CAP-5993 Homework 3
Deadline: 3:30PM on 3/2 (Thursday)

1. Exercise 5.68 from textbook. (10 pts)
2. Exercise 7.1 from textbook. (10 pts)
3. Exercise 7.2 from textbook. (10 pts)
4. Exercise 7.3 from textbook. (10 pts)
5. Exercise 7.4 from textbook. (10 pts)
6. Exercise 7.20 from textbook. (10 pts)
7. Exercise 7.22 from textbook. (10 pts)
8. Exercise 7.25 from textbook. (10 pts)
9. Exercise 7.28 from textbook. (10 pts)
10. Exercise 13.1 from textbook. (10 pts)
11. Exercise 13.8 from textbook. (10 pts)
12. In lecture we saw the "support enumeration" algorithm for computing a Nash equilibrium in two-player general-sum strategic-form games. The algorithm works by iterating over all possible strategy supports for both players, and checking whether an equilibrium exists with the given support (and outputting one if so). Recall that the *support* of a mixed strategy σ_i is the set of pure strategies that are played with nonzero probability under σ_i. For example, in Rock-Paper-Scissors, if σ_i is the strategy that plays 60% Rock and 40% Paper, then the support of σ_i (denoted $\text{supp}(\sigma_i)$) is {Rock, Paper}.

    In class we argued that this algorithm runs in exponential time, because there are exponentially many possible supports to consider for each player (each support corresponds to a subset of $\{1, \ldots, n\}$, where n is the number of pure strategies, and the total number of subsets of $\{1, \ldots, n\}$ is 2^n). However, for each guess of a support for both players, there exists a linear programming formulation for computing a Nash equilibrium with the given support (if one exists). Write down a linear program formulation for computing a Nash equilibrium given supports S_1, S_2 for both agents (if one exists), and prove that it is correct. That is, if there exists a Nash equilibrium with supports S_1, S_2, then prove that the solution to your formulation will be one, and conversely that if there does not exist a Nash equilibrium with supports S_1, S_2, then your formulation will have no solution. (20 pts)

13. Consider the following game, which is similar to the Split or Steal game we saw in lecture:

	Split	*Steal*
Split	$(60, 60)$	$(0, 100)$
Steal	$(100, 0)$	$(5, 5)$

    The following questions will refer to this game. (40 pts)

    (a) List all the strictly-dominated and weakly-dominated strategies for each player. (5 pts)

    (b) What are all of the Nash equilibria of this game? (5 pts)

    (c) Now suppose that the players are repeating this game k times. What are all of the subgame-perfect equilibria of the repeated game? Hint: try solving the last repetition first (i.e., set $k = 1$). With a simultaneous-moves game like this one, this backward-induction-like strategy doesn't always work, but when it does it is often the best way to solve a game. (10 pts)

    (d) Now suppose that at each time step, we will repeat the game with probability 0.9, and end the game with probability 0.1. Consider the following strategy, which is known as *tit for tat*: split in the first round, and at each future round play the strategy selected by the other player in the previous round. Prove that the strategy profile in which both players play tit for tat is a Nash equilibrium of this game. (10 pts)

(e) Now let $p \geq \frac{8}{19}$ be arbitrary and suppose that at each time step, we will repeat the game with probability p and end the game with probability $1 - p$. Please find two different Nash equilibria of this game, and justify your answer. Note that you must find two equilibria for each p, and that your equilibria could be different for different values of p. (10 pts)

Hint: to construct an equilibrium of the repeated game, consider the idea of *punishment*: suppose the two players agree to play a particular way, and then one (say the row player) decides to do something different from the agreement. What could the column player do on future rounds to make the row player regret her choice?

14. This question will refer to the following game. (30 pts)

	L	R
U	$(9,4)$	$(2,9)$
M	$(4,7)$	$(14,6)$
D	$(19,11)$	$(2,9)$

(a) Formulate the problem of computing a Nash equilibrium in this game as a linear complementarity problem. Note that this requires specifying the values of 6 matrices, A, B, E, e, F, and f. (10 pts)

(b) Find all Nash equilibria in this game using Gambit. (5 pts)

(c) Showing your steps, apply the Lemke Howson algorithm to this game to obtain a Nash equilibrium (which should hopefully be one of the equilibria you computed in part b). (10 pts)

(d) Had you instead used the support enumeration algorithm, would you have found the same Nash equilibrium as the previous one? If not, which one would you have found instead? (5 pts)

