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A multi-scale grid strategy in time domain electromagnetic modelling

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SUMMARY

The finite difference time domain (FDTD) method is versatile in time domain electromagnetic modelling. Its Yee cell simulate the progress of electromagnetic in space domain and time domain with alternate sampling in the earth. However, the Yee cell requires a cube or cuboid grid to discrete the Maxwell equation. Although the non-uniform meshing can enlarge the grid size in modelling to form a larger computation area, the side length ratio for each cell should be controlled in a limited number because of the stability condition and computation errors. We present a multi-scale grid strategy to solve this problem. In the space domain, a coarse grid is established firstly to simulate the background homogeneously or in-homogeneously. A fine grid is then set only in the focused area inside a grid. The fine grid can be set as several levels to simulate very small targets. In the time stepping, there are also two sets of calculation progress. A large time step is used to simulate the electromagnetic progression in coarse grids while a small time step inside the large time step is used to simulate the fine grids. For the boundary grid of the coarse and fine area, the electromagnetic data are transferred by updating the electric field in the coarse grid. This strategy will reduce the time cost as the fine grids are limited. Moreover, some small articles such as UXO shell, steel casing or metallic ore vein etc. and the strategy can be applied in three dimensional footprint inversion of time domain electromagnetic data.

Keywords: forward modeling, finite difference time domain (FDTD), multi-scale grid, time domain electromagnetic

INTRODUCTION

The finite difference time domain (FDTD) method is widely applied in time domain electromagnetic modelling and inversion. The first attempt in two dimensions is described by Oristaglio & Hohmann (1984) to solve a diffusion equation with Du Fort-Frankel finite-difference scheme. After that, Adhidjaja & Hohmann (1989) tried but failed to solve a 2nd order equation derived from the Maxwell's equation in three dimensions. Then, Wang & Hohmann (1993) succeed to establish the three dimensional algorithm to solve the Maxwell equation directly in time domain for transient electromagnetic modelling with a staggered-grid technique. They use a 1st order Maxwell equation. After that, many research use FDTD in time domain electromagnetic modelling. In this extended abstract, we present a multi-scale grid strategy in time domain electromagnetic modelling based on our previous research with normal FDTD. We test the accuracy with other methods in literature.

METHODOLOGY

Govering equation and multi-sacle grid discretation

The Maxwell equation is the basic formulation of the electromagnetic induction in the earth as follows,

$$\begin{cases} \nabla\times H = \dfrac{\partial D}{\partial t} + J \\ \nabla\times E = -\dfrac{\partial B}{\partial t} \\ \nabla\times E = 0 \\ \nabla\times H = 0 \end{cases} \tag{1}$$

where **E** and **H** are the electric field intensity and magnetic field intensity, respectively, **B** is magnetic induction, D is the Electric displacement vector, J is the source current density, and t is time.

We take the Ex as an example, the finite difference time domain stepping equation for the normal Yee grids is as follows,

$$E_x^{n+1}(i+\frac{1}{2},j,k)=\frac{2\gamma-\sigma\Delta t}{2\gamma+\sigma\Delta t}\cdot E_x^{n}(i+\frac{1}{2},j,k)+\frac{2\Delta t}{2\gamma+\sigma\Delta t}\cdot \left[\frac{H_z^{n+1/2}(i+\frac{1}{2},j+\frac{1}{2},k)-H_z^{n+1/2}(i+\frac{1}{2},j-\frac{1}{2},k)}{\Delta y}-\frac{H_y^{n+1/2}(i+\frac{1}{2},j,k+\frac{1}{2})-H_y^{n+1/2}(i+\frac{1}{2},j,k-\frac{1}{2})}{\Delta z}\right] -\frac{2\Delta t}{2\gamma+\sigma\Delta t}J_s^{\,n+1/2}, \tag{2}$$

In the fine grids, as shown in Figure 1, there exist another meshed Yee grids inside a coarse grid to establish a fine grids. We define a coefficient n_f to describe the number of time steps for the computation of fine grinds inside a coarse. The electromagnetic stepping equation is very similar to the coarse one as follows,

$$Ex^{n+1}(i+\frac{1}{2},j,k)=\frac{2n_f\gamma-\sigma(i+\frac{1}{2},j,k)\Delta t}{2n_f\gamma+\sigma(i+\frac{1}{2},j,k)\Delta t}Ex^{n}(i+\frac{1}{2},j,k) \quad (3)$$

$$+\frac{2\Delta t}{2n_f\gamma+\sigma(i+\frac{1}{2},j,k)\Delta t}\bullet\left[\frac{Hz^{n+\frac{1}{2}}(i+\frac{1}{2},j+\frac{1}{2},k)-Hz^{n+\frac{1}{2}}(i+\frac{1}{2},j-\frac{1}{2},k)}{\Delta y}\right.$$

$$\left.-\frac{Hy^{n+\frac{1}{2}}(i+\frac{1}{2},j,k+\frac{1}{2})-Hy^{n+\frac{1}{2}}(i+\frac{1}{2},j,k-\frac{1}{2})}{\Delta z}\right]$$

Inside boundary and fine grid meshing

Take a one coarse with four fine grids as an example in Figure 1, the electromagnetic field of the fine grid will sample at the inside boundary edge, eg. E_{11}. For this problem, the homogeneous damped wave equation is satisfied for the electromagnetic induction in the inside boundary. Hence we derive the electromagnetic exchange equation between the coarse and fine grids in the inside boundary area.

