# Fast 3D simulation of semi-airborne transient electromagnetic responses by model reduction method

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## SUMMARY

The Semi-airborne Transient Electromagnetic (SATEM) method is a kind of survey method using an exciting source on the earth surface and a flying aircraft to receive induced electromotive forces in the air. In this extended abstract, we simulate the electromagnetic responses of SATEM excited by a grounded-wire using a model reduction technique. The simulation method is based on the frequency domain solutions at a sufficient number of discrete frequencies using a finite difference (FD) scheme, which results are synthesized to the time-domain by means of cosine transform. To accelerate the 3D modelling in frequency-domain for multiple discrete frequencies, the model reduction technique which is based on rational Krylov subspace projection algorithm is used to mitigate the large-scale 3D modelling. For a rational approximation of the transfer function is used to reduce the dimension of the calculation subspace, the calculation results of FD numerical method can be obtained at a very low cost. The acceleration level of this method is well higher than that obtained by the conventional FD scheme, especially for a 3D numerical problem with a wide frequency band like the 3D forward of SATEM. Finally, numerical results have been provided to give an insight into the reliability of the simulation method for the 3D responses of the SATEM using the model reduction technique.

**Keywords**: model reduction technique, Krylov subspace, finite difference, semi-airborne TEM

## INTRODUCTION

The concept of semi-airborne systems was proposed in 1970s and many systems have been applied in geophysical exploration (Bosschart and Seigel, 1972; Elliot, 1998; Mogi, T., et al, 1998), such as TURAIR system (in frequency domain), FLAIRTEM system (in time-domain), TerraAir (in time-domain), GREATEM system (in time-domain) and so on. This kind of airborne electromagnetic (EM) method allows for application of a large-moment source and use of a long transmitter-receiver distance, expanding the survey depth about 800m (Mogi et al., 2009). For SATEM survey method, using a high efficient aircraft platform to measure the EM responses, there are so many transients at receiver locations need to be calculated that the inversion is especially expansive accordingly for an extremely large and ill-conditioned Jacobian matrix arises in this case. Therefore, a fast and robust forward scheme is crucial for the inversion efficiency.

In this extended abstract, the Model Reduction (MR) technique of rational Krylov subspace projection (Antoulas, 2005; Druskin, 2010) is applied to simulate the 3D responses of SATEM by rational functions of lower order. The discretized frequency-domain Maxwell system for a suitably chosen reference frequency is projected onto a rational Krylov subspace of low dimension. Based on this subspace, a sufficiently accurate approximation of the rational function can be obtained for the other discretized frequencies at very little cost. Finally, these responses in frequency-domain are synthesized to the time evolution using a cosine transform. 3D model studies for the responses of a SATEM with a grounded-wire source after a current shut-off are given blow.

## ALGORITHM

According to the Maxwell's equation in quasi-static approximation, the time-harmonic electric field in a computation domain $\Omega \subset \mathrm{R}^{\mathbf{3}}$ should be satisfied

$$\nabla \times \nabla \times \mathbf{E} + i\omega\mu\sigma\mathbf{E} = \mu\mathbf{J} \qquad (1)$$

$$n \times \mathbf{E}\big|_{\Gamma} = \mathbf{0} \qquad (2)$$

where $\mu$ denotes the magnetic permeability and it is supposed as a constant in the whole computation domain, $\omega$ denoting angular frequency of the excited field, $\sigma$ the conductivity, **J** external source current density.

Finite-difference (FD) discretization of (1) on Lebedev's grid (Davydycheva, et al. 2003) behaves as

$$(\mathbf{P} + i\omega\mu\mathbf{M})\mathbf{u} = \mathbf{f} \qquad (3)$$

Here $\mathbf{u} \in C^N$ is the solution vector of the unknown coefficient of electric fields at discrete nodes, and $N$ is dimension of the electric field, $\mathbf{P}$ the nonnegative symmetric matrix, **f** the result of the right-hind side of (1), and $\mathbf{M}$ is known as mass matrix.

