Pre-Distribution: Bargaining over Incentives with Endogenous Production

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Abstract

This article sets forth a model of multilateral negotiations with alternating offers and voting in which members bargain over the distribution of equity shares prior to making investment decisions that determine the surplus to distribute. We challenge the classical assumption embedded in the divide-the-dollar setting that the fund to distribute is exogenoulsy given. Bargaining over productive incentives is fundamentally a different problem that yields an allocation of the surplus which usually does not coincide with the exogenous fund case. The proposer faces a trade-off between rent-extraction and rent-generation subject to the constraint of receiving a majority vote. If he attempts to offer small equity shares to coalition partners, the size of the pie might shrink, rendering his large ownership percentage unfruitful. In equilibrium, the proposer forms a minimum winning coalition by offering two types of shares. Some members are incentivized to invest in the common project and they receive a share that yields a payoff greater than the ex ante value of the game, thus the proposer's rent-seeking incentives are mitigated by the production constraints. The remaining coalition partners are offered a share that *buys* their vote without inducing investment. We provide a new view on efficiency in bargaining games which relates to the approved incentive scheme, and not only the timing of the approval.

I. Introduction

It is quite common that people explicitly or implicitly agree to a rent-sharing scheme prior to engaging in mutually beneficial activities such as starting a business partnership, fighting a common enemy by forming a military coalition, or coauthoring a book. In many cases, the agreements that govern the relationship between two or more parties are the result of a negotiation process and the agreements that are reached determine simultaneously productive incentives and profit-sharing rules.[1] In this article, we set forth a game of *pre-distributive* multilateral bargaining in which economic agents determine property rights prior to engaging in productive activities. Our aim is to model a situation where all players are part of the bargaining process and production occurs after a profit-sharing agreement has been reached. We seek to analyze how the bargaining institutional variables such as the committee size, voting rule, and discount factor affect the trade-off between rent-seeking and rent-generating incentives.

While contracting theory has mostly been focused on how to provide the right incentives to create a surplus, bargaining theory has been mostly concerned with how to redistribute an existing surplus. In fact, the classical models of structured bargaining with alternating offers (Rubinstein 1982, Baron and Ferejohn 1989) and the extensive generalizations that have followed[2], have assumed the existence of a fund which is to be divided via bargaining. This assumption might be appropriate in settings such as legislatures where committee members bargain over the distribution of a budget that has been generated by others, but it is certainly

[1] One can think of situations in which there is no prior negotiation because a default rule is implicitly agreed upon. For example, in certain disciplines it is a common practice to list the names of coauthors in descending order of importance and those involved in the project know ex ante who is the lead researcher. In other disciplines, authors are listed in alphabetical order implying an equal split of the intellectual credit.

[2] For an extensive treatment of bilateral bargaining with alternating offers see Osborne and Rubinstein (1990). For a generalization of the Baron and Ferejohn (1989) model see Eraslan (2000). See Jackson and Moselle (2002) for a model that combines private and public goods in the multilateral bargaining setting and Yildirim (2007) for rent-seeking behavior in proposal rights. See Merlo and Wilson (1995) for a model of stochastic bargaining. Montero (2007) for the incorporation of other-regarding preferences. For a brief review of experimental results regarding the Baron and Ferejohn game see Palfrey (2005), Frechette, Kagel, and Morelli (2005), and Baranski (2016).

not suitable for the partnership context which we seek to study here.[3] Here we show that bargaining over productive incentives is, indeed, a different problem because it poses a trade-off between productive and appropriative incentives which is absent in the classical setting. In our model, the two approaches coincide only for a restricted set of parameters in the limiting case when players are perfectly patient. We derive novel results regarding the distribution of the surplus and provide a new view on efficiency in structured bargaining games.

The model developed in this article employs the Baron and Ferejohn (1989; BF hereafter) game of multilateral bargaining as the negotiations protocol for establishing equity shares. In the original BF bargaining game[4], players negotiate with alternating offers and voting over the allocation of an exogenous fund.[5] The BF game is a cornerstone model of political economy and multilateral bargaining because it offers very clear predictions about the equilibrium bargaining outcome.[6] First, proposers disburse funds only to the smallest coalition of members required for approval of the proposal and those members included in the coalition are offered a share that exactly matches the continuation value of the game. Second, the proposer keeps a larger portion of the fund which is decreasing in the voting quota. Third, bargaining outcomes are efficient in the sense that proposals are accepted without delay.

