

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exam style paper
Exercise A, Question 1

Question:

Use the binomial theorem to expand $\frac{1}{(2+x)^2}$, $|x| < 2$, in ascending powers

of x , as far as the term in x^3 , giving each coefficient as a simplified fraction. (6)

Solution:

$$\begin{aligned}
 (2+x)^{-2} &= 2^{-2} \left(1 + \frac{x}{2} \right)^{-2} \\
 &= 2^{-2} \left[1 + \binom{-2}{1} \left(\frac{x}{2} \right) + \frac{(-2)(-3)}{1 \times 2} \left(\frac{x}{2} \right)^2 + \right. \\
 &\quad \left. \frac{(-2)(-3)(-4)}{1 \times 2 \times 3} \left(\frac{x}{2} \right)^3 + \dots \right] \\
 &= 2^{-2} \left(1 - x + \frac{3}{4}x^2 - \frac{1}{2}x^3 + \dots \right) \\
 &= \frac{1}{4} - \frac{x}{4} + \frac{3x^2}{16} - \frac{x^3}{8} + \dots
 \end{aligned}$$

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Exam style paper
Exercise A, Question 2

Question:

The curve C has equation

$$x^2 + 2y^2 - 4x - 6yx + 3 = 0$$

Find the gradient of C at the point $(1, 3)$. (7)

Solution:

$$x^2 + 2y^2 - 4x - 6yx + 3 = 0$$

Differentiate with respect to x :

$$2x + 4y \frac{dy}{dx} - 4 - \left(6x \frac{dy}{dx} + 6y \right) = 0$$

At the point $(1, 3)$, $x = 1$ and $y = 3$.

$$\therefore 2 + 12 \frac{dy}{dx} - 4 - \left(6 \frac{dy}{dx} + 18 \right) = 0$$

$$\therefore 6 \frac{dy}{dx} - 20 = 0$$

$$\therefore \frac{dy}{dx} = \frac{20}{6} = \frac{10}{3}$$

\therefore the gradient of C at $(1, 3)$ is $\frac{10}{3}$.

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Exam style paper
Exercise A, Question 3

Question:

Use the substitution $u = 5x + 3$, to find an exact value for

$$\int_0^3 \frac{10x}{(5x+3)^3} dx \quad (9)$$

Solution:

$$u = 5x + 3$$

$$\therefore \frac{du}{dx} = 5 \text{ and } x = \frac{u-3}{5}$$

$$\therefore \int \frac{10x}{(5x+3)^3} dx = \int \frac{2(u-3)}{u^3} \frac{du}{5}$$

$$= \frac{2}{5} \int \frac{u-3}{u^3} du$$

$$= \frac{2}{5} \int \frac{u}{u^3} - \frac{3}{u^3} du$$

$$= \frac{2}{5} \int u^{-2} - 3u^{-3} du$$

$$= \frac{2}{5} \left[-u^{-1} + \frac{3}{2}u^{-2} \right]$$

Change the limits: $x = 0 \Rightarrow u = 3$ and $x = 3 \Rightarrow u = 18$

$$\therefore \text{Integral} = \frac{2}{5} \left[-\frac{1}{18} + \frac{3}{2 \times 18^2} - \left(-\frac{1}{3} + \frac{3}{2 \times 3^2} \right) \right] = \frac{5}{108}$$

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Exercise A, Question 4

Question:

(a) Find the values of A and B for which

$$\frac{1}{(2x+1)(x-2)} \equiv \frac{A}{2x+1} + \frac{B}{x-2} \quad (3)$$

(b) Hence find $\int \frac{1}{(2x+1)(x-2)} dx$, giving your answer in the form $y = \ln f(x)$. (4)

(c) Hence, or otherwise, obtain the solution of

$$\left(2x+1\right) \left(x-2\right) \frac{dy}{dx} = 10y, y > 0, x > 2$$

for which $y = 1$ at $x = 3$, giving your answer in the form $y = f(x)$. (5)

Solution:

$$(a) \frac{1}{(2x+1)(x-2)} \equiv \frac{A}{(2x+1)} + \frac{B}{(x-2)} \equiv \frac{A(x-2) + B(2x+1)}{(2x+1)(x-2)}$$

$$\therefore A(x-2) + B(2x+1) \equiv 1$$

$$\text{Substitute } x = 2, \text{ then } 5B = 1 \Rightarrow B = \frac{1}{5}$$

$$\text{Substitute } x = -\frac{1}{2}, \text{ then } -\frac{5}{2}A = 1 \Rightarrow A = -\frac{2}{5}$$

$$(b) \therefore \text{Integral} = \int \frac{-\frac{2}{5}}{2x+1} + \frac{\frac{1}{5}}{x-2} dx$$

