More Curve Sketching

Pg 256 # 24

$$g(x) = x\sqrt{9-x^2}$$

x & y intercepts $x=0$ $y=0$

$y=0$ $x=0, \pm 3$

The graph is symmetric in the origin

since $g(-x) = -x\sqrt{9-x^2} = -g(x)$

Domain $D = \{x \mid -3 \le x \le 3\}$

Range – TBD

The function is cont's on $[-3,3]$

No VA or HA

Diff – we need the derivative here

$$g'(x) = 1\sqrt{9-x^2} + x\frac{(-2x)}{2\sqrt{9-x^2}}$$

$$= \sqrt{9-x^2} - \frac{x^2}{\sqrt{9-x^2}} = \frac{9-x^2-x^2}{\sqrt{9-x^2}}$$

$$= \frac{9-2x^2}{\sqrt{9-x^2}}$$

$g'=0$ when $x = \pm\frac{3}{\sqrt{2}}$ & DNE at $x = \pm 3$

$= \pm\frac{3\sqrt{2}}{2} = \pm 2.1213$ (inside interval)

$$g'' = \frac{x(2x^2-27)}{(9-x^2)^{3/2}}$$

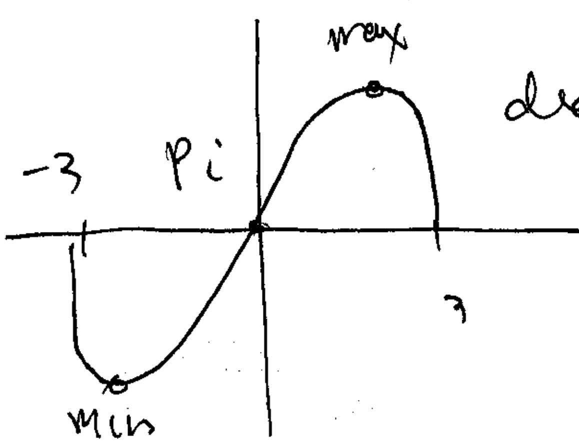
$g''=0$ $x=0$ $x = \pm\frac{3\sqrt{3}}{\sqrt{2}} = \pm\frac{3\sqrt{6}}{2} = \pm 3.674\ldots$ which is outside on interval

so $x=0$ is the only possible inflection pt

Sign chart

x	-3		$-\frac{3\sqrt{2}}{2}$		0		$\frac{3\sqrt{2}}{2}$		3
$9-2x^2$	−	−	0	+	+	+	0	−	−
$\sqrt{9-x^2}$	0	+	+	+	+	+	+	+	0
$\frac{9-2x^2}{\sqrt{9-x^2}}$	∞	−	0	+	+	+	0	−	∞
slope	|	\	_	/	/	/	‾	\	|
x	−	−	−	−	0	+	+	+	+
$2x^2-27$	−	−	−	−	−	−	−	−	−
$\frac{x(2x^2-27)}{(9-x^2)^{3/2}}$	∞	+	+	+		−	−	−	∞
c/v					Pi				

increasing $\left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$



dec. $\left(-3, -\frac{3\sqrt{2}}{2}\right)$ $\left(\frac{3\sqrt{2}}{2}, 3\right)$

concave ↑ $(-3, 0)$

concave ↓ $(0, 3)$

min $\left(-\frac{3\sqrt{2}}{2}, -9/2\right)$

max $\left(\frac{3\sqrt{2}}{2}, 9/2\right)$

Pi$(0,0)$

pg 250 # 48

$$y = \frac{x^3}{24} - \ln x, \quad x > 0$$

$x = 0$ is a VA no HA

intercept - max y - we will see.

Extrema

$$y' = \frac{3x^2}{24} - \frac{1}{x} = \frac{x^3 - 8}{8x}$$

$y' = 0$ when $x = 2$
y' DNE at $x = 0$
(VA)

Note $x^3 - 8 = (x-2)(x^2+2x+4)$

$$y'' = \frac{x}{4} + \frac{1}{x^2} = \frac{x^3+4}{x^2}$$

$y'' \neq 0$ so no PI also for $x > 0$ $y'' > 0$

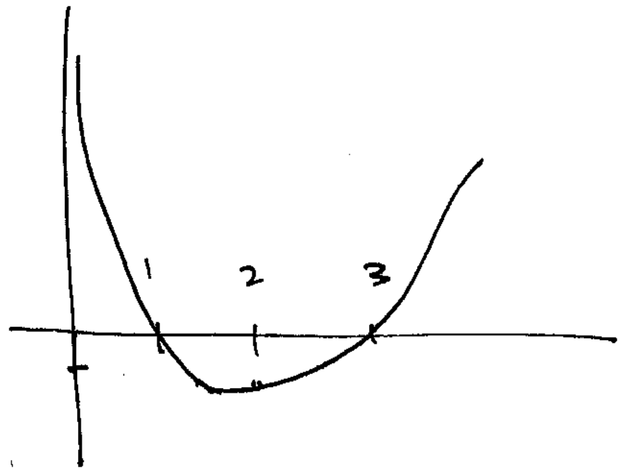
so always concave ↑

x		2	
$x-2$	$-$	0	$+$
y'	$-$	0	$+$
slope	\	—	/

so $x = 2$ is a min

$x = 2$ $y = \frac{8}{24} - \ln 2 = \frac{1}{3} - \ln 2$

$= -.3598$



So there are 2 places where the function = 0

x = 1.0493, 2.9660 found numerically