TeachSpin's Quantum Analogs
A Conceptual Introduction to the Experiment

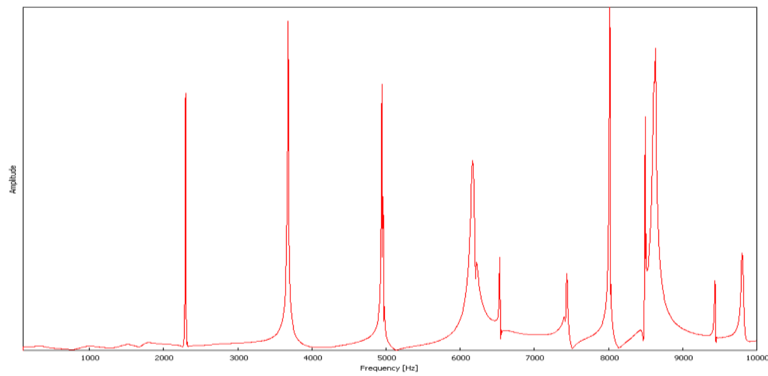
Given the importance and applicability of the one-particle Schrödinger Equation to so many problems in quantum physics, it is worth thinking about the process by which physicists gain intuition about the solution of this equation. Students need to see beyond the mathematical details of partial differential equations to some physical concepts, such as eigenvalue and eigenfunction, which might be entirely novel to them. TeachSpin's apparatus, Quantum Analogs, is intended to assist in this process of 'intuition formation', by exploiting a powerful mathematical analogy between the behavior of matter waves in various potentials, and sound waves in air inside various resonant structures.

The eigenstates of the Schrödinger Equation solve a particular differential equation, subject to certain boundary conditions, with simple harmonic time dependence, and with a characteristic spatial dependence called an 'eigenstate'. Happily, ordinary sound waves in air, resonant inside a confined structure, *also* solve a (similarly-structured) partial differential equation, subject to analogous boundary conditions, with sinusoidal time dependence, and a characteristic spatial dependence called a 'normal mode'. The theoretical basis of the analogy is carefully set out in the manual and leads smoothly into the experimental activities.

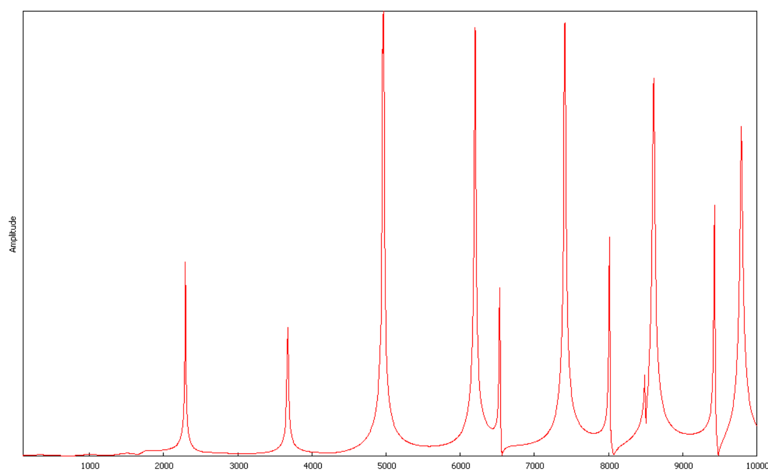
The Quantum Analogs set-up makes it possible to explore the acoustic side of this analogy in considerable detail for resonant structures of two general topologies -- spherical resonators, and one-dimensional spatially-periodic resonators. All the experiments done with this apparatus occur at ordinary audio frequencies (1-10 kHz), and all involve quite ordinary (though miniaturized) audio loudspeakers and microphones to create and to sample the acoustic wave field. Because of the importance of building intuition about resonant frequencies and resonant modes, the system also comes with software tools that make it easy to perform scans in frequency, using any computer's sound card for driving the speaker and receiving the microphone signals. The same signals can be monitored with an oscilloscope.

In the topology of **spherical resonators**, you'll see a pair-of-hemispheres that, together, defines a high-quality spherical volume of air (or other gas). A speaker and a microphone are positioned to lie on the sphere's surface, at a relative angle in space, θ, that can be varied continuously from 90° to 180°.

For any relative angle, a frequency scan will reveal a succession of 'resonant modes' of the acoustic wavefield in the sphere. A repeat of such a scan, at a new setting of the relative angle of speaker and microphone, reveals the *same* list of characteristic frequencies (pointing to the excitation of the same resonant modes), but with a *different* set of relative intensities.



