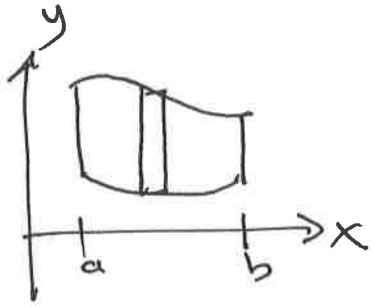
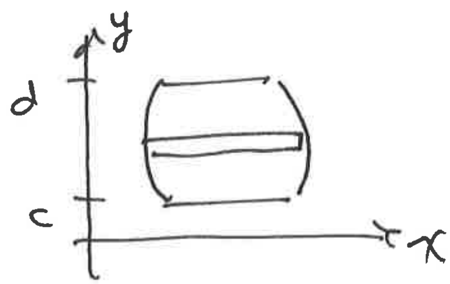
Math 2471 - Calc 3

Double Integrals

$$\iint_R f(x,y)\,dA$$

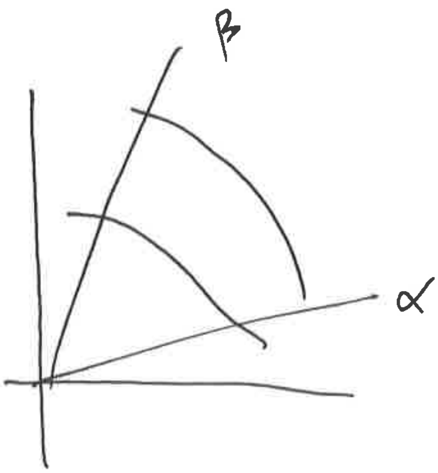


$$\int_a^b \int_{g(x)}^{h(x)} f(x,y)\,dy\,dx$$



$$\int_c^d \int_{G(y)}^{H(y)} f(x,y)\,dx\,dy$$

Polar



$$\int_\alpha^\beta \int_{g(r)}^{h(r)} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

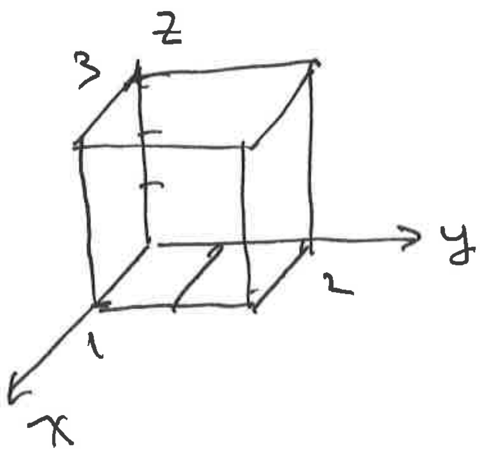
Now we move to triple integral

$$\iiint_V f(x,y,z)\,dv$$

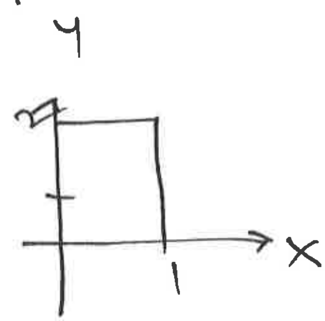
double integrals integrate of a region of integration in 2D

triple integrals integrate of a volume.

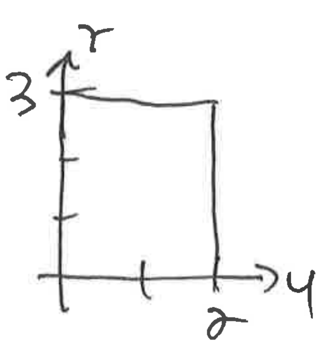
Simplest



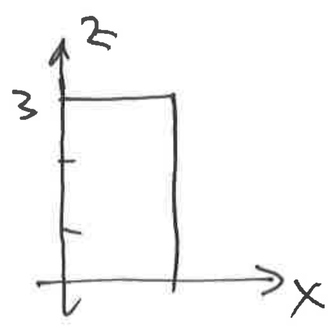
top view



Front



Side



so the limits of integration

$$\int_{x=0}^{1}\int_{y=0}^{2}\int_{z=0}^{3} f(x,y,z)\,dz\,dy\,dx$$

$$\int_{pt}^{pt} \quad \int_{curve}^{curve} \quad \int_{surface}^{surface}$$

so if $f = x+z$

$$\int_0^1\int_0^2\int_0^3 (x+z)\,dz\,dy\,dx = \int_0^1\int_0^2 xz+\frac{z^2}{2}\Big|_0^3 dy\,dx$$

