Fuzzy Reliability Prediction of Extra Link Multistage Interconnection Network

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Abstract- The computation of reliability is of paramount importance in parallel processing environment where thousands of processors cooperate with each other to solve a complicated problem. However, there lies a large degree of uncertainty in system failure and therefore, the conventional methods of reliability evaluation for large parallel computer system may not be appropriate to get a realistic value. So the effects of uncertainty in a system can be handled in a better way by using fuzzy logic. This paper presents a new and simple method for evaluating the fuzzy reliability of Extra link multistage interconnection networks (ELMIN). The proposed method is supported by an algorithm to evaluate the fuzzy reliability.

Index Terms- Reliability, Fuzzy set, Multistage Interconnection network

Notations

- X a set containing a space of points in the probability domain
- x an element of X
- p_i fuzzy probability of an event i
- $\overline{p}_i$ complement of fuzzy probability of an event i
- $\mu_{p_i}(p)$ membership function of fuzzy probability p_i
- N number of nodes of the MIN
- R fuzzy reliability of MIN
- G reliability logic graph
- V vertex set
- E edge set
- S system success containing all paths between the source node(s) to destination node (t)
- P_i path at the i^{th} step
- $W, \overline{W}$ indicator variables

Assumptions

1. Initially, all components of the system are in good conditions.
2. The link failure and link success probability is assumed to be fuzzy numbers

I. INTRODUCTION

A major part of parallel computer systems is its interconnection network, which is used to interconnect a large number of standalone processors. As the industry develops fault-tolerant systems with high reliability and safety are required. This development of the fault-tolerant system required the study of failures, and it is known that fault, error and failure have close relation in a system. As fault can cause error and error can cause failure. This leads to one of the important engineering task in design and development of a technical system i.e. reliability evaluation. Under such condition, one of the tools to cope with imprecision of available information in reliability analysis is *fuzzy set theory* [1], which is based on uncertainties like vagueness, ambiguity and imprecision.

In the conventional methods [2-4], it is required to find the minimized expression of system reliability using Boolean algebra. However, these expressions cannot be used in fuzzy set theory because of non-applicability of complementary laws. The expression used for fuzzy reliability of parallel systems has to be different from the expression of conventional probability analysis for obvious reason. Keller and Kara-Zaitri [5] presented a method for assessment of reliability of a non-series parallel network using fuzzy logic. Soman and Misra analysed fuzzy fault tree using resolution identity [6], Tanaka et al [7] and Misra and Weber [8] showed how fuzzification can be carried out for the quantitative analysis of fault tree. Chowdhury and Mishra [9] evaluated the reliability of a non-series parallel network. Bastani et al [10] considered the reliability modeling continuous process-control system. Patra et al [11] presents a method for evaluating fuzzy reliability of a communication network with fuzzy element capacities and probabilities. But none of methods discussed above considers the multistage interconnection networks and suggests a general method of evaluating fuzzy reliability of multistage interconnection networks where there lies a large degree of uncertainty in system failure.

Tripathy et al [12] have proposed a method to evaluate fuzzy reliability of MINs. However, the method is not general and can not be applied as such for all MINs. So, there is always a need to search for a general and efficient method to evaluate the fuzzy reliability of such systems.

The main objective of this paper is to explain how the fuzzy set concepts can be applied in evaluating the system reliability. A general and efficient method has been proposed to find an expression of fuzzy system reliability of fault-tolerant Extra link multistage interconnection networks taking in to consideration the special requirements of fuzzy sets. The supporting algorithm enumerates all the path sets from the source node to destination node. Then the system fuzzy reliability is expressed in terms of fuzzy probability of the disjoint terms of all path sets. The proposed method only uses two operations i.e. multiplication and complementation in evaluating system reliability.

II. CONCEPT OF FUZZY THEORY

A. Basic Concepts

Given a classical (crisp) set A, it's characteristic function assigns a value either 1 or 0 each element x in the universal set X, discriminating between members and non members of A. As fuzzy sets are extensions of crisp sets, this function can be generalized so that values assign to elements of the universal set fall within a continuous range of real numbers in the intervals [0, 1]. Such function is called a membership function. A general fuzzy set will be written in the form $\tilde{A}$ and its membership function denoted by $\mu_{\tilde{A}}(x)$. The (α-cut of a fuzzy set $\tilde{A}$ is the crisp set $\tilde{A}$ that contains all the elements of the universal set X whose membership grades in $\tilde{A}$ are greater or equal to given value of a α (Fig.1). A particular type of fuzzy sets occurs when a single point defines the set; in this case, the fuzzy set is called fuzzy singleton.