Scan at Scale Reading α = 0, Polar Angle θ = 90°

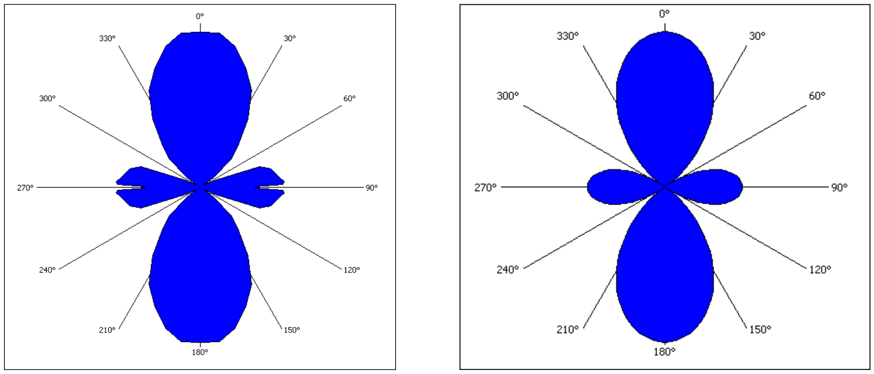


Scan at Scale Reading α = 180, Polar Angle θ = 180°

Figure 1: Received sound amplitude as a function of frequency, for two locations of the microphone.

'Parking' the excitation frequency at the value appropriate to one particular resonant mode then allows a different kind of 'scan': this time, over the angular dependence of the acoustic resonant mode. By this technique, the angular structure of the acoustic mode can be mapped, and compared to the structure predicted from the partial differential equation. The same 'spherical harmonics' Y_{lm} crucial to the Schrödinger Equation in cases of spherical symmetry *also* emerge in this spherically-symmetric acoustical problem.

The computer program supplied with the apparatus converts the scale reading to the appropriate polar angle and then plots the amplitude of the sound signal on a polar grid. Below we see a comparison of a polar plot of acoustical amplitude made with Quantum Analogs and the computed spherical harmonics for the equivalent eigenstate.



Wavefunction of the peak near 3700Hz (left) and computed spherical harmonics for $l = 2$, m = 0 (right)

Figure 2: Comparison of experimental and theoretical polar plots.

It is instructive to see how the Quantum Analogs apparatus accomplishes its task.

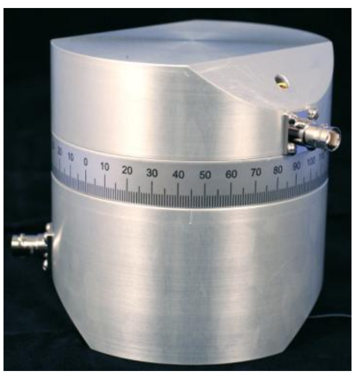


Figure 3: "Atom" analog

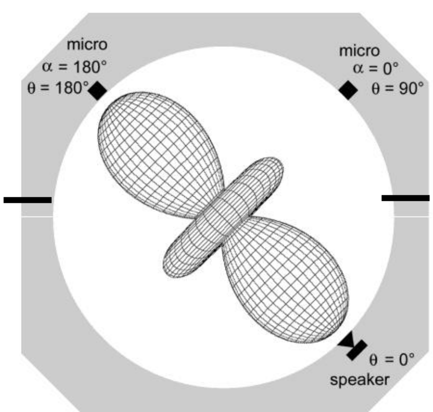


Figure 4: Polar plot of sound amplitude inside the sphere indicating the orientation of an eigenstate

The precisely machined spherical cavity is within the aluminum apparatus in Figure 3. The BNC connectors indicate the location of microphones. Most explorations use the microphone in the upper section. A scale is mounted at the intersection of the upper and lower hemispheres. The speaker is located at the lower right, as shown in Figure 4. Figure 4 also shows a plot of the spherical harmonic within the sphere for a particular harmonic frequency.

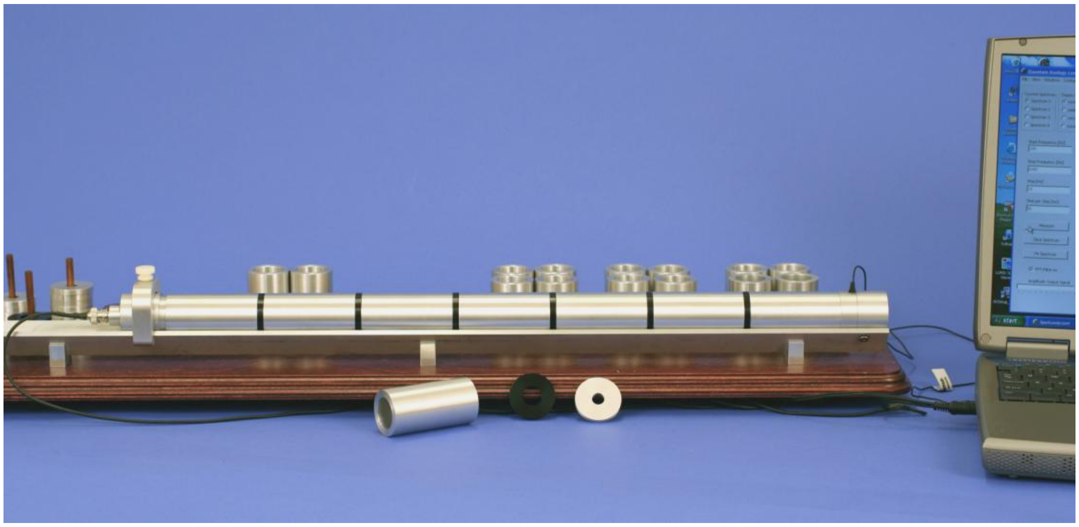
The location of the speaker determines the axis of symmetry and the polar angles, θ, are measured with reference to the speaker itself. The wavefunction for one possible resonance is shown aligned along the speaker axis.