15. In lecture we saw how the problem of computing a Nash equilibrium in two-player zero-sum extensive-form games of imperfect information can be formulated as a linear program by using the "sequence form" representation. For the following game that we saw the last homework, formulate two linear programs; one for computing a Nash equilibrium strategy for player 1, and one for computing an equilibrium strategy for player 2. Note that this will involve specifying the values of 5 matrices, A, E, e, F, and f. (20 pts)

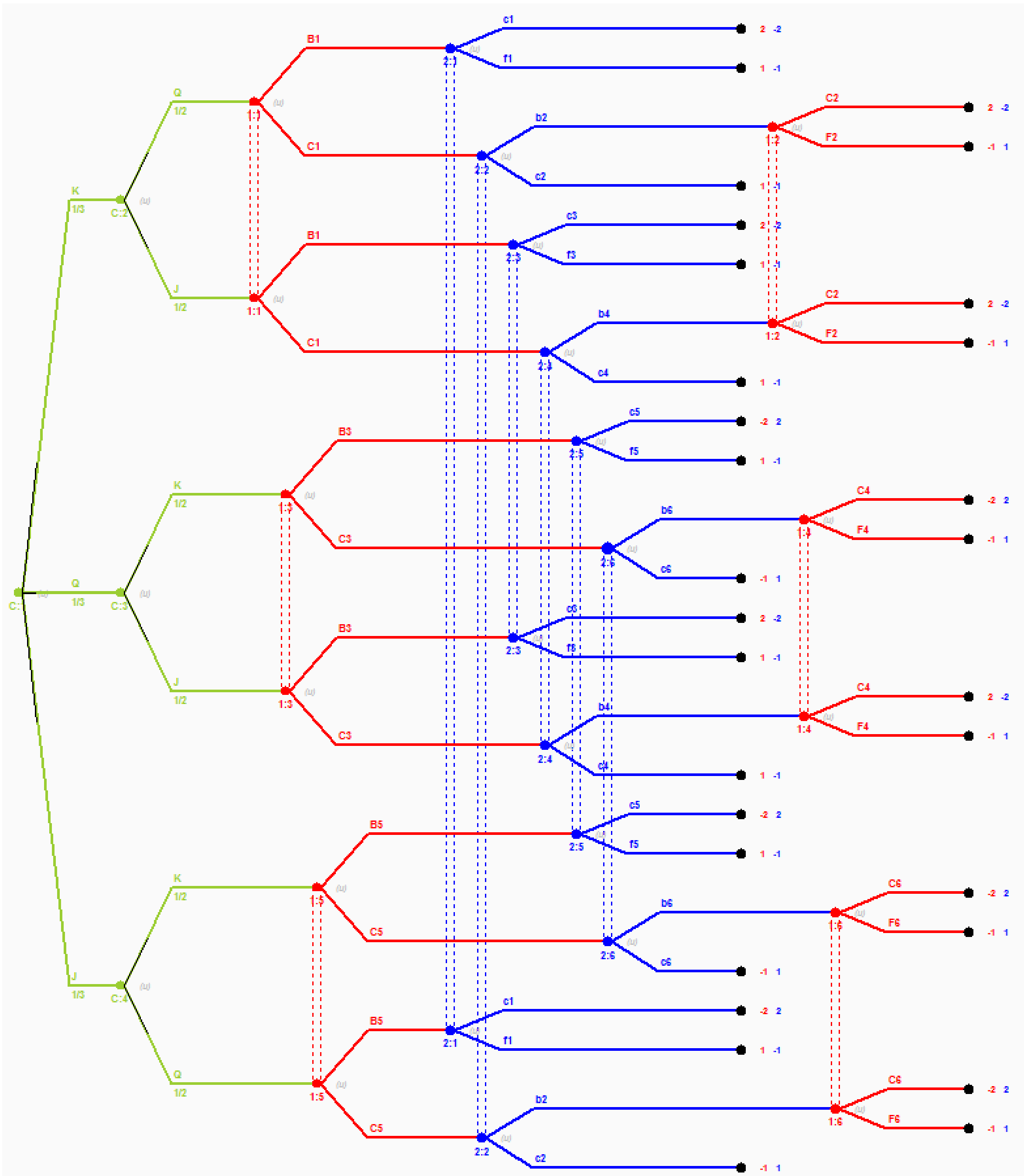

Figure 1: Game tree for problem 15.