them to respond with pencil and paper or they should solve it mentally. Suppose students must evaluate $\left(\frac{-b}{2a}\right)^2$ and produce a value in rational form. Pencil and paper will work fine, but students quite often make mistakes when evaluating it. Chapter Nine offers a technology-based method that will probably be less prone to error. The instructor should use both methods so students see they have a choice. Examples 8 and 9 from Section 9.5 demonstrate how multiple methods are modeled in the text. ***Example 8:*** Find the exact solution to $\sqrt{x-3} - \sqrt{x+1} = -2$. ***Solution:*** Add $\sqrt{x+1}$ to both sides before squaring each side. $\sqrt{x-3} - \sqrt{x+1} + \sqrt{x+1} = -2 + \sqrt{x+1}$ addition property $\sqrt{x-3} = \sqrt{x+1} - 2$ simplification $(\sqrt{x-3})^2 = (\sqrt{x+1} - 2)^2$ power property Remember $(a+b)^2 = a^2 + 2ab + b^2$ $x-3 = x+1-4\sqrt{x+1}+4$ $x-3 = x+5-4\sqrt{x+1}$ subtract x and 5 $-8 = -4\sqrt{x+1}$ divide by –4 $2 = \sqrt{x+1}$ square both sides $2^2 = (\sqrt{x+1})^2$ simplify $4 = x+1$ $x = 3$ check for extraneous solution $\sqrt{x-3} - \sqrt{x+1} = -2$ $\sqrt{3-3} - \sqrt{3+1} = -2$ $\sqrt{0} - \sqrt{4} = -2$ $-2 = -2$ The solution is 3. Remember: A graphical check may be a better choice. ***Example 9:*** Solve $\sqrt{x-3} - \sqrt{x+1} = -2$ again. ***Solution:*** Since the exact solution is not required, you may want to use a graphical method. It too may give the exact solution however. Add 2 to both sides and graph the related function. Graph $y = \sqrt{x-3} - \sqrt{x+1} + 2$ and find values for x where the function has a value of 0.



The solution is exactly 3.

Remember, you have five methods for solving an equation. In addition to the three graphical methods and the analytical method, you may also use the numeric method.

Below is the numerical method for solving the above equation.

Table 9.5.1

x	3	4	5	6	7	8
$y=\sqrt{x-3}-\sqrt{x+1}+2$	0	0.76	0.96	1.09	1.17	1.24

The domain is [3, ∞), thus there are no zeros left of 3. The function is increasing; therefore, there are no zeros to the right of 3. The only solution is 3.

- **Mathematical concepts are introduced in the context of real-world situations.**

About 70% of the topics developed start with a problem that promotes the mathematics. Here is a sample taken from Section 3.3.

The Alberta Clipper

When a fast moving "Alberta Clipper" cold front drops down from Alberta, Canada and sweeps across a relatively small path through the Northeast quarter of the US, the temperature drops quickly by as much as 25° and then recovers quickly. Below is the numeric representation of the temperature (T) in Columbus, Ohio during an "Alberta Clipper". The day the front arrived, the daily high had been 18° at 4 PM. Time (t) zero is midnight.

t	0	1	2	3	4	5	6	7	8	9	10
T	2	0	-2	-4	-6	-8	-6	-4	-2	0	2

If a meteorologist wants to model this data with a function, what function can be used? The numeric data suggests that the linear function cannot be used because the temperature (the function) is both decreasing and increasing -- not a behavior of the linear function. Can the data be modeled by a quadratic function? Quadratic functions both decrease and increase on the normal domain. However, the quadratic function may not be a good choice because the average rate of change of the quadratic is not constant. If you check the rate of change of the function in the numeric representation, you will find that from midnight to 5 am the temperature decreased by 2° per hour and after 5 am it increased by 2° per hour. This is not one of the behaviors of the quadratic function. If not linear and if not quadratic, then what? The graphical representation of the data may give a clue as to the type of function that can be used to model the temperature.

The shape matches that of the absolute value function studied in Sections1.2 and 2.2. The figure shows the data and the graph of the absolute value function $2|t-5|-8$ on the same coordinate system. The absolute value function $2|t-5|-8$

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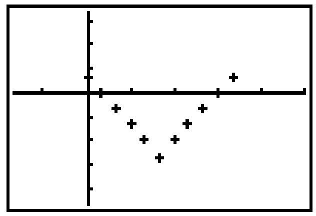
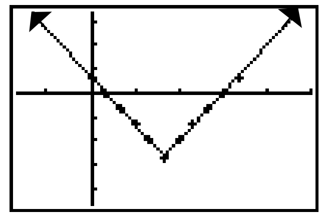
<tr><td></td><td>matches the data. The shape is a V and the graph of any absolute value function of the form $d|x+e|+f$ will always have this shape. You may want to confirm this conjecture by graphing several absolute value functions of the form $2|t-5|-8$. When an absolute value function looks like $d|x+e|+f$, it is in standard form and the function parameters are d, e, and f.

 </td></tr>
<tr><td>• Thorough analysis of applications.</td><td>Many application situations have extended exercise questions. The above example has one related exercise in the exercise set—it is listed below.

7. The temperature in the "Alberta Clipper Problem" was 18° when it started and the temperature dropped to –8° 13 hours later, this is a drop of –26°. If a similar front moves through with the 4 PM beginning temperature of 33°, the model for the temperature becomes $2|t-5|+7$. For this clipper, find the minimum temperature, when the temperature reaches the minimum, when the temperature is dropping (decreasing), when the temperature is rising (increasing), how fast the temperature is decreasing and then increasing, when the temperature is zero, when it is negative, and when it is positive?

Many sections have more. For example, Section 7.5 starts with the problem of mixing ground-up peanut shells with the cleaning agent in laundry detergent. The exercise set contains six related problems.

This idea of using exercises related to problems developed in the section should make students realize that a "real-world" situation gives rise to multiple mathematical problems. Not unlike in the real world. This approach forces students to read the textbook in a new way. It becomes a reference book because they must refer to the initial situation for information to help solve exercise problems.</td></tr>
<tr><td>• Guided discovery exercises and explorations</td><td>Many sections contain discovery exercise sets and all sections contain exploration exercises. Below is a sample guided discovery exercise set from Section 9.1.