Vector $\mathbf{v} \in C^q : \{v_j = u_{k_i}, j = 1, 2, \cdots, q; k_i \in Z^N.\}$ is a subset of **u,** and the solutions at the measure points are included. Then, the solution **u** and subset **v** satisfy

$$\mathbf{G}^T\mathbf{u}=\mathbf{v} \quad (4)$$

where extension operator $\mathbf{G}\in \mathrm{R}^{\mathrm{N}\times \mathrm{q}}$ is

$$\left[\mathbf{G}_{\mathbf{i},\mathbf{j}}\right]=\begin{cases}1.0, & if \;\; j=k_i \\ 0 & otherwise\end{cases} \quad (5)$$

It can be seen easily that $\mathbf{v}$ is a reduced vector, and the solution of electric field can be valued efficiently in a subset of the computational domain $\Omega$.

A vector-valued function $\mathbf{R}\in C^q$ for the frequency $\omega$ is defined as

$$\mathbf{R}=\mathbf{R}(\omega)=\mathbf{G}^T(\mathbf{P}+i\omega\mu\mathbf{M})^{-1}\mathbf{f} \quad (6)$$

$\mathbf{R}$ is known as a transfer function and is used to approximate the solutions with significantly fewer degrees of freedom than N, and consequently a fast 3D forward can be realized.

We rewite (6) at a reference frequency $\omega_0$

$$\mathbf{R}=\mathbf{R}(s)=\mathbf{G}^T\left[\mathbf{P_0}+s\mathbf{M}\right]^{-1}\mathbf{f} \quad (7)$$

where $\mathbf{P_0}=\mathbf{P}+i\omega_0\mathbf{M}$ , $s=i(\omega-\omega_0)$ . Setting $\mathbf{r}=\mathbf{P}_0^{-1}\mathbf{f}$ and $\mathbf{K}=\mathbf{P_0^{-1}M}$, the transfer function is

$$\mathbf{R}(s)=\mathbf{G}^T\left(\mathbf{I}+s\mathbf{K}\right)^{-1}\mathbf{r} \quad (8)$$

we can see that $\mathbf{R}(s)$ is a rational function of *s*, then the result of the reference frequency $\omega_0$ is the one when *s*=0. The model reduction method is applied here to solve (8) based on the way of the Krylov subpace projection (Freund, 2003; Antoulas, 2005; Börner, 2008). we apply an orthogonal projection onto a Krylov space based on Arnoldi's method, and an m-dimensional Krylov subspace based on the initial vector $\mathbf{r}$ and matrix $\mathbf{A}$ is introduced

$$\aleph_{\mathrm{m}}(\mathbf{A},\mathbf{r})=span\{\mathbf{r},\mathbf{Ar},\cdots,\mathbf{A}^{m-1}\mathbf{r}\}$$

Firstly, an orthonormal basis of $\aleph_{\mathrm{m}}(\mathbf{A},\mathbf{r})$ and an upper $m\times m$ Hessenberg-matrix $\mathbf{H}_m$ generate using m steps of the Arnoldi process successively. $\mathbf{V}_m\in C^{N\times m}$ is formed from the *m* orthonormal bases which are the m columns of $\mathbf{V}_m$ and satisfies the relation $\mathbf{H}_m\approx\mathbf{V}_m^T\mathbf{A}\mathbf{V}_m$ . Then the transfer function (8), projected onto $\aleph_{\mathrm{m}}$, can be rewritten as

$$\mathbf{R}_m(s)=\beta\mathbf{G}^T\mathbf{V}_m\left(\mathbf{I}_m+s\mathbf{H}_m\right)^{-1}\mathbf{e}_1 \quad (9)$$

where $\mathbf{e}_1$ denotes the 1th unit vector belonging to $R^{\mathrm{m}}$. Because *m* is very small compared to *N*, the computational cost is decrease naturally, especially in the case of a wide computational frequency band.