The balck circle corresponds to the sampling nodes in coarse area. For the fine area in space, the hallow circle corresponds to the added sampling nodes. The sampling nodes in the overlapping parts between coarse and fine parts are also added to the fine grids sampling space.

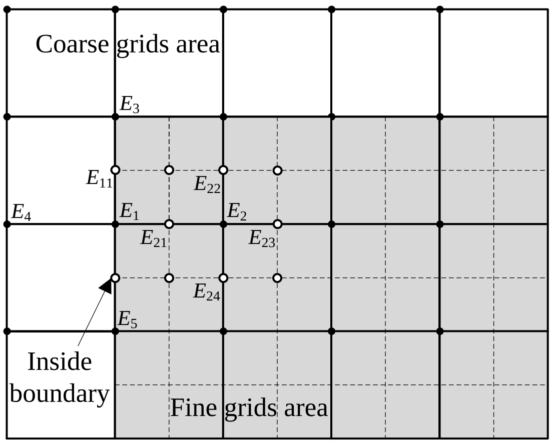

Figure 1. The multi-scale Yee grid in two dimensions.

Sync for the modeling

The computation for normal FDTD modeling is an alternate sampling in time. We calculate the electric field in time n and magnetic field in time n +1/2, then we obtain the electric field in time n+1 as shown in Figure 2 (a). For the area with fine grids, the time stepping is very similar but divided into more parts. In Figure 2 (b), we give an example of n_f=2. The time from n to n+1 are divided into 4 parts with n+1/4 and n+3/4 added. Then, the normal part of the model in space will follow the coarse time steps and the fine part follow the fine time steps at the corresponding time range.

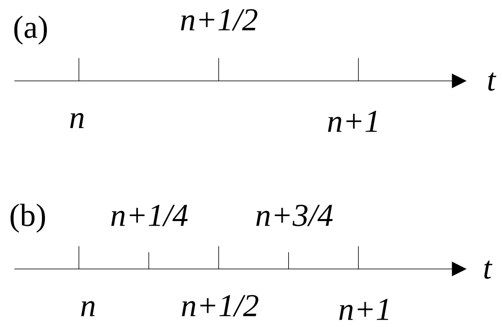

Figure 2. Diagram of time stepping in (a) coarse grid and (b) fine grid.

Modelling comparison

We use a three dimensional model in literature to test the multi-scale grid algorithm. As shown in Figure 3, the modeling result are in good agreement with normal FDTD solutions and integral equation solutions.

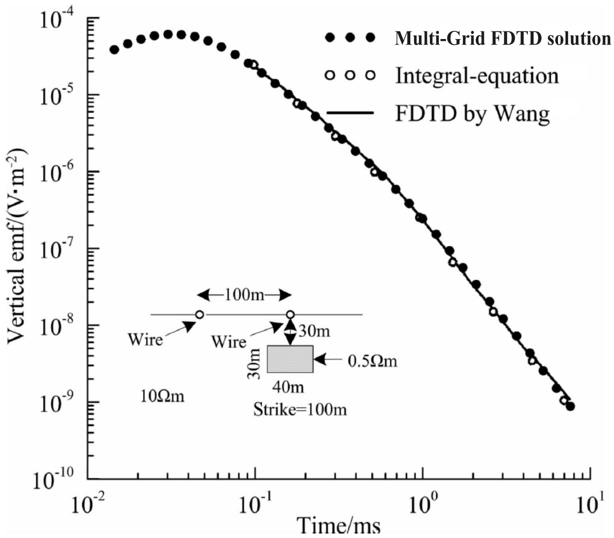

Figure 3. Modelling results comparison between our multi scale grid scheme and the normal FDTD, IE solutions.

RESULTS AND DISCUSSION

We present a multi scale grid scheme in FDTD to modeling the time domain electromagnetic problems. The strategy involves two types of grids: the coarse grids are used to mesh the background and the large abnormal bodies; the fine grids are set inside one or more coarse grids to simulate the small targets with fine shape and details. The two types of grids alternating in time following its own stepping limitations. The modeling results fit well with normal FDTD and IE results.

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