In our model, players instead bargain over the distribution of equity shares which determine simultaneously productive incentives and a profit-sharing rule. Once an agreement is reached, partners proceed to an investment stage in order to determine the total fund. We

[3] Baranski (2016) and Frankel (1995) endogenize the origin of the fund prior to bargaining à *la* Baron and Ferejohn and Rubinstein respectively. See Battaglini and Coate (2007) and (2008) for dynamic models in which taxes and spendings are determined via legislative bargaining as in Baron and Ferejohn. The tax choice gives rise to an endogenous labor supply which generates the budget avaiable for spending. These papers are examined in the literature review section.

[4] Under the closed amendment rule, proposals on the floor are voted on without the possibility to modify them. Under the open rule, a proposal has to be seconded to take the floor for voting. We focus on the closed rule because it has received much attention in the theoretical and experimental literature.

[5] In the BF game, a randomly selected member is called to propose a distribution of the common fund and then all members proceed to vote. If a majority of partners votes in favor, the game ends and payoffs are realized. If the voting quota is not met, the process repeats itself (and usually the fund is discounted).

[6] In this article we will focus on the stationary subgame perfect equilibrium refinement (SSPE) and refer to it as *equilibrium.*

consider a very simple production technology (linear and symmetric) because our aim is to describe the strategic interaction between the bargaining environment and the provision of productive incentives, and avoid the complications that might arise with a richer production context.

When bargaining over equity, the proposer faces a trade-off between rent-sharing and rent-generation. If he attempts to offer a small share of equity to coalition partners, the size of the pie might shrink, rendering his large ownership percentage unprofitable. In equilibrium, the proposer forms a coalition by offering two types of shares: Some members are incentivized to invest (denoted hereafter as *productive members*) and others are simply offered a share that *buys* their vote without inducing investment. Productive members receive a payoff that is greater than the continuation value of the game, while non-productive coalition partners receive a share that induces a payoff equal to the ex ante value of the game. Hence, we are able to show that the proposer's rent-extraction aspirations are mitigated by the provision productive incentives. Redundant members (those whose vote is not required for approval) receive zero equity. In an extension of the model, we consider the case in which the total fund has an exogenous component. We show that the existence of an exogenous portion can crowd out productive incentives and enhance the agenda setter's rents. In general, the proposer finds it more profitable to increase own equity holdings (at the expense of losing investing partners) as the exogenous portion of the fund increases.

Pre-distributive bargaining also provides a new perspective on the analysis of efficiency in bargaining games. In the redistributive bargaining context, efficiency has typically been measured by how costly it is to reach a decision, where the cost of delay is parametrized through the discount factor (shrinking of the total surplus) or players' impatience levels.[7] Both BF and Rubinstein (1982) predict efficient equilibria in the sense that agreements are reached in the first round of negotiations. Moreover, the institutional variables such as the committee size and voting rule play no role in this regard. In the present setting, efficiency

[7] Rubinstein (1982) also considers a fixed cost of bargaining.

is not only about the timing of approval, but also about the agreement *per se* because the total fund will depend on the approved equity scheme. We find that efficiency, measured as total production, is inversely related to the voting requirement all else constant. With higher voting quotas, too much equity is diverted to *buying* votes and little is devoted to incentivize production. For two commonly used voting rules, unanimity and simple majority, efficiency is inversely related to the size of the committee. Thus, we provide a rationale for the existence of inefficiencies in committee decision-making processes which are not due to the costs of bargaining (discounting), but are a consequence of the distributional compromises that must be reached *a priori* in order to implement a common project.

The equity feasibility constraint, which states that the sum of ownership percentage shares must equal one, acts as a production technology constraint by setting a limit on the size of the fund that can be produced in equilibrium. Also, the fact that shares cannot be conditioned on productive decisions could be suppressing efficiency. For these reasons, we consider a variant of the model in which the proposer can condition compensations on members' contributions (we call this model contract bargaining). Here, the fully efficient outcome can be achieved regardless of the committee size, the voting rule, or the discount factor. In equilibrium, certain members are offered a compensation that makes them slightly better off by contributing but not enough to match the continuation value of the game. Hence, full efficiency does not entail unanimous approval.