$$= -\frac{1}{5} \ln \left| 2x+1 \right| + \frac{1}{5} \ln \left| x-2 \right| + C$$

$$= \ln \left[k \left(\frac{|x-2|}{|2x+1|} \right)^{\frac{1}{5}} \right]$$

(c) Separate the variables to give

$$\int \frac{dy}{y} = \int \frac{10 dx}{(2x+1)(x-2)}$$

$$\therefore \ln y = 2 \ln |x - 2| - 2 \ln |2x + 1| + C$$

$$y = 1 \text{ when } x = 3 \Rightarrow C = 2 \ln 7 = \ln 49$$

$$\therefore y = 49 \left(\frac{|x - 2|}{|2x + 1|} \right)^2$$

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Exam style paper
Exercise A, Question 5

Question:

A population grows in such a way that the rate of change of the population P at time t in days is proportional to P .

(a) Write down a differential equation relating P and t . (2)

(b) Show, by solving this equation or by differentiation, that the general solution of this equation may be written as $P = Ak^t$, where A and k are positive constants. (5)

Initially the population is 8 million and 7 days later it has grown to 8.5 million.

(c) Find the size of the population after a further 28 days. (5)

Solution:

$$(a) \frac{dP}{dt} \propto P$$

$$\therefore \frac{dP}{dt} = mP$$

$$(b) \int \frac{dP}{P} = \int m \, dt$$

$$\therefore \ln P = mt + C$$

$$\therefore P = e^{mt + C}$$

$$= Ae^{mt} \quad \text{where } A = e^C$$

$$= Ak^t \quad \text{where } k = e^m$$

$$(c) \text{ When } t = 0, P = 8 \quad \therefore A = 8$$

$$\text{When } t = 7, P = 8.5 \quad \therefore 8.5 = 8k^7$$

$$\therefore k^7 = \frac{8.5}{8}$$

$$\text{When } t = 35,$$

$$P = 8k^{35}$$

$$= 8 (k^7)^5$$

$$\begin{aligned} &= 8 \left(\frac{8.5}{8} \right)^5 \\ &= 10.8 \text{ million (to 3 s.f.)} \end{aligned}$$

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Exam style paper
Exercise A, Question 6

Question:

Referred to an origin O the points A and B have position vectors $i - 5j - 7k$ and $10i + 10j + 5k$ respectively. P is a point on the line AB .

(a) Find a vector equation for the line passing through A and B . (3)

(b) Find the position vector of point P such that OP is perpendicular to AB . (5)

(c) Find the area of triangle OAB . (4)

(d) Find the ratio in which P divides the line AB . (2)

Solution:

(a) $AB = 9i + 15j + 12k$ (or $BA = -9i - 15j - 12k$)

\therefore the line may be written

$$r = \begin{pmatrix} 1 \\ -5 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 9 \\ 15 \\ 12 \end{pmatrix} \quad \text{or} \quad r = \begin{pmatrix} 10 \\ 10 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix} \quad \text{or equivalent}$$

$$(b) \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} +1 + 9\lambda \\ -5 + 15\lambda \\ -7 + 12\lambda \end{pmatrix} = 0$$

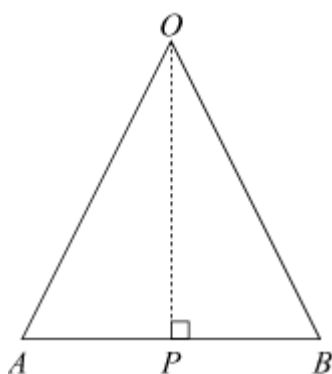
$$\therefore +3 + 27\lambda - 25 + 75\lambda - 28 + 48\lambda = 0$$

$$\therefore 150\lambda - 50 = 0$$

$$\therefore \lambda = \frac{1}{3}$$

$$\therefore \text{the point } P \text{ has position vector } \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$$

$$(c) |OP| = 5 \text{ and } |AB| = \sqrt{9^2 + 15^2 + 12^2} = 15\sqrt{2}$$



$$\text{Area of } \triangle OAB = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 15\sqrt{2} \times 5 = \frac{1}{2} \times 75\sqrt{2}$$

$$\begin{aligned} \text{(d) } AP &= \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} - \begin{pmatrix} 1 \\ -5 \\ -7 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix} \text{ and } PB = \begin{pmatrix} 10 \\ 10 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} \\ &= \begin{pmatrix} 6 \\ 10 \\ 8 \end{pmatrix} \end{aligned}$$

$$\therefore PB = 2AP$$

i.e. P divides AB in the ratio $1 : 2$.