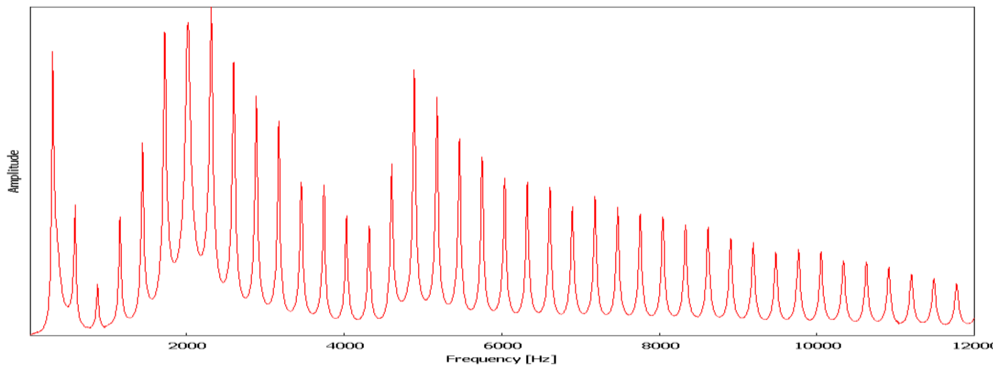
The system is configured so that when the reading on the scale, α, is 0, the actual angle θ between the speaker and microphone is 90°. The upper half of the system is rotated, the scale reading, α, goes from 0° to180° and the microphone moves from the upper right to the upper left in the drawing. With the microphone at the upper left, the angle between speaker and microphone, θ, is 180°.

In the topology of **one-dimensional spatially-periodic structures**, you'll be able to see the acoustic analogy for the important Kronig-Penney model for the emergence of electronic band structure in crystalline solids. Here the starting point is an 'organ pipe' structure constructed out of sections of hollow tube, and two end plates, one bearing the speaker and the other the microphone.

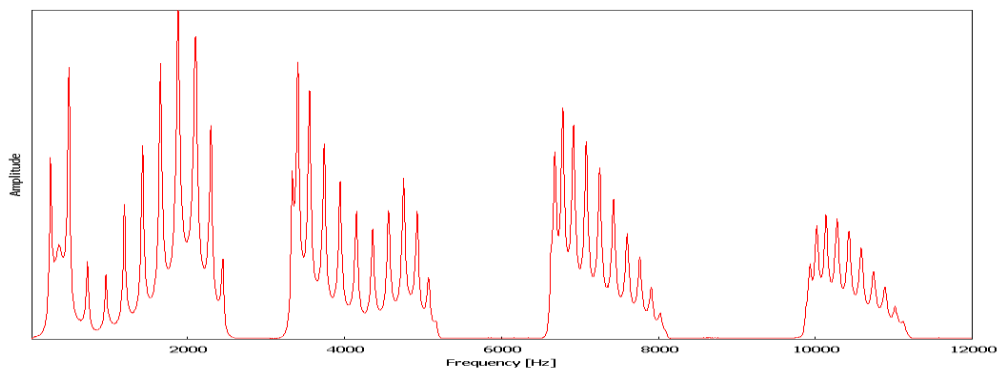
The same computer-based frequency-scanning system will display a whole series of resonant modes, evenly spaced in frequency.

Now the 1-d organ-pipe structure can be perturbed, by interspersing 'irises', i.e. smaller-diameter constrictions, at a series of equally-spaced locations along the pipe. Clearly, these irises will interact strongly with sound waves whose wavelengths lie near the inter-iris spacing. And a frequency scan over the 'mode structure' of this now spatially-periodic structure does indeed reveal this interaction. The resonant modes, formerly evenly spaced in frequency, are now concentrated instead into 'bands' in frequency space, and eliminated from 'gaps' in the frequency spectrum.





Spectrum measured in a tube made from 12 pieces each 50 mm long without irises.



Spectrum measured in a tube made from 12 pieces each 50 mm long and 11 irises which have a diameter of 16 mm.

It's easy to find the three relevant independent variables (iris spacing, iris diameter, and number of repetitions), and to see by hands-on experiment how they are connected to the details of the band/gap structure. In particular, it's possible to vary independently the frequency locations of the band's centers, the frequency width of the bands, and the number of modes per band, and to see which of these three dependent variables is affected by which choice of those three independent variables. And, of course, you can see what happens when you design more complicated cells by combining a repeating series made up of differently sized cylinders and irises or even create a 'defect' by disturbing the symmetry! The explorations are limited only by your imagination and the time you are allowed for working with the apparatus. One thing we guarantee, it will be hard to pass this experiment on to the next student willingly.