$$= \int_0^1\int_0^2 \left(3x+\frac{9}{2}\right)dy\,dx = \int_0^1 3xy+\frac{9}{2}y\Big|_0^2 dx$$

$$= \int_0^1 (6x+9)\,dx = 3x^2+9x\Big|_0^1 = 3+9 = 12$$

As with double integral we have 2 types

$$dA = dx\,dy = dy\,dx$$

we have 6 types with triples

$$dV = dx\,dy\,dz = dx\,dz\,dy$$
$$= dy\,dx\,dz = dy\,dz\,dx$$
$$= dz\,dx\,dy = dz\,dy\,dx$$

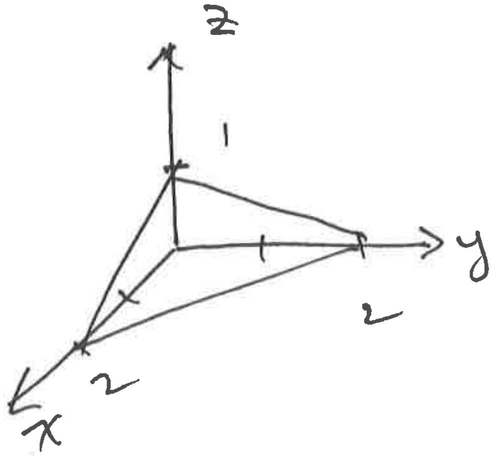
inside integral is surface to surface and moves in direction of the d( ) variable

so $\iiint f\,dz$ surface move in z direction

remaining 2 integral is a projection onto the remaining 2 variable space.

so here (x,y) plane.