Find the minimum or maximum value of the following functions; also, find the value of x at the minimum or maximum.

21. $f(x)=5.6\sqrt{x-3.2}+11.2$

22. $y=-2\sqrt{x+7}+11.2$

23. $f(x)=-3\sqrt{x-5}-11.2$

24. $y=0.2\sqrt{x+4.4}-11.2$

25. What are the coordinates of the maximum (or minimum) point on the graph of</td></tr>
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	$y = d\sqrt{x+e} + f$? What this and other guided exercises do is to make students think about what they have just learned in the exercises. Many times, students think of exercises as "just mindless practice." The fact is that students can learn from this kind of exercise. Upon successful completion of an exercise like this, students have learned mathematics on their own – one of the best learning experiences available. Below are two explorations from Section 4.5 where students have learned how to factor. 41. Pick numbers at random for the parameters *a*, *b*, and *c* in the trinomial of the form $ax^2 + bx + c$ and try to find the factors. How many are factorable? 43. Find the zeros of the following functions. a. $f(x) = (x-5)(x-1)(x+3)$ b. $f(x) = \|(x-5)(x-1)(x+3)\|$ c. $f(x) = \|(x-5)\|(x-1)(x+3)$ d. $f(x) = (x-5)\|(x-1)\|(x+3)$ e. $f(x) = (x-5)(x-1)\|(x+3)\|$ f. $f(x) = \|(x-5)(x-1)\|(x+3)$ g. $f(x) = (x-5)\|(x-1)(x+3)\|$ Explorations are usually extensions of material in the section. Most explorations in the text require a calculator. There is considerable variety in the types of explorations. In the second example above, students are expanding on the relationship between zeros and factors. They have never seen what these graphs look like, but they certainly should learn more about the connection between factors and zeros.
• **Reduced emphasis on the use of symbol manipulation and increased emphasis on the use of function as a central theme.**	While the number of pages and the number of exercises dedicated to the practice of symbol manipulation has been limited, this is not to say that these traditional topics have been eliminated. Traditional algorithms have a place in the reform curriculum. They are no longer "the" curriculum. What you won't see in this text is exercises or examples of simplification of expressions with three or four levels of exponents with three or four bases. You won't see many exercises with complicated radicals to be simplified. Nor are there sums or differences of fractions with denominators containing five or six linear factors. What is in the text is enough for courses that follow and enough for surviving in the world outside academia. One of the main features of this text, as recommended by current standards documents, is the use of function as a central theme. A traditional approach is to use equations as a central theme, but this has been a problem in mathematics education because it requires considerable symbol manipulation ability, if no technology is

	used. If technology alone is used to solve equations, there is little math to include in the text. However, the major reason for using a function approach is that: • Function behavior is much more prevalent in the world than are equations. (That is, we aren't just teaching engineers and scientists anymore.) • Equations are a subset of the study of functions. So the scientists and engineers still learn equation setup and solving. • All the traditional mathematics can be developed from the study of functions. • New topics like mathematical modeling and data analysis follow much more naturally. • Technology allows for the study of functions as they commonly appear – as data pairs and as graphs. If you think that the study of functions is too complicated for beginning and intermediate algebra students, you may be thinking of function notation. Even this can be developed smoothly because of technology. But, when function is used as a central theme, there are many opportunities to develop the concept throughout a course. Because functions permeate a course and are no longer in just one topic of a course, the exposure is much deeper and richer than when using an equation/manipulation approach.
• **Three methods (numeric, graphic, and algebraic) of representing a function.**	As students make the transition from number based thinking about mathematics to a symbolic-based approach it is highly desirable to keep referring to numeric representations. The first cycle of mathematics in the text explaining the behavior of functions is numeric. The second cycle is the use of graphical representations. The third is to use symbols. This is developed in Chapters Two and Three. From Chapter Two to the end, all three representations of relationships are used as appropriate for the situation. However, when analyzing a new idea, the text almost always starts with numeric information. This follows the pattern established in their life experiences with mathematics.
• **Distributed learning has been incorporated.**	For example, the idea of function is introduced in Section 2.0 as a data relationship. This intuitive idea of a function is further developed in Chapter Two. From the data relationships, algebraic expressions that model data relationships are developed in Section 2.2. Symbolic representation is then used in Chapter Three. Chapter Four continues with formal function notation, and functions are used in equation and inequality solving in Chapters Six, Seven, Eight, Nine, Ten, Thirteen and Fourteen. These chapters also offer analysis of individual function types. Each of these chapters starts with a relative through investigation of a particular function. The mathematical level of the work with functions in Chapter Two is relatively low. Chapter Three raises the level slightly, and the remaining chapters finish the distributed learning.
• **A variety of methods for solving problems are encouraged. Students are encouraged to explore on their own.**	The use of technology provides for a natural flow of using a variety of methods for solving problems as does using a function approach. Many of the worked-out sample problems show a variety of methods with a discussion as to why each is used. Below is an excerpt from Section 4.6 showing how students are encouraged to explore on their own. "This example is significant because the absolute value function has had its domain altered while the shape of the graph has remained the same. Hopefully, you have already come to expect that different symbolic representations of functions have different graphical representations. In the case of $\lvert x-3\rvert-4+0\sqrt{x}$, the graphical representation is still a **V** with a different domain. Explore a little with your