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Exam style paper
Exercise A, Question 7

Question:

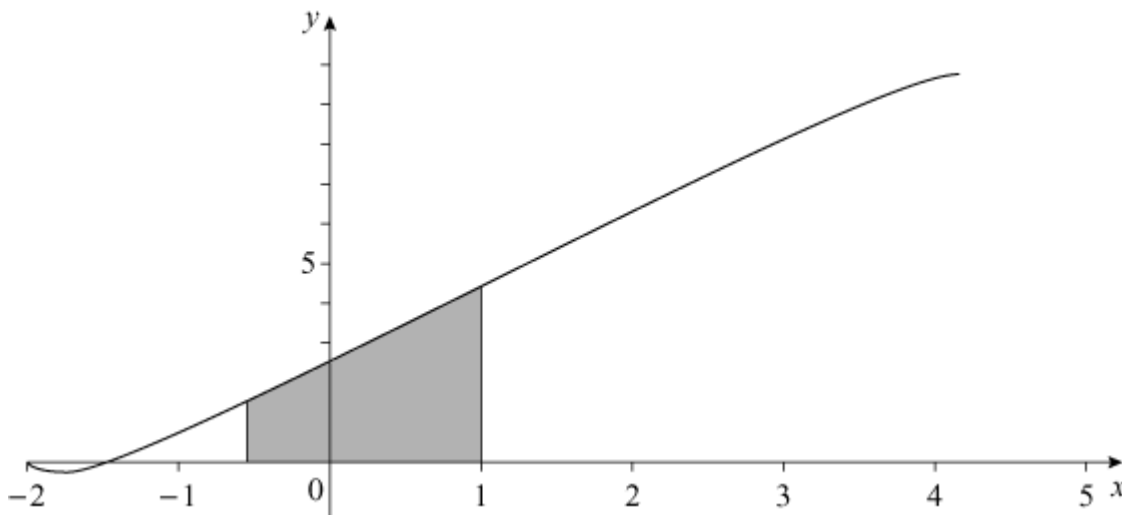
The curve C , shown has parametric equations
 $x = 1 - 3 \cos t, y = 3t - 2 \sin 2t, 0 < t < \pi$.

(a) Find the gradient of the curve at the point P where $t = \frac{\pi}{6}$. (4)

(b) Show that the area of the finite region beneath the curve, between the lines $x = -\frac{1}{2}, x = 1$ and the x -axis, shown shaded in the diagram, is given by the integral

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9t \sin t \, dt - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 12 \sin^2 t \cos t \, dt. \quad (4)$$

(c) Hence, by integration, find an exact value for this area. (7)



Solution:

(a) $x = 1 - 3 \cos t, y = 3t - 2 \sin 2t$

$$\frac{dx}{dt} = 3 \sin t \text{ and } \frac{dy}{dt} = 3 - 4 \cos 2t$$

$$\therefore \frac{dy}{dx} = \frac{3 - 4 \cos 2t}{3 \sin t}$$

$$\text{When } t = \frac{\pi}{6}, \frac{dy}{dx} = \frac{3-2}{\left(\frac{3}{2}\right)} = \frac{2}{3}$$

(b) The area shown is given by $\int_{t_1}^{t_2} y \frac{dx}{dt} dt$

Where t_1 is value of parameter when $x = -\frac{1}{2}$

and t_2 is value of parameter when $x = 1$

$$\text{i.e. } 1 - 3 \cos t_1 = -\frac{1}{2}$$

$$\therefore \cos t_1 = \frac{1}{2}$$

$$\therefore t_1 = \frac{\pi}{3}$$

$$\text{Also } 1 - 3 \cos t_2 = 1$$

$$\therefore \cos t_2 = 0$$

$$\therefore t_2 = \frac{\pi}{2}$$

The area is given by

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(3t - 2 \sin 2t \right) \times 3 \sin t dt$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\pi}{2} 9t \sin t dt - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\pi}{2} 6 \times 2 \sin t \cos t \sin t dt \quad \text{Using the double}$$

angle formula

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\pi}{2} 9t \sin t dt - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\pi}{2} 12 \sin^2 t \cos t dt$$

$$\text{(c) Area} = \left[-9t \cos t \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\pi}{2} 9 \cos t dt - \left[4 \sin^3 t \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \left[-9t \cos t + 9 \sin t - 4 \sin^3 t \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \left(9 - 4 \right) - \left(-\frac{3\pi}{2} + \frac{9\sqrt{3}}{2} - 4 \times \frac{3\sqrt{3}}{8} \right)$$

$$= 5 - 3\sqrt{3} + \frac{3\pi}{2}$$

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