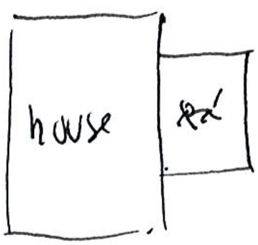
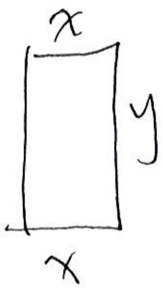
Math 1496 Calc I

Optimization - 1

Ex 1 You are to build rectangular pen for your dog along the house. You will use the house for 1 side of the pen. If you have 80 ft of fence. Determine the dimension that maximizes the area of the pen



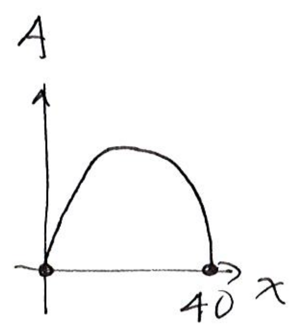


$A = xy \qquad P = 2x + y = 80$

so $y = 80 - 2x$

so $A = x(80 - 2x)$

$= 80x - 2x^2$



$A' = 80 - 4x$

$A' = 0$ when $80 - 4x = 0 \quad x = \frac{80}{4} = 20$

$A'' = -4$ so max

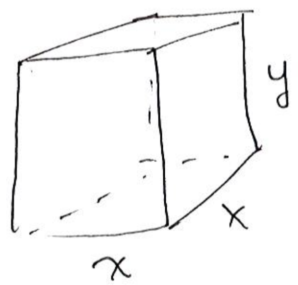
$y = 80 - 2(20) = 80 - 40$

$= 40$

so dimension

$40' \times 20'$

Ex 2 You wish to make a box with a square bottom. You have 96 sq inches. Find the dimension that maximizes volume



$$V = x^2 y \qquad A = 2x^2 + 4xy = 96$$

so $y = \dfrac{96 - 2x^2}{4x}$

$$V = \cancel{x^2}\left(\frac{96-2x^2}{4\cancel{x}}\right) = \frac{96}{4}x - \frac{1}{2}x^3$$

$$= 24x - \frac{1}{2}x^3$$

$V' = 24 - \frac{3}{2}x^2 \qquad V' = 0 \qquad 24 - \frac{3}{2}x^2 = 0 \qquad x^2 = \dfrac{2(24)}{3} = \dfrac{2(8)\cancel{(3)}}{\cancel{3}}$

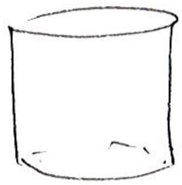
$x^2 = 16 \qquad x = \pm 4$ only pos. case

$x = 4, \quad V'' = -3x^2 < 0$ when $x = 4$ ~~min~~ $\cap$ max.

$$y = \left.\frac{96 - 2x^2}{4x}\right|_{x=4} = \frac{96 - 2(4)^2}{4(4)} = 4$$

so box is $4'' \times 4'' \times 4''$

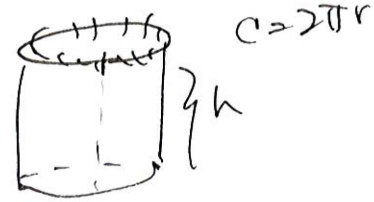
Ex 3



The can is to hold $1000\ cm^3$

find the dimension that min Area

$V = \pi r^2 h.$ $SA = 2\pi r h + 2\pi r^2$



$h = 1000/\pi r^2$

$$A = 2\pi r\left(\frac{1000}{\pi r^2}\right) + 2\pi r^2$$

$$= \frac{2000}{r} + 2\pi r^2$$

$$A' = -\frac{2000}{r^2} + 4\pi r$$

$A' = 0$ when $4\pi r = \dfrac{2000}{r^2}$

$$r^3 = \frac{2000}{4\pi} = \frac{500}{\pi}$$

$$r = \sqrt[3]{\frac{500}{\pi}}$$

$$h = \frac{1000}{\pi\left(\frac{500}{\pi}\right)^{2/3}} = 2\left(\frac{500}{\pi}\right)^{1/3}$$

Note! $A'' = \dfrac{2000}{r^3} + 4\pi > 0$ when $r = \sqrt[3]{\dfrac{500}{\pi}}$