	calculator by entering any function of your choosing and then add the function $0\sqrt{x}$. Find the domain of your function. The beauty of adding $0\sqrt{x}$ is that you are adding 0 to the function. This is why the graph does not change, just the domain. Try graphing any function on your calculator. Now add $0\sqrt{x}$ to it and graph it again. Note that there is no change in shape, only a change in the domain. See the Explorations exercises for a further investigation of this idea."
• **Higher level thinking skills are encouraged through projects, open-ended questions, and concept questions.**	Every section contains open-ended questions and concept-questions. Most chapters contain extended projects. Below are typical exercises from Section 5.3 that are based on concepts rather than skills. 36. What information about the graphical representation of a linear function is needed to find the symbolic representation? 37. If the only thing known about the graph of a linear function is its slope, can one distinct symbolic representation be found? Explain your answer. 38. Can more than one symbolic representation be found for a line through (2, 1)? Explain your answer. 39. Can a single symbolic representation be found for a line whose *y*-intercept is 6? Explain. 40. If the graph of $y = ex + f$ is perpendicular to $y = gx + h$, what is the relationship between e and g? 41. If the graph of $y = ex + f$ is parallel to $y = gx + h$, what is the relationship between e and g? Are f and h related? 42. Make a list of applications of the linear function. Include background material and uses of the linear function. 43. What is the meaning of the words "point-slope form of a linear function?" 44. What conditions must be met before two lines can be perpendicular? 45. Describe how you put the function $y = -2(x+4) - 5$ in slope-intercept form. Do not actually put it in slope-intercept form. 46. Describe how you can put $y = 2x + 5$ in point-slope form. Do not actually do it. Below are examples of open-ended questions found in typical exercise sets. These are taken from Section 3.4. 22. Create a square root function that is increasing. 23. Create a square root function that is decreasing. 24. Create a square root function that has a maximum of 7. 25. Create a square root function that has a minimum of 2. 26. Create a square root function that has a domain of $[-3, \infty)$. 27. Create a square root function that has a range of $[-3, \infty)$. 28. Create a square root function that has a zero at 4. 29. Give an example of a square root function whose graph crosses the *x*-axis. 30. Give an example of a square root function whose graph does not cross the *x*-axis. 31. Give an example of a square root function that starts on the *x*-axis.
• **Sample problems are checked numerically or graphically and students are encouraged to do likewise.**	Many times they are checked algebraically because they are solved by technology-based methods. Below is an example from Section 9.5. ***Example 7:*** Find the exact solution to $x - 3 = \sqrt{2x+2}$. ***Solution:*** $x - 3 = \sqrt{2x+2}$ given $(x-3)^2 = (\sqrt{2x+2})^2$ power property of equality

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$x^2 - 6x + 9 = 2x + 2$ simplification

$x^2 - 8x + 7 = 0$ subtraction property of equality

$(x - 1)(x - 7) = 0$ factorization

$x = 1$ $x = 7$ relationship between factors and zeros

Check for extraneous solutions:

$x - 3 = \sqrt{2x + 2}$ $x - 3 = \sqrt{2x + 2}$

$1 - 3 = \sqrt{2 \cdot 1 + 2}$ $7 - 3 = \sqrt{2 \cdot 7 + 2}$

$-2 = \sqrt{4}$ $4 = \sqrt{16}$

$-2 = 2$ FALSE $4 = 4$ TRUE

$x \neq 1$ $x = 7$ This is the only solution.

Instead of checking analytically, you may find it simpler to check graphically. Figure 8.5.9 shows the graphs of the functions $x - 3$ and $\sqrt{2x + 2}$. As you can see the intersection method only shows one solution to the equation.

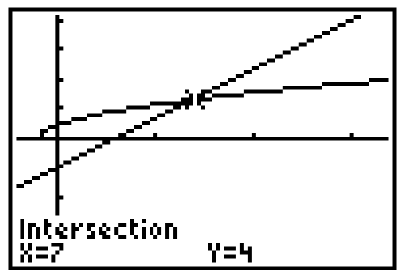

Figure 9.5.9

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- **Exercise sets contain low-level difficulty problems that prime students for future topics.**

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To cause students to think about future topics and to instill a sense of comfort with the topics, each exercise set starts with a series of four (usually simple) questions that foreshadow what is to come. One use of the questions is for discussion at the end (beginning) of each class period. Below is an example from Section 7.1.

-4. Simplify $(x{\cdot}x{\cdot}x{\cdot}x)$ $(x{\cdot}x{\cdot}x{\cdot}x)$ $(x{\cdot}x{\cdot}x{\cdot}x)$ $(x{\cdot}x{\cdot}x{\cdot}x)$ $(x{\cdot}x{\cdot}x{\cdot}x)$
-3. Is $6 = 2^{x+1} - 4$ an equation or a function?
-2. Give an example of something that grows exponentially.
-1. What is the strength of a solution that is 12% alcohol?

Question number -4 is to prepare students for the next section that is on the properties of exponents. Question -3 is a reminder of the difference between equations and functions. Topic 7.3 is on solving exponential equations. Question –2 is to start students thinking about applications of the exponential function studied in 7.4. Finally, question -1 is to prime students for the rational function developed in the first section of the next chapter.

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- **Exercise sets contain high-level difficulty problems that review topics previously taught.**

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These six exercises found in every section are usually designed to enhance skills from the previous four sections. It is a simple device that helps students remember skills longer through distributive learning. Below are the exercises from Section 10.1, which is on the quadratic function.

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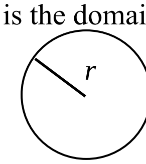
1. The radius of a circle can be found by the function $r = \dfrac{\sqrt{A}}{\sqrt{\pi}}$. This simplifies to $r = 0.5642\sqrt{A}$. Is the function increasing? What is the domain of the function? Find A when r = 10 inches.



2. Imagine a coiled spring hanging from the ceiling, with a mass attached to the spring. If the mass is pulled down a small amount and released, it will bounce up and down and have a period of $T = \dfrac{2\pi}{\sqrt{k}}\sqrt{m}$. The period is T seconds for a mass of m. The value of k is the spring constant and is different for each spring.
3. Suppose the spring constant of a certain spring is 0.01. The function then simplifies to $T = 20\pi\sqrt{m}$, or simply $T = 62.83\sqrt{m}$. If a mass of 7 grams is set in motion, what is its period? What is the period for a 14-gram mass?

3. In the above situation, what mass (in grams) can be hung from the spring to maintain a period of 2 seconds?

4. Solve $-\sqrt{2x-6}+3=0$

5. Solve $\sqrt{x+4}+\sqrt{4-x}=3$

6. Simplify $\left[\left(256^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$

These six exercises are set at the beginning of each exercise set to emphasize the importance of reviewing previous material. Three exercises are from the previous section, two are from the second previous section and one is from the third previous section.