After all the transfer function for $N_f$ frequencies $\omega_k\,(k=1,2,\cdots N_f)$, the responses at the points of interest in time-domain can be transformed by a cosine transform of the imaginary part of $\mathbf{E}$

$$\mathbf{e}(t)=\frac{2}{\pi}\int_0^\infty \mathrm{Im}(\mathbf{E})\frac{\cos\omega t}{\omega}d\omega \quad (10)$$

## NUMERICAL EXAMPLES

We consider the responses of a SATEM system with a 1km long grounded-wire as a transmitter. As is shown in Figure 1, the conductivity structure we consider consists of a 3D object at a depth of 150m embedded in a uniform halfspace of 0.02S/m. The entire 3D mesh in FD scheme consists of 70,560 cells, which corresponds to 222,052 degrees of freedom. The induced electromotive force responses measured in the air after a driving current of 60A is shut off at *t*=0 are computed using the modelling method with the model reduction technique. The effective area of the receiving coil is 200m$^2$, the circle number of coil 200. We simulate the responses of this model for Krylov subspaces of dimension *m*=6000. Figure 2 shows the three component responses calculated by conventional FD and MR scheme respectively, and the responses are subtracted by the background responses of the homogeneous model with resistivity 0.02S/m to show the relative between the abnormal response and the 3D object underground clearly.

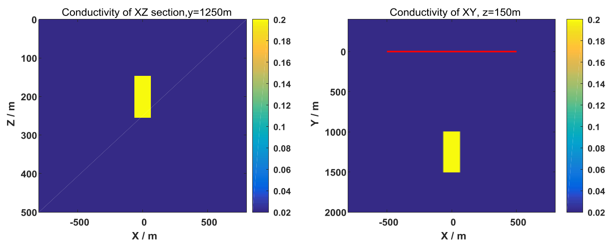



**Figure 1.** Section and top view of the conductivity structure of the 3D model. The red line is projection of the grounded-wire source.

Figure 2 shows the y-component responses at different height H using the Krylov subspaces of dimension *m*=6000 and the relative error between the responses with *m*=4500 and *m*=6000. We can see that the amplitudes of the responses on the earth surface (H=0m) are the largest one and the amplitude decreases as the observed height increases slowly. The difference of these responses decreases gradually at the late time. Moreover, the results for *m*=4500 are almost the same as those for *m*=6000, whose amplitude of the relative errors are less than 4%. It means that we can simulate the responses using MR method with a very small dimension and then leads to a fast 3D forward method.

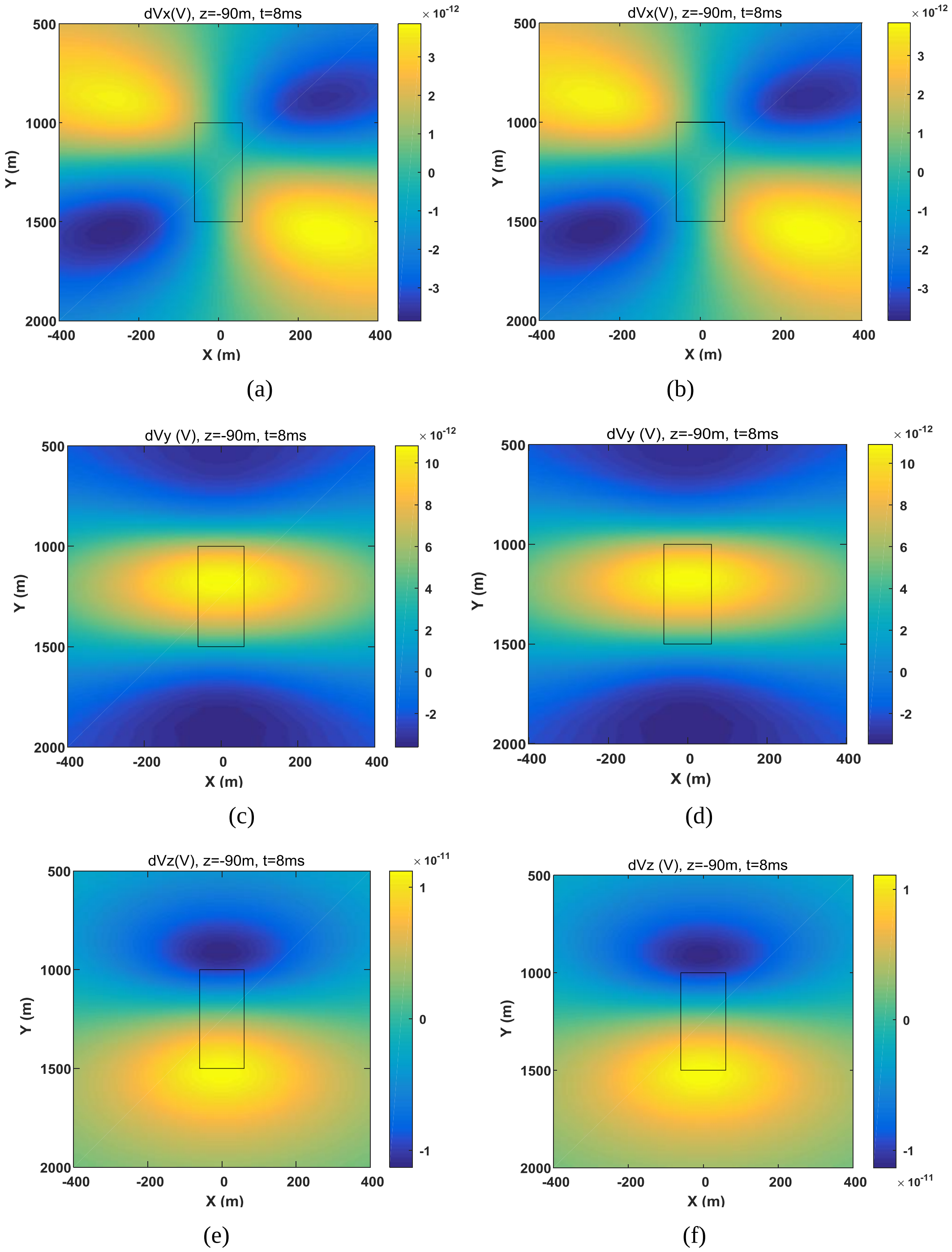
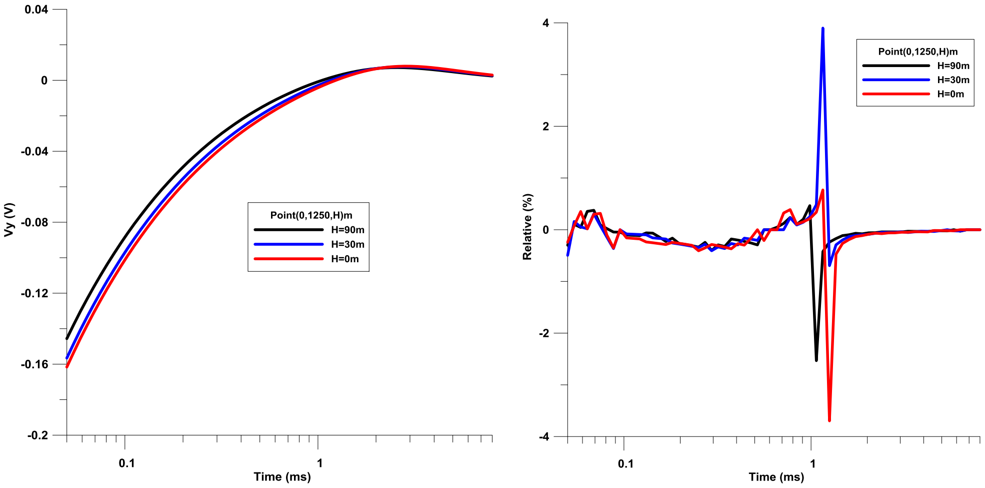



**Figure 2.** The snapshots of the abnormal parts of three-component transient responses at 8ms after the current is cut off and the height of the receiver is 90m. The results of the left column (a,c,e) and the right column (b,d,f) are computed using conventional FD scheme and the scheme based on MR technique with the Krylov subspaces of dimension $m$=6000 respectively.




(a) (b)

**Figure 3.** Y-component responses at different heights calculated using the model reduction method with the Kyrlov subspaces of dimension *m*=6000 (a) and the relative error curves of the corresponding results of *m*=4500 compared with those of *m*=6000 (b).

## Conclusion

The model reduction algorithm for simulating the responses of the SATEM survey method is implemented in this extended abstract. The system of equations arising from the FD discretization of equation in frequency-domain is projected onto a low-dimensional subspace using a Krylov subspace projection technique. Numerical test of 3D object in a homogeneous model excited by a SATEM system is performed on a PC to show the effect of the simulation. The results show the efficiency of the simulation method. The MR method is especially suitable for simulating electromagnetic fields with a wide frequency band and obviously can be applied to the large-scale, 3D, inversion of SATEM method.

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