Math 2471 – Calc 3   ST2 – V2

1. $\bar{F} = \langle 2xy + 3z^3, x^2+1, 9xz^2 \rangle$

we check for conserivativeness

$$\bar{\nabla} \times \bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy+3z^3 & x^2+1 & 9xz^2 \end{vmatrix} = \langle 0, -(9z^2-9z^2), 2x-2x \rangle$$

$= \vec{0}$ so yes ✓

so ϕ exists such that

$$\phi_x = 2xy + 3z^3 \Rightarrow \phi = x^2y + 3xz^3 + A(y,z)$$

$$\phi_y = x^2+1 \Rightarrow \phi = x^2y + y + B(x,z)$$

$$\phi_z = 9xz^2 \Rightarrow \phi = 3xz^3 + C(x,y)$$

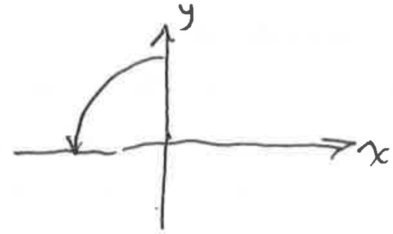
So $\phi = x^2y + y + 3xz^3$ (no constant)

so $\int_c (2xy+3z^3)\,dx + (x^2+1)\,dy + 9xz^2\,dz$

$$= x^2y + y + 3xz^3 \Big|_{(0,0,0)}^{(1,-1,1)} = (1)^2(-1) - 1 + 3(1)(1)^3$$

$= 1$

#2 Evaluate $\int_C xy\,ds$ where C is ccw along $x^2+y^2=4$ from $(0,2)\to(-2,0)$



we parametrize the curve C

$x=2\cos t,\ y=2\sin t$

so $t: \pi/2 \to \pi$

$\frac{dx}{dt}=-2\sin t,\quad \frac{dy}{dt}=2\cos t$ so $ds=\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

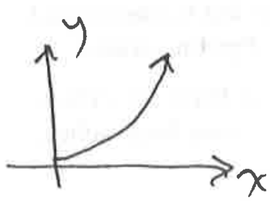
$=\sqrt{4\sin^2 t+4\cos^2 t}\,dt$

the line integral becomes $= 2dt$

$$\int_{\pi/2}^{\pi} 2\cos t\, 2\sin t\, 2dt = 8\int_{\pi/2}^{\pi} \sin t\cos t\,dt = 4\sin^2 t\Big|_{\pi/2}^{\pi}$$

$$= -4$$

#3 Evaluate $\int_C dx+(x+y)dy$ where C is $y=x^2$ $(0,0)\to(1,1)$



if $y=x^2$ then $dy=2x\,dx$

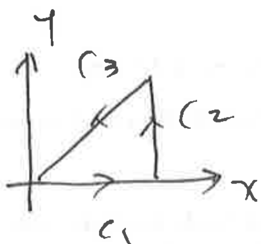
so $\int_0^1 1\,dx+(x+x^2)(2x)\,dx$

$$=\int_0^1 (1+2x^2+2x^3)dx = x+\frac{2x^3}{3}+\frac{x^4}{2}\Big|_0^1$$

$$= 1+\frac{2}{3}+\frac{1}{2}-(0)=\frac{13}{6}$$

#4 Verify Green's Th^{m} if $\bar{F} = \langle x^2+2y, x^2 \rangle$

$$\int_C P\,dx + Q\,dy = \iint_R (Q_x - P_y)\,dA$$ & R is bound by $y=0,\ x=1,\ y=x$ in Q I



$\underline{C_1}$: $y=0,\ dy=0 \quad x: 0 \to 1$

so $\displaystyle\int_{C_1} (x^2+2y)\,dx + x^2\,dy = \int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$

$\underline{C_2}$: $x=1,\ dx=0$ so $y: 0 \to 1$ &

$$\int_0^1 dy = y\Big|_0^1 = 1$$

$\underline{C_3}$: $y=x$ so $dy=dx \quad x: 1 \to 0$ so line $\int$ is

$$\int_1^0 (x^2+2x)\,dx + x^2\,dx = -\int_0^1 (2x^2+2x)\,dx = -\left(\frac{2x^3}{3} + x^2\Big|_0^1\right)$$

$$= -1 - 2/3$$

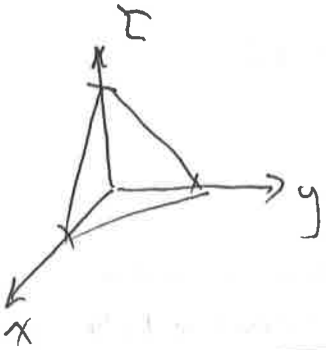
so $\displaystyle\int_C P\,dx + Q\,dy = \frac{1}{3} + 1 - 1 - 2/3 = -1/3 \;/\!/$

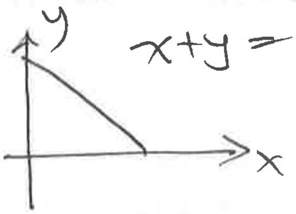
$\underline{\text{Part 2}}$ $\quad Q_x - P_y = 2x - 2$

$$\iint_R (2x-2)\,dA = \int_0^1 \int_0^x (2x-2)\,dy\,dx = \int_0^1 (2xy - 2y)\Big|_0^x = \int_0^1 (2x^2 - 2x)\,dx$$

$$= \frac{2x^3}{3} - x^2\Big|_0^1 = \frac{2}{3} - 1 = -1/3 \;/\!/$$ Same!

\# 5 Evaluate $\iint_S z\,dS$ where S: $2x+2y+z=2$ Q1





$x+y=1$

$z = 2-2x-2y = f$

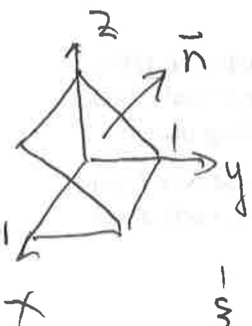
$f_x=-2,\ f_y=-2$

$dS = \sqrt{1+f_x^2+f_y^2}\,dA = \sqrt{1+(-2)^2+(-2)^2}\,dA = 3\,dA$

$\int_0^1\int_0^{1-x}(2-2x-2y)\,dy\,dx = \int_0^1 (2y-2xy-y^2)\Big|_0^{1-x} dx$

$= \int_0^1 2(1-x)-2x(1-x)-(1-x)^2\,dx = \int_0^1 (1-2x+x^2)\,dx = 1/3$

\#6 Find the flux $\iint_S \bar F\cdot\hat n\,dS$ through $y+z=1$



$z=1-y=f$ $\quad f_x=0,\ f_y=-1$ so

$\bar n = \langle -f_x,-f_y,1\rangle = \langle 0,1,1\rangle$

$\therefore\ \hat n = \frac{\langle 0,1,1\rangle}{\sqrt2}$ $\quad dS=\sqrt{1+f_x^2+f_y^2}\,dA=\sqrt2\,dA$

$\bar F\cdot\hat n\,dS = \langle y,z,z^2\rangle\cdot\frac{\langle0,1,1\rangle}{\sqrt2}\sqrt2\,dA = (z+z^2)\,dA$

$\therefore\ \iint_S \bar F\cdot\hat n\,dS = \int_0^1\int_0^1 (1-y)+(1-y)^2\,dy\,dx = \int_0^1\int_0^1 (2-3y+y^2)\,dy\,dx$

$= \int_0^1 2y-\frac32 y^2+\frac{y^3}{3}\Big|_0^1 dx = \int_0^1 \frac56\,dx = \frac56$