- **Exercise sets contain writing questions. The more modes of teaching we can use, the more likely our students will <u>correctly</u> remember what we teach.**

Every exercise set has writing questions. They can be assigned as part of a regular assignment. They may be used as journal entries. While some writing questions, like those below, are standard to every section. Each section also has a unique set of questions.

60. After reading this section, make a list of questions that you want to ask your instructor.
61. Continue in your daily journal and make an entry. In addition to your normal entry on thoughts about the mathematics in this section, list at least two positive comments about what you have learned about this topic.
62. In paragraph format, summarize the material in this section of the text in your daily journal.
63. Describe how your classroom instructor made this topic more understandable and clear.
64. After reading the text and listening to your instructor, what do you not understand about this topic?

	Below are writing questions specific to Section 14.1. 56. Explain why the base of the logarithmic function cannot be a negative number. 57. In the function in Exercise 53, can $(x+e)$ be a negative number? Why? 58. In the function in Exercise 53, can y be a negative number? Why? 59. What happens when you try to find log (-5) on your calculator? Explain why you think the calculator does what it does.
• **Exercise sets contain exploration problems. These are in activity format in the ancillary book.**	Many of the explorations can be used for group work. Many can be used as portfolio exercises. Below are samples from Section 5.4. Please note that students have not learned about transformation in Chapter Five. 25. From Exercise 14 - the waiter problem - graph the salary model followed by a model that shows a wage of $90 plus tips. Of $100 plus tips. Of $110 plus tips. How are all of the graphs related? From Exercise 14 - the waiter problem - graph the salary model followed by a model that shows a wage of $80 plus tips at a rate of $3.75 per hour. At $4.00 per hour. At $4.25 per hour. At $4.50 per hour. How are the graphs related? 26. From Exercise 12 - the gas-tank problem – graph the gas model followed by a model that shows a tank size of 13 gallon. Of 13.5 gallons. Of 16 gallons. How are all of the graphs related? From Exercise 12 - the gas tank problem – graph the gas model followed by a model that shows a rate of 35 mpg. Of 37 mpg. Of 40 mpg. Of 43 mpg. How are the graphs related? What is the problem domain for each of these models?
• **Exercise sets contain open-ended questions. These are in activity format in the ancillary book.**	Every exercise set contains open-ended questions. They are quite challenging to many students. You may use the questions as take-home quizzes. Below are a few examples from Section 6.3. 30. In the inequality $\lvert x + 3\rvert + \mathbf{?} < 0$, what value can the "**?**" be replaced with so the inequality has no solution? (Hint: There are many answers.) 31. In the inequality $-2\lvert x + 4\rvert + \mathbf{?} > 0$, describe all possible numbers that "**?**" can be replaced with so there is no solution. 32. In the inequality $-2\lvert x + 4\rvert\ \mathbf{?}\ 0$, what inequality symbol can "**?**" be replaced with so the inequality has no solution? 33. Create an equation containing an absolute function that has a solution of -3. 34. Find an equation containing an absolute function that has a solution of -4 and -2. 35. Develop an inequality containing an absolute function that has a solution of $[-4, -2]$. 36. Make an inequality containing an absolute function that has a solution of $(-\infty, -4] \cup [-2, \infty)$. 37. Write any inequality containing an absolute function that has no solution. 38. Create any inequality containing an absolute function that has $(-\infty, \infty)$ as a solution.
• **Extended laboratory projects on modeling are included with the text. These are in**	The modeling projects have been tested over a period of six years and have proved to be extremely valuable, both as an alternative assessment tool and as a means of getting students to talk about mathematics. They have been used by a variety of people at several colleges.

activity format in the ancillary book.	The modeling projects provide you with an opportunity to ask your students to apply the mathematics that is in the text. Mathematics such as using the relationships between the parameters in a function and the behavior of the function, geometric transformations, arithmetic operations of functions and the resulting change in the domain, and behavior near the zeros of functions. Students must recognize the shapes of the basic elementary functions. They must know how arithmetic operations of functions change the geometry of the graphical representation of the functions. Your students must use three different representations of functional relationships found in the data. Each project contains Either the numeric representation or the graphical representation of a functional relationship. The main goal of the modeling projects is to find a good symbolic representation of any function that models the data. Just as an engineer, physicist, business person, or astronomer may solve a problem by following a prescribed procedure, your students are directed through this problem solving process by being asked a series of questions about the data. Once they have solved the main problem and have developed the symbolic representation of the function (mathematical model), they are asked to defend their solution by explaining the limitations of their model. This includes giving a situation when their model does not apply. They must explain their thinking on how they developed the model. Students are given the opportunity to use their model when asked questions about data not available in the given information. They are also asked to conjecture on what type of professional person might use their model. And finally, they must identify the references and resource person(s) used to help them in the problem solving process. Class time may be used to work the projects or they provide a good assignment outside of class. Depending on the level of mathematical sophistication of your students, the projects may take from three to fifteen or so hours. Group work is encouraged. As you move away from evaluating your students by traditional testing methods, the modeling projects should provide a large percent of your assessment of student understanding of mathematics. It is very useful to discuss various student answers during class time. Students also should be given the answers you develop.

e-Activities for *Foundations*
(directions for teachers)

The Texas Instruments (TI) StudyCard app (free calculator software) and companion StudyCard Creator (free computer software) help create and facilitate the e-activities described below. The StudyCard stacks are called e-activities because they are all electronically based; no hard copies are needed. They are provided to you and your students in electronic form, they are executed in electronic form on the calculator, you can edit my work with StudyCard Creator-electronically, and you distribute them to your students electronically via the calculator-to-calculator link cable, or the graph link cable from your computer.

The e-activities have three uses, as a teaching tool – by the teacher as a lesson plan and used during class (like a Power Point presentation), by the student before attending class, and the other use is as a summative assessment. If used by the student, either outside class or during class, it is recommended that students work in a team of two. In addition to the benefits of collaborative work, students also have access to the functionality of the TI-83/84 Plus (or SE). The purpose and type of each activity is marked in the table below. Those marked as summative assessments are used after a particular topic is developed in class and/or through a teaching activity. Those marked as teaching activities are used to help you teach a topic. Use as a teaching tool is described below.

Imagine asking your students a question. You might have 3 – 4 students raise their hands to answer and you will ask one student to respond. So, what you know for sure is that one student knows the answer to your question. But, what about everyone else? Can you assume they all know the answer? Even after the one student tells the answer, do the others understand? We often incorrectly assume that they do. When you use the e-activities, every student must answer all of the questions. Further, they can answer the questions in the e-activities in the privacy of their two-person group. That in its self is a powerful teaching method.

The teaching e-activities consist of a series of questions about a particular topic. They are arranged as in guided discovery learning, or as you might structure a lesson plan. But they are more than just a series of multiple-choice questions. In addition to the responses to the question being carefully selected, the backside of each study card (where you go, after a response is selected) usually contains an explanation of the correct response, or why incorrect responses are incorrect. Sometimes the backside of the card provides information that leads to the next question in the activity, or guides the student in another direction. On occasion, the backside asks them if they used the calculator for help in answering the question. While the e-activity is scored (inside the calculator), the intent of the teaching activities is learning, not assessment. However, since the score is recorded, you may use it as part of your overall assessment if you choose.

Teaching through asking a series of questions is a well-established and successful teaching method. If you choose to use the e-activities as your lesson plan, they will guide your in-class presentation of a mathematical topic. Because you project what is on your calculator screen to the full class, the topic will be at the center of the discussion, just like when you use a Power Point presentation. You may choose, or require your students to follow along with the e-activity on their calculators, so they are engaged too and not just watching your screen. Of course, you may put a question on the screen and ask them to answer it on their calculators. If you use TI Navigator, you would know how every student answered – immediately. If you do not use TI Navigator, you may ask for a show of hands, before moving on to the next question. The direction you take in class depends on how all students respond. After displaying the backside of the card, you may want, or need, to expand on the information before going on to the next question.

If you assign the teaching activities to be executed right before your next class starts, your students will be ready for your classroom lesson plan. This approach is suggested so as to not use valuable class time. On the other hand, the logistics may be too complicated. If this is the case, then you may choose to give your students 10 minutes at the beginning of class to do an activity – perhaps while you take attendance or other housekeeping tasks. The e-activities are not meant to replace the print copy explorations, concept quizzes, investigations, or modeling projects, but rather, to supplement the over-all teaching tools that are significant part of *Foundations for College Mathematics 3e*.

Due to the limitations of the app, proper mathematical notation cannot always be used on the calculator. For example, square root symbols, the symbol for infinity, two-line fractions, etc. Please have your students write e-activity questions with pencil and paper if the notation in the activity is too complicated. Notation used: sqr for square root, inf for infinity, / for division, * for multiplication, belongs to for $\in$, ^ for exponentiation, and log2(x) for $\log_2 x$.

Research in learning with hand-held technology is clear and consistent, if you want the full benefit of the technology, students must have access to the technology both in class and out. Research is also clear and consistent on increased test scores for those using the technology. The point is, that we have provided you with the activities (both the e-activities and the print activities) to assist you in teaching algebra for understanding, but to take advantage of these, students must have access to the technology at all times. Teaching from *Foundations* and its ancillaries means you have a wide variety of educational tools with which to engage students and teach for mathematical understanding. In addition to the text, which was written for students, you also have all the student activities from *Explorations, Concept Quizzes, Investigations, Writing Mathematics, and Modeling Projects for Foundations for College Mathematics.* You now also have 48 StudyCard activities that can be used in three ways.

The study card files are called AppVar files (on your calculator). The free study card app (StudyCrd.8Xk) from Texas Instruments is used to process the AppVars on the calculator. In particular, these AppVars are StudyCard stacks, and the app is the StudyCrd software. These can be used on any TI-83 Plus, TI-83 Plus SE, TI-84 Plus, TI-84 Plus SE calculator. They can be copied to your calculator through TI-Connect. Texas Instruments TI-Connect is free computer software that allows your PC to communicate with your calculator through the USB cable.

All of the stacks are editable through the use of the free computer software StudyCard Creator. It can be found on the Texas Instruments web page.

To execute a stack on the TI-83 Plus (or other), from the app key, select the StudyCard app and then follow the on-screen directions to open a stack. The front or back of each card may often be more than one calculator screen. If it is, the down arrow cursor movement key on the calculator can be used to see the remaining card. Flipping the card from front to back will be considered as missing the question. Cards can be re-played using the left arrow cursor movement key.

Below is a list of the stacks and a discussion of how to use them. These lessons will be available as apps for the iPad, Kindle, Nook, iPhone and Android phones by November, 2012.

Computer file name	Calculator file name (what your students see)	Chapter-Section	Suggestions on when to use	Description
BEHAVABS.8xv	Behaviors-AbsValue	Chapter Three, Section Three; & perhaps Chapter Six, Section Three	As a lead-in to 3.3. As a review before you teach 6.3. If you teach the parameter-behavior connection, use it then too.	A teaching activity for the absolute value function that includes an absolute value data relationship, symbolic form, and absolute value behaviors such as opening up or down, increasing/decreasing, slope of each branch, location of the vertex, maximum, minimum, and range. The emphasis is on making the connection between the function parameters and the resulting geometric behaviors of the absolute value function.
BEHAVEXP.8xv	Behaviors-Exp	Chapter Seven, Section One	As a lead-in to 7.1 – either to students right before (or at the beginning of) class, or as your presentation.	A teaching activity for the exponential function that includes exponential data relationships, symbolic form, and exponential behaviors such as increasing/decreasing, horizontal asymptotes, and range. The emphasis is on making the connection between the function parameters and the resulting geometric behaviors of the exponential function.
BEHAVLIN.8xv	Behaviors-Linear	Chapter Three, Section One; Chapter Five, Section One	As a lead-in to 3.1 or 5.1 – either to students right before (or at the beginning of) class, or as your presentation.	A teaching activity for the linear function that includes linear data relationships, symbolic form, and linear behaviors such as increasing/decreasing, introduction to rate of change, *y*-intercept, and zeros. The emphasis is on making the connection between the function parameters and the resulting geometric behaviors of the linear function.
BEHAVRTN.8xv	Behaviors-Rational	Chapter Eight, Section One	As a lead-in to 8.1 – either to students right before (or at the beginning of) class, or as your presentation.	A teaching activity for rational functions that includes a rational data relationship, symbolic form, and rational behaviors such as increasing/decreasing, domain, range, horizontal and vertical asymptotes, and zeros. The emphasis is on making the connection between the function parameters and the resulting geometric behaviors of the rational function.

BEHAVSQR.8xv	Behaviors-SquareRt	Chapter Three, Section Four; Chapter Nine, Section One	As a lead-in to 3.4 or 9.1 – either to students right before (or at the beginning of) class, or as your presentation.	A teaching activity for square root functions that includes a square root data relationship, symbolic form, and square root behaviors such as increasing/decreasing, domain, range, and maximum/minimums. The emphasis is on making the connection between the function parameters and the resulting geometric behaviors of the square root function.
BEHVQUAD.8xv	Behaviors-Quadrati	Chapter Three, Section Two; Chapter Ten, Section One	As a lead-in to 3.2 or 10.1 – either to students right before (or at the beginning of) class, or as your presentation.	A teaching activity for quadratic functions that includes quadratic data relationships, symbolic form, and quadratic behaviors such as opening up or down, increasing/decreasing, location of the vertex, maximum, minimum, and range. The emphasis is on making the connection between the function parameters and the resulting geometric behaviors of the quadratic function.
CHANGE.8xv	Change	Chapter Two, Section Three; Chapter Five, Section One	As a lead-in to 2.3 or 5.1 – either to students right before (or at the beginning of) class, or as your presentation.	A teaching activity that demonstrates that the constant rate of change idea is present in many situations outside the mathematics classroom.
CREATABS.8xv	CreateAbsoluteValu	Chapter Three, Section Three; & maybe Chapter Six, Section Three	This is a summative assessment and should be used at the end of 3.3 and maybe the end of 6.3.	A summative assessment activity that tests students on the behaviors of the absolute value function. Students are also asked to create their own absolute function that meets the criteria listed in each card. Behaviors addressed are opening up or down, increasing/decreasing, slope of each branch, location of the vertex, maximum, minimum, and range. The emphasis is on assessing knowledge of the connection between the function parameters and the resulting geometric behaviors of the absolute value function. Students are also encouraged to use the graphing calculator to help answer the questions as needed.
CREATEXP.8xv	Create Exponential	Chapter Seven	Use at the end of Chapter Seen.	A summative assessment activity that tests students on the behaviors of the exponential function. Students are also

				asked to create their own exponential function that meets the criteria listed in each card. Behaviors addressed are increasing/decreasing, horizontal asymptotes, and range. The emphasis is on assessing knowledge of the connection between the function parameters and the resulting geometric behaviors of the exponential function. Students are also encouraged to use the graphing calculator to help answer the questions as needed.
CREATLIN.8xv	Create Linear	Chapter Five	Use at the end of Chapter Five.	A summative assessment activity that tests students on the behaviors of the linear function. Students are also asked to create their own linear function that meets the criteria listed in each card. Behaviors addressed are increasing/decreasing, rate of change, and *y*-intercept. The activity emphasis is on assessing knowledge of the connection between the function parameters and the resulting geometric behaviors of the linear function. Students are also encouraged to use the graphing calculator to help answer the questions as needed.
CREATQDR.8xv	Create Quadratic	Chapter Ten or maybe after Chapter Three, Section Two	Use at the end of Chapter Ten.	A summative assessment activity that tests students on the behaviors of the quadratic function. Students are also asked to create their own quadratic function that meets the criteria listed in each card. Behaviors addressed are increasing/decreasing, location of the vertex, maximum, minimum, range, positive/negative, and zeros. The activity emphasis is on assessing knowledge of the connection between the function parameters and the resulting geometric behaviors of the quadratic function. Students are also encouraged to use the graphing calculator to help answer the questions as needed.
CREATRTN.8xv	Create Rational	Chapter Eight	Use at the end of Chapter Eight.	A summative assessment activity that tests students on the behaviors of the rational function. Students are also asked to create their own rational function that meets the criteria listed in

				each card. Behaviors addressed are increasing/decreasing, domain, range, and horizontal and vertical asymptotes. The activity emphasis is on assessing knowledge of the connection between the function parameters and the resulting geometric behaviors of the rational function. Students are also encouraged to use the graphing calculator to help answer the questions as needed.
CREATSQR.8xv	CreateSquareRoot	Chapter Nine or maybe Chapter Three, Section Four	Use at the end of Chapter Nine, but also maybe at the end of 3.4.	A summative assessment activity that tests students on the behaviors of the square root function. Students are also asked to create their own square root function that meets the criteria listed in each card. Behaviors addressed are increasing/decreasing, domain, range, and maximum/minimums. The activity emphasis is on assessing knowledge of the connection between the function parameters and the resulting geometric behaviors of the square root function. Students are also encouraged to use the graphing calculator to help answer the questions as needed.
DOMRANGE.8xv	Domain & Range	Chapter Two, Section Two; Chapter Three, Section One	As a lead-in to 2.2 or 3.1 – either to students right before (or at the beginning of) class, or as your presentation.	This teaching activity uses real-world contexts to teach the concepts of independent and dependent variables, and then domain and range. It includes a couple exercise-type examples at the end.
FACTOR1.8xv	Factoring-#1	Chapter Four, Section Five	Use it before you teach pencil and paper factoring!	A teaching activity that makes the equivalence and zeros connection between, for example, the function $y = (x - 3)(x + 2)$ and $y = x^2 - x - 6$. All quadratics included have a leading coefficient of 1.
FACTOR2.8xv	Factoring-#2	Chapter Four, Section Five, or maybe in section Four	Use it before you teach pencil and paper factoring!	A teaching activity that makes the equivalence and zeros connection between, for example, the function $y = (2x - 3)(5x + 2)$ and $y = 10x^2 - 11x - 6$. All quadratics included have a leading coefficient of something other than 1.