B. Fuzzy Probability

Fuzzy set theory was first introduced by Zadeh [1]. A *fuzzy set* can be defined as follows:

Let X be a space of points and an element of X be denoted by x, i.e. *X={x}*. A fuzzy set A in X is characterized by a membership function $\mu_A(x)$, which is a real number in the interval [0, 1] and represents the degree of membership of x in A.

Fuzzy probability represents a fuzzy number between zero and one, assigned to the probability of an event. One can choose different type of membership functions for fuzzy probability. For instance, a fuzzy probability may have a trapezoidal membership function. The fuzzy probabilities of an event *i* can then be denoted by a four parameter functions i.e.

$$p_i = (\alpha_{i1}, \alpha_{i2}, \beta_{i2}, \beta_{i1}) \quad (1)$$

The membership function is given by

$$\mu_{p_i}(p) = \begin{cases} 0, & 0 \le p \le \alpha_{i1} \\ 1 - \dfrac{\alpha_{i2} - p}{\alpha_{i2} - \alpha_{i1}} & \alpha_{i1} \le p \le \alpha_{i2} \\ 1 & \alpha_{i2} \le p \le \beta_{i2} \\ 1 - \dfrac{p - \beta_{i2}}{\beta_{i1} - \beta_{i2}} & \beta_{i2} \le p \le \beta_{i1} \\ 0 & \beta_{i1} \le p \le 1 \end{cases} \quad (2)$$

C. Fuzzy Numbers and Fuzzy Arithmetic

Fuzzy numbers are fuzzy sets defined on the set *of* real numbers and have special significance. They represent the intuitive conception of *approximate numbers*, such as "*numbers close to a given real number*". Arithmetic operations on fuzzy numbers can be defined in terms of arithmetic operations on their *α-cuts* (arithmetic operations on closed intervals). The addition, subtraction and multiplication arithmetic operations on closed intervals are defined as follows:

$$[a,b] + [c,d] = [a+c,\, b+d],$$
$$[a,b] - [c,d] = [a-d,\, b-c],$$
$$[a,b] \cdot [c,d] = [\min(ac, ad, bc, bd),\, \max(ac, ad, bc, bd)]. \quad (3)$$

Particularly, when both intervals are in the form *[r , r]*, the result is a standard arithmetic operation of real numbers

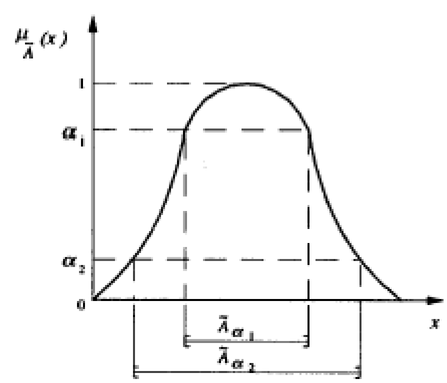

Fig.1: Example of a Membership Function and *α-cuts*.

D. Operation used in computing fuzzy reliability

Let p_i and p_j be two fuzzy sets that have membership functions given by $\mu(p_i)$ & $\mu(p_j)$, respectively. The operations used in fuzzy reliability evaluation, i.e. multiplication and complementation can be defined as follows:

1. Multiplication

$$p_i.p_j = \text{product of } p_i \text{ and } p_j$$
$$= \mu_{p_i p_j}(p) = \mu_{p_i}(p).\mu_{p_j}(p) \quad (4)$$

However, Tanaka et al [7] provided an approximation of the multiplication procedure by defining

$$p_{ij} = p_i.p_j = (\alpha_{i1}.\alpha_{j2}, \alpha_{i2}.\alpha_{j2}, \beta_{i2}.\beta_{j2}, \beta_{i1}\beta_{j1}) \quad (5)$$