ex



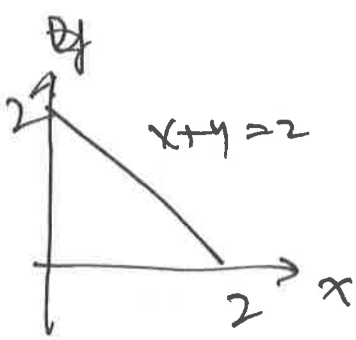
$x+y+2z=2$

bottom surface is $z=0$ top $z=\frac{2-x-y}{2}$

$$\iint_{R_{xy}} \int_{z=0}^{\frac{2-x-y}{2}} f(x,y,z)\,dz\,dA_{xy}$$

~~nextw~~ next 2 integrals involve xy plane

project down

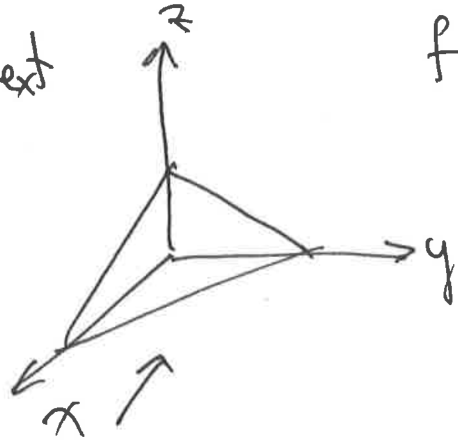


$$\int_{x=0}^{2}\int_{y=0}^{2-x}\int_{z=0}^{\frac{2-x-y}{2}} f\,dz\,dy\,dx$$

set $z=0$ $x+y=2$

$$\int_{y=0}^{2}\int_{x=0}^{2-y}\int_{0}^{\frac{2-x-y}{2}} f\,dz\,dx\,dy$$

Next



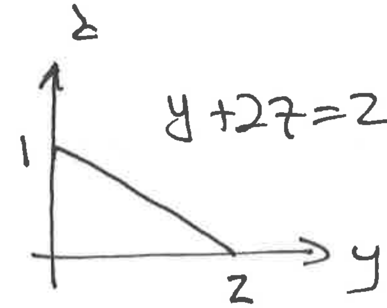
front & back surface
look down x axis

back $x=0$

front $x=2-y-2z$

$$\iint_{R_{yz}}\int_{x=0}^{2-y-2z} f\,dx\,dA_{yz}$$

set $x=0$

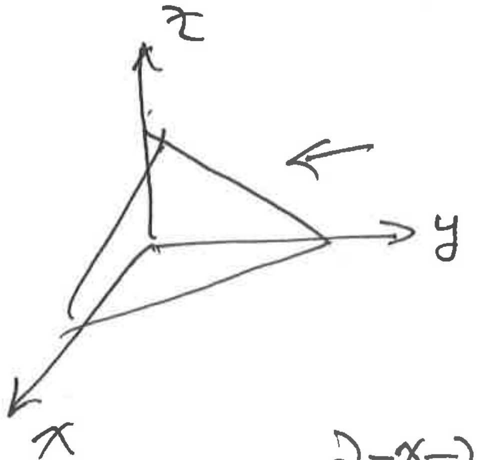


$y+2z=2$

$$\int_{y=0}^{2}\int_{z=0}^{\frac{2-y}{2}}\int_{x=0}^{2-y-2z} f\,dx\,dz\,dy$$

$$\int_{z=0}^{1}\int_{y=0}^{2-2z}\int_{x=0}^{2-y-2z} f\,dx\,dy\,dz$$

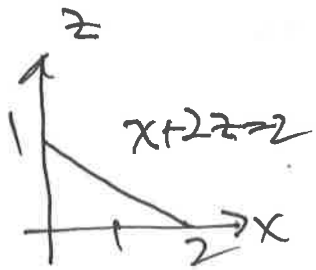
Now left & right



left $y=0$ right $y=2-x-2z$

$$\iint_{R_{xz}} \int_{y=0}^{2-x-2z} f(x,y,z)\,dy\,dA_{xz}$$

Set $y=0$

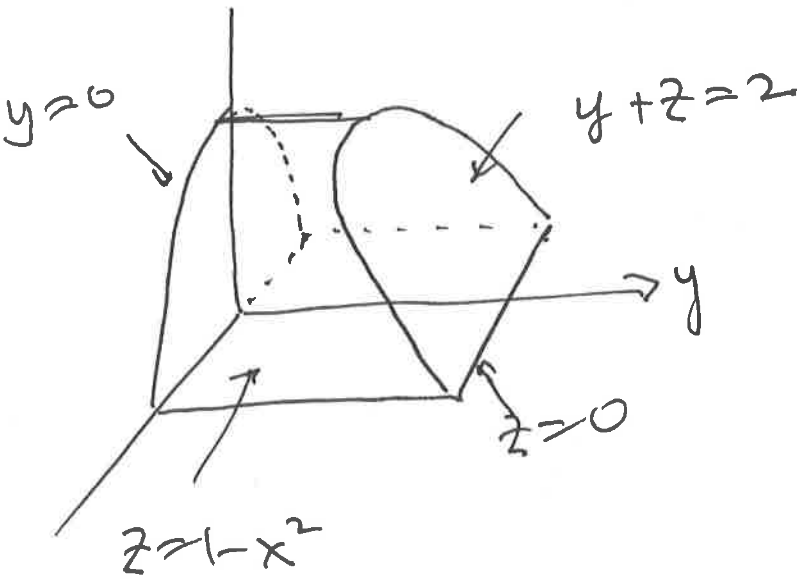


$$\int_{x=0}^{2} \int_{z=0}^{\frac{2-x}{2}} \int_{0}^{2-x-2z} f(x,y,z)\,dy\,dz\,dx$$

$$\int_{z=0}^{1} \int_{x=0}^{2-2z} \int_{y=0}^{2-x-2z} f(x,y,z)\,dy\,dx\,dz$$

Ex 2 The Volume is the solid bound by

$z = 0, \ z = 1 - x^2, \ y = 0, \ y + z = 2$



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