FACTOR3.8xv	Factoring-#3	Chapter Four, Section Five, or maybe in section Four	Use it before you teach pencil and paper factoring!	A teaching activity that makes the argument that if you know the zeros of, for example, $y = (2x - 1)(x + 3)$, (whether through the parameter-zero connection or using a graphing calculator), and you know the equivalence to $y = 2x^2 + 5x - 3$, then if you start with a function like $y = 2x^2 + 5x - 3$ and find the zeros using a graphing calculator, you know it is equivalent to $y = (2x - 1)(x + 3)$. Thus, students can now factor with the graph of the function.
FUNCTION.8xv	FunctionNotation	Chapter Four, Section One	As a lead-in to 4.1 – either to students right before (or at the beginning of) class, or as your lesson.	A teaching activity to help understand the meaning of the notation $f(x)$. In addition, cards also address finding, for example, $f(2)$ given $f(x)$, and the connection to the point on the graph of $f(x)$. The graphing calculator is used extensively.
INTERVAL.8xv	IntervalNotation	Chapter One, Section Three	As a lead-in to 1.3 – either to students right before (or at the beginning of) class, or as your lesson.	A teaching activity on understanding interval notation. It uses functions and function behaviors as the context for needing and using interval notation.
LAWSEXPO.8xv	LawsOfExponents	Chapter Seven, Section Two; or maybe parts in Chapter One, Section Four	As a lead-in to 7.2 – either to students right before (or at the beginning of) class, or as your presentation.	This activity is not on the laws of exponents, but rather, on the definitions of Natural number exponents, zero, and negative integer exponents. The activity uses student's understanding of behaviors of functions. In particular, students need to know the connection between the symbolic form and the graphical form of basic functions. The stack is instructive in nature. The title is used to associate it with the laws of exponents.
LAWSEXI.8xv	LawsOfExponentsI	Chapter Seven, Section Two; or maybe parts in Chapter One, Section Four	As a lead-in to 7.2 – either to students right before (or at the beginning of) class, or as your presentation.	This is a teaching activity on the First Law of Exponents and is based in the connection between functions in symbolic and graphic forms. This is to say that the first law is discovered by students through the use of multiplication of functions – comparing graphical representations with symbolic forms. Later confirmation of products is accomplished through comparing

				numeric representations of the problem and student answers.
LAWSEXII.8xv	LawsOfExponentsII	Chapter Seven, Section Two	As a lead-in to 7.2 – either to students right before (or at the beginning of) class, or as your presentation.	This is a teaching activity on the Second Law of Exponents and is based in the connection between functions in symbolic and graphic forms. This is to say that the second law is discovered by students through the use of division of functions – comparing graphical representations with symbolic forms.
LAWEXIII.8xv	LawsOfExponentsIII	Chapter Seven, Section Two	As a lead-in to 7.2 – either to students right before (or at the beginning of) class, or as your presentation.	This is a teaching activity on the Third Law of Exponents and is based in the connection between functions in symbolic and graphic forms. This is to say that the third law is discovered by students through the use of raising power functions to powers – comparing graphical representations with symbolic forms. Later confirmation of powers of powers is accomplished through comparing numeric representations of the problem and student answers.
MAXMINID.8xv	Max/Min & Inc/Dec	Chapter Two, Section Three, but it could be used in Chapter Three, 5.1, 7.1, 8.1, 9.1 and 10.1 as a review of the behaviors	As a lead-in to 2.3 – either to students right before (or at the beginning of) class, or as your presentation.	This is a teaching activity that uses real-world contexts to assist students in understanding the concepts of maximum, minimum, increasing, and decreasing.
POLYADD.8xv	PolynomialAddSubt	Chapter Four, Section Two	As a lead-in to 4.2 – either to students right before (or at the beginning of) class, or as your presentation.	Addition and subtraction of polynomials is based in the context of discovering that when linear functions are added, the slope of the sum is equal to the sum of the slopes, likewise for *y*-intercepts. Once students discover how to add (or subtract) linear polynomials (in the form of polynomial functions) the activity moves on to quadratic and up polynomials. Confirmation of sums or differences is accomplished through comparing numeric representations of the problem and the student answer.

POLYMULT.8xv	PolynomialMult	Chapter Four, Section Three	As a lead-in to 4.3 – either to students right before (or at the beginning of) class, or as your presentation.	Students discover that the product of linear functions (polynomial) is usually quadratic. They then move on to discovering the exact product. This leads to the more traditional methods for multiplying polynomials.
RATNLADD.8xv	RationalAddSubt	Chapter Eight, Section Four	As a lead-in to 8.4 – either to students right before (or at the beginning of) class, or as your presentation.	The activity is based in arithmetic to convince students they cannot add fractions by adding numerators and denominators. Further discovery focuses on adding (or subtracting) numerators with examples from simple to complex denominators. The teaching activity ends with sums or differences of rational functions with different denominators. Confirmation of sums or differences is accomplished through comparing numeric representations of the problem and student answers. A discussion of domain is included.
RATNLMUL.8xv	RationalMultiply	Chapter Eight, Section Three	As a lead-in to 8.3 – either to students right before (or at the beginning of) class, or as your presentation.	Rational expression multiplication and division algorithms are based in arithmetic, but then progress to function operations. Guided discovery concepts are used to develop ideas for the operations. Confirmation of products or quotients is accomplished through comparing numeric representations of the problem and student answers. A discussion of domain is included.
RATNLRED.8xv	RationalReduce	Chapter Right, Section Two	As a lead-in to 8.2 – either to students right before (or at the beginning of) class, or as your presentation.	This is a guided discovery teaching activity that starts with arithmetic examples and continues through algebra. The activity uses student's understanding of behaviors of functions. In particular, students need to know the connection between the symbolic form and the graphical form of basic functions. Confirmation of the correctness of the reduced form is accomplished through comparing numeric representations of the problem and student answers. A discussion of domain is included.
REALPROP.8xv	Real # Properties	Chapter One, Section One	As a lead-in to 1.1 – either to students right before (or at the beginning of) class, or as your presentation.	A teaching activity on understanding the commutative properties, associative properties and the distributive property. Examples from arithmetic are used to lead to the properties.

SCIENTIF.8xv	ScientificNotation	Chapter One, Section Four	As a lead-in to 1.4 – either to students right before (or at the beginning of) class, or as your presentation.	The case is built for the need of scientific notation by using very large numbers (and numbers near zero) from real-world contexts. Further, a guided discovery argument is use to get the students to the proper form. This is followed by examples.
SCIENTOP.8xv	SciTifOperations	Chapter One, Section Four	As a lead-in to 1.4 – either to students right before (or at the beginning of) class, or as your presentation.	The four basic operations are discussed in guided discovery fashion. Understanding is assessed by reversing the problem – given the answer to a (× or ÷), find two numbers.
SLOPE.8xv	Slope	Chapter Five, Section One	As a lead-in to 5.1 – either to students right before (or at the beginning of) class, or as your presentation.	A teaching activity on understanding the meaning of slope as well as the evaluation of slope using $\Delta y/\Delta x$ definition. The activity starts with the function concept of increasing & decreasing and this leads to "how fast" is it increasing or decreasing. A connection is made to the linear function parameter that controls slope. It is somewhat more traditional in nature.
SOLVEABS.8xv	SolveAbsValueEqn	Chapter Six, Section Three	As a lead-in to 6.3 – either to students right before (or at the beginning of) class, or as your presentation.	This teaching activity begins with "what is an equation?" and continues with a meaning of solving an equation. The initial method discussed is trace, as related to the absolute value function. That is, the function approach is used to solve the equations. An understanding of absolute value function behaviors is required.
SOLVEEXP.8xv	SolveExponentialEq	Chapter Seven, Section Three	As a lead-in to 7.3 – either to students right before (or at the beginning of) class, or as your presentation.	This teaching activity begins with "what is an equation?" and continues with a meaning of solving an equation. The function approach is used to solve the equations and the exponential function behaviors are needed to help answer some questions.
SOLVELIN.8xv	SolveLinearEquatio	Chapter Six, Section One	As a lead-in to 6.1 – either to students right before (or at the beginning of) class, or as your presentation.	This teaching activity begins with "what is an equation?" and continues with a meaning of solving an equation. A discovery approach is used to learn how to solve linear equations using the graph of the related linear function(s). The trace and zeros methods are developed – with emphasis on the connection between roots and zeros of the related function.

SOLVELOG.8xv	SolveLogEquations	Chapter Fourteen, Section Three	As a lead-in to 14.3 – either to students right before (or at the beginning of) class, or as your lesson.	This teaching activity begins with "what is an equation?" and continues with a meaning of solving an equation. The activity continues by developing the connection between points on the graph of the related function and a solution to an equation. Examples follow.
SOLVERTN.8xv	SolveRationalEqn	Chapter Eight, Section Five	As a lead-in to 8.5 – either to students right before (or at the beginning of) class, or as your lesson.	This teaching activity begins with "what is an equation?" and continues with a meaning of solving an equation. The activity continues by developing the connection between points on the graph of the related function and a solution to an equation. Examples follow.
SOLVESQR.8xv	SolveSqrRootEqn	Chapter Nine, Section Five	As a lead-in to 9.5 – either to students right before (or at the beginning of) class, or as your presentation.	This teaching activity begins with "what is an equation?" and continues with a meaning of solving an equation. The activity continues by developing the connection between points on the graph of the related function and a solution to an equation. Examples follow, and extraneous roots are discussed.
SOLVINEQ.8xv	SolveInequalities	Chapter Six, Section Two	As a lead-in to 6.2 – either to students right before (or at the beginning of) class, or as your presentation.	The teaching activity starts with a discussion of the meaning of a solution to an inequality. A graphical method is assumed. The lesson ends with several examples.
SOLVQUAD.8xv	SolveQuadraticEqn	Chapter Ten, Section Two	As a lead-in to 10.2 – either to students right before (or at the beginning of) class, or as your presentation.	This teaching activity begins with "what is an equation?" and continues with a meaning of solving an equation. The activity focuses on "understanding" the solving process and the solution. Examples follow.
SQRADSUB.8xv	SqrRootAddSubtract	Chapter Nine, Section Three	As a lead-in to 9.3 – either to students right before (or at the beginning of) class, or as your presentation.	A direct connection is made from square root functions to adding and subtracting square root expressions. The teaching activity is based on the discovery approach and uses students' knowledge of function behaviors. The activity ends with several examples.
SQRMULT.8xv	SqrMultiplication	Chapter Nine, Section Three	As a lead-in to 9.3 – either to students right	A brief discussion of the product property is followed by several "instructive" examples. Functions are

			before (or at the beginning of) class, or as your presentation.	used in early examples because students can use the calculator to confirm statements made.
SQRSIMP.8xv	SqrRootSimplifying	Chapter Nine, Section Two	As a lead-in to 9.2 – either to students right before (or at the beginning of) class, or as your presentation.	This is a teaching activity that starts by developing a need for simplification of square roots. Both numbers with perfect square factors and fractions are included in the sample exercises. It ends with finding square roots of numbers squared – leading to the square root of $\sqrt{x^2}$ being $\|x\|$.
VARDIRCT.8xv	Variation-Direct	Chapter Six, Section Four	As a lead-in to 6.4 – either to students right before (or at the beginning of) class, or as your presentation.	A teaching activity that connects the linear function $y = mx$ to the more traditional direct variation concepts. There are examples with explanations.
VARINVAR.8xv	Variation-Inverse	Chapter Eight, Section Five	As a lead-in to 8.5 – either to students right before (or at the beginning of) class, or as your presentation.	This is a teaching activity that uses real-world contexts to teach the idea of inverse variation. Inverse variation is then connected to the rational function and it ends with an example.
ZEROSPN.8xv	Zero's, Pos & Neg	Chapter Two, Section Three, or Chapter Three, Section One	As a lead-in to 2.3 – either to students right before (or at the beginning of) class, or as your presentation. May be reused in 3.1	In this teaching activity, several situations are included to teach the concepts of zeros, and when functions are positive or negative.