The article proceeds as follows. Section 2 presents a literature review mainly focused on multilateral bargaining games. Section 3 introduces the model. Section 4 solves the equilibrium and provides the comparative statics. Section 5 analyzes the setting in which partners can bargain over contracts contingent of investments. Section 6 concludes the paper. Appendices A and B contain the main proofs. Appendix C extends the equity bargaining model by endogenizing the partnership size.

II. Literature Review

The standard BF model under the closed amendment rule[8] has been generalized in many directions and a comprehensive survey is provided in Eraslan and McLennan (2013). Eraslan (2002) considers the game in which players differ in their probability of being recognized as the proposer and hold different discount factors. It is shown that for a given parameter configuration, stationary subgame perfect equilibrium strategies (SSPE) need not be unique but the ex ante values of the game are. This implies that all equilibria yield the same vector of payoffs. Importantly, a player's ex ante value of the game is increasing in the probability of recognition and the discount factor. Eraslan and McLennan (2013) show uniqueness of payoffs in a setting where the set of winning coalitions may vary for each proposer.

The BF game has also been studied in an environment in which the surplus to distribute fluctuates stochastically over time. Merlo and Wilson (1995) restrict attention to the unanimity voting rule and show that efficient delay might arise in equilibrium because players are better off by waiting for a larger fund. Eraslan and Wilson (2002) consider any voting rule and show that when not every member's vote is required for approval, delay can also arise but it will be inefficient because agreements can be reached too soon. Eraslan and Merlo (2014) consider a case where each proposer *brings* a total fund of different magnitude to be distributed (i.e. the proposer and fund stochastic processes are perfectly correlated). In our model the fund may vary because anyone can contribute to expand the total fund, but the size of the fund is uniquely determined by the incentives embedded in the equity agreement.

In a closely related article to the game of pre-distribution, Baranski (2016) presents a game with the reverse timing: Players first engage in a production stage followed by a profit-sharing game of bargaining.[9] Investments can be considered a sunk cost at the beginning of

[8] The closed amendment rule refers to the situation in which a proposal on the floor is voted as is. In the open amendment rule protocol, another player can either challenge the proposal by submitting a new one or second the current proposal which then leads to a voting stage.

[9] See Frankel (1998) for a model of production prior to bargaining in the Rubinstein (1982) bilateral bargaining framework.

the bargaining stage (if strategies are restricted to be history-independent) and the resulting equilibrium prediction is that no one should contribute to the common fund. The reason is that the ex ante value of the bargaining game under the SSPE is equal to the average fund, which induces the same payoff structure as the linear public goods game.

Another application of the BF bargaining protocol to the firm context can be found in Britz, Herings, and Predtetchisnki (2013). In their setting, partners must agree unanimously over a future production plan for the firm in the midst of uncertainty. Differences in the risk attitudes of the committee members result in conflicts about which production plan to choose, but transfers that are payable prior to the realization of the state of nature are used to *grease the wheels* of the bargaining process. Their main result is that the payoffs resulting from the equilibrium production plan and transfer scheme determined in the bargaining game are equivalent to the payoffs specified by a generalized Nash bargaining solution where the relative bargaining power weights are given by the probability of being the proposer for each player.[10] Their model can be interpreted as a bargaining game of risk sharing, an aspect that is not present in the game of equity bargaining which we pose here.

Battaglini and Coate (2007, 2008) study a dynamic model in which taxing and spending decisions are determined via legislative bargaining in a finite horizon version of the Baron and Ferejohn game.[11] The tax level uniquely determines optimal behavior in the labor market (i.e. how much leisure to consume and hours to work) which in turn determines the total budget available for public spending. In this sense, Battaglini and Coate endogenize the surplus to distribute in the bargaining game. Their setting is dynamic because policymaking is linked across periods via the accumulation of a durable public good. Besides differences in the focus of the research question and the dynamic nature the Battaglini and Coate settings, our models are quite distinct in other dimensions as well. For example, workers receive

[10] Once they characterize the equilibrium in the payoff space, they show that there is a unique production plan and vector of transfers satisfying the Nash solution. