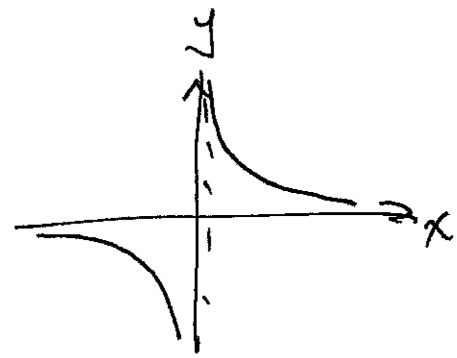
Math 1496 - Calc I

2.5 Infinite Limits

consider $f(x) = \frac{1}{x}$

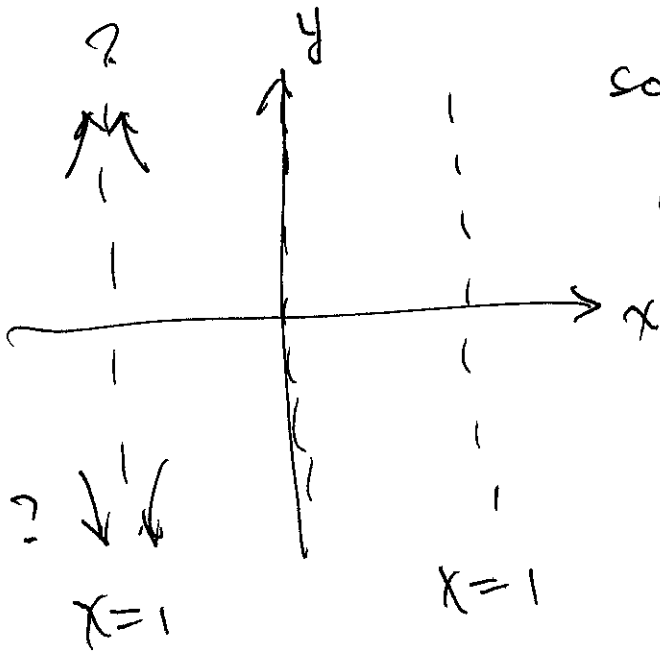


as $x \to 0$ f grows unbounded

$\lim_{x \to 0^-} \frac{1}{x} = -\infty$ $\lim_{x \to 0^+} \frac{1}{x} = +\infty$ $x = 0$ VA

consider $f(x) = \frac{1}{x(x-1)(x+1)}$

So we see that f(x) is unbounded at $x = 0, \pm 1$ and these are VA but does $f \to +\infty$ or $-\infty$

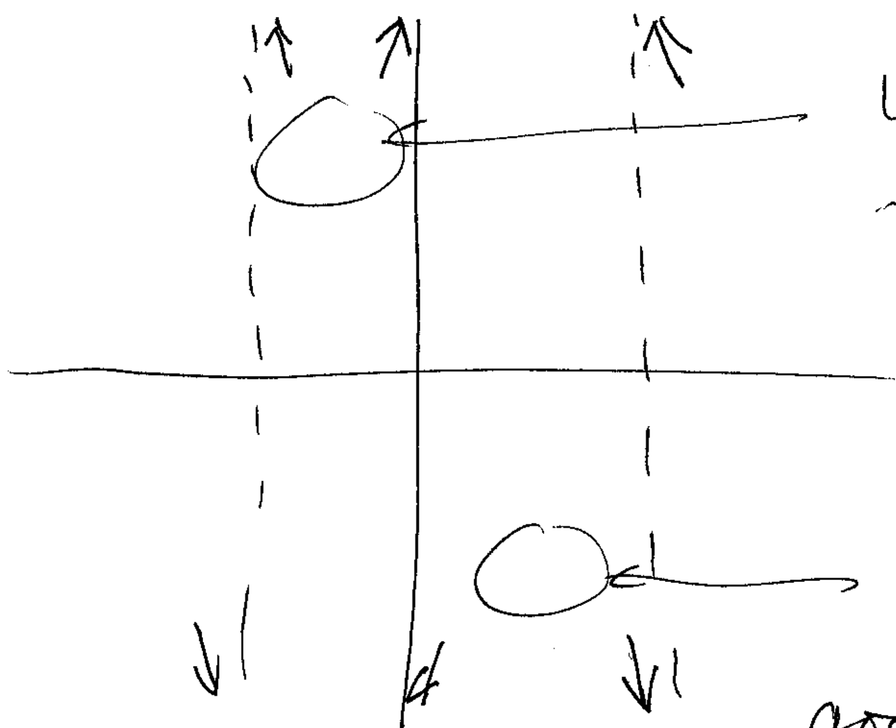


so we know we'll approach ∞ but how

– sign chart

let's consider the sign of each term

x		-1		0		1	
$x+1$	$-$	0	$+$	$+$	$+$	$+$	$+$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	$-$	$-$	0	$+$
$x(x-1)(x+1)$	$-$	0	$+$	0	$-$	0	$+$
$\frac{1}{x(x-1)(x+1)}$	$-$	∞	$+$	∞	$-$	∞	$+$



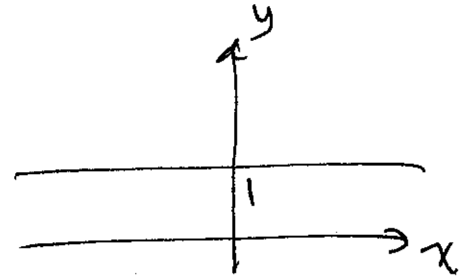
what happens here

and here

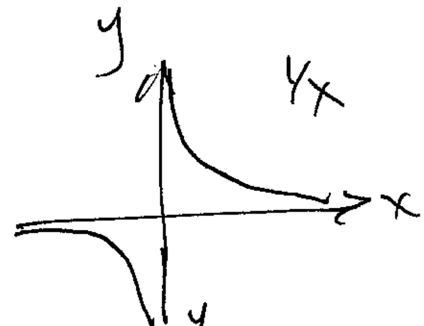
good question!

A couple of basic limits first

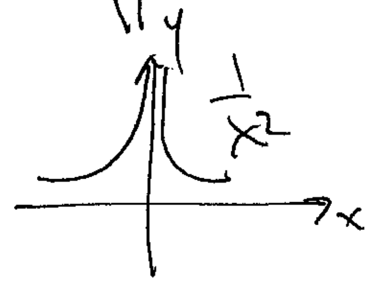
(1) $$\lim_{x \to \pm\infty} 1 = 1$$



(2) $$\lim_{x \to \pm\infty} \frac{1}{x} = 0$$



(3) $$\lim_{x \to \pm\infty} \frac{1}{x^n} = 0, \quad n > 0$$



So consider

$$\lim_{x \to \infty} \frac{x+1}{2x+1} = \lim_{x \to \infty} \frac{\frac{x}{x} + \frac{1}{x}}{\frac{2x}{x} + \frac{1}{x}}$$

$$\lim_{x \to \infty} \frac{1 + \frac{1}{x}}{2 + \frac{1}{x}} = \frac{\lim_{x \to \infty} 1 + \lim_{x \to \infty} \frac{1}{x}}{2 \lim_{x \to \infty} 1 + \lim_{x \to \infty} \frac{1}{x}}$$

$$= \frac{1+0}{2+0} = \frac{1}{2}$$

consider

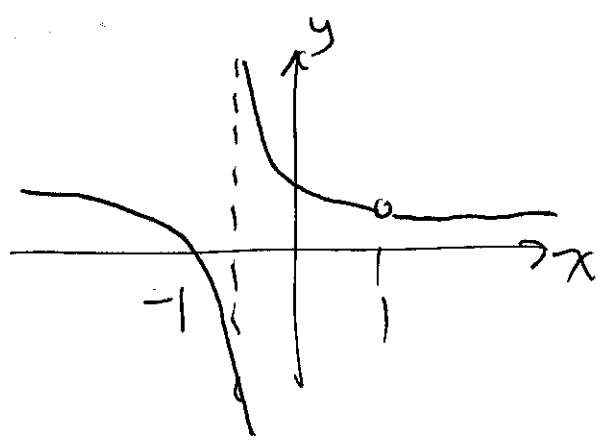
$$f(x) = \frac{x^2 - 1}{2x^2 + x - 1} = \frac{x^2-1}{(2x+1)(x-1)}$$

One might say that we have 2 VA

$x = -1/2, 1$

but at $x=1$ "$\frac{0}{0}$" so it might have a removable disc^{ty}

$$f(x) = \frac{(x-1)(x+1)}{(2x+1)(x-1)}$$



So now we consider what happens when x gets large – i.e. $\lim\limits_{x \to \pm\infty} f(x)$

what does the $f(x)$ do?

if $\lim\limits_{x \to +\infty} f(x) = L$

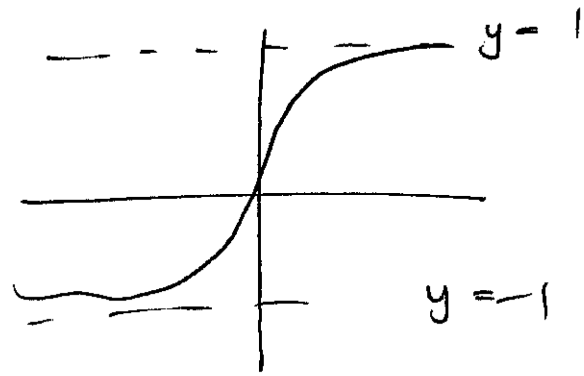
we say $y = L$ is a horizontal asymptote (HA)

similarly $\lim\limits_{x \to -\infty} f(x) = K$

then $y = K$ is a HA

sometimes $L = K$ sometimes not

Ex $f(x) = \dfrac{x}{\sqrt{x^2+1}}$



$$\lim_{x \to \infty} \frac{x}{\sqrt{x^2+1}}$$

$$\lim_{x \to \infty} \frac{x}{\sqrt{x^2(1+\frac{1}{x^2})}} = \lim_{x \to \infty} \frac{\cancel{x}}{\cancel{x}\sqrt{1+\frac{1}{x^2}}} = \frac{1}{\sqrt{1+0}} = 1$$

$$\lim_{x \to -\infty} \frac{x}{\sqrt{x^2(1+\frac{1}{x^2})}} = \lim_{x \to -\infty} \frac{x}{|x|\sqrt{1+\frac{1}{x^2}}}$$

$$= \lim_{x \to} \frac{x}{-x} \cdot \frac{1}{\sqrt{1+\frac{1}{x^2}}} = \frac{-1}{\sqrt{1+0}} = -1$$