2. Complementation

The complementation of any fuzzy set p_i will be given by

$$\bar{p}_i = 1 - \mu_{p_i} \quad (6)$$

For example, in case of trapezoidal membership function, one can obtain

$$\bar{p}_i = (1-\alpha_{i1}, 1-\alpha_{i2}, 1-\beta_{i2}, 1-\beta_{i1}) \quad (7)$$

III. PROPOSED METHOD FOR FUZZY RELIABILITY EVALUATION

First, the multistage interconnection network is converted into the equivalent reliability logic graph G{V,E}, where V is the vertex set and E is edge set. The edge (link) success and edge failure probability is assumed to be fuzzy numbers. Let P_i be the i^{th} path generated from the given source (s) to the given destination (t). Let S be the union of all the paths generated from the source (s) to destination (t). The system success *S* on disjointing gives $(S)_{dis}$. Fuzzy reliability can then be obtained on replacing all indicator variables by their fuzzy probabilities and logical sum and product operator by their fuzzy arithmetic counterparts.

$$R = (S_{dis})_{\{W_i, \overline{W}_i, \cup, \cap\} \to \{p_i, q_i, +, .\}} \quad (8)$$

IV. PROPOSED ALGORITHM

[1]. Convert the multistage interconnection network to a reliability logic graph with V vertices and E set with source(s) and destination (t) node.

[2]. Generate Trapezoidal membership functions for each edge e $\in$ E of the graph.

[3]. $S = \phi$, i= 1;

[4]. while (P_i not a cycle and the end points u, v $\in P_i$ are s and t)

  i. {
  ii. Generate the Path P_i
  iii. $S = S \cup P_i$
  iv. Next P_i
  v. i=i+1;

b. }

[5]. repeat steps 6-7 for i =1 to 4

[6]. Find $(S_i)_{dis}$ by the edge success and edge failure of each edge in S as i^{th} parameter of the membership function and i^{th} parameter of membership function in its complement form.

[7]. The system fuzzy reliability is then expressed as

$$R_i = (S_i)_{dis\{\cup, \cap \to +, \times\}}$$

V. RESULT AND DISCUSSION

The fuzzy reliability of Extra link Multistage Interconnection Network (ELMIN) have been evaluated by the proposed approach. The membership functions of the said multistage interconnection network is plotted against the probability in figure . The parameter function of the said MIN is presented in Table 1. The inference that can be drawn from the Table is that, the fuzzy reliability of ELMIN lie between the limit 0.558-0.702 respectively with a 100% possibility.

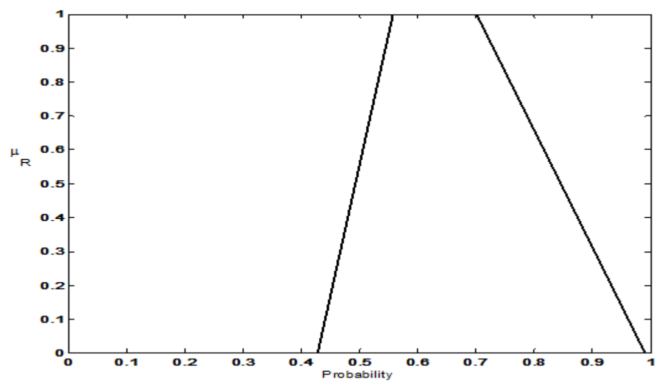

Fig.2: Fuzzy reliability of Extra Link Multistage Interconnection Network (8×8)

MIN	α_1	α_2	β_2	β_1
ELMIN	0.4280	0.5580	0.7028	0.991

Table1. Parameter functions of the multi stage interconnection network

VI. CONCLUSION

In a parallel computing environment there lies large degree of uncertainty in system failure and therefore, conventional methods of reliability evaluation may not be appropriate to get a realistic value. Under such situations it is most appropriate to use the concept of fuzzy set. Fuzzy theory concepts are discussed in the introduction of this paper. The importance of fuzzy reliability and its evaluation methods have been presented. The method is followed by mathematical basis, algorithm. Results have been obtained for fault-tolerant Extra Link multistage interconnection network. This method can be used to predict fuzzy reliability of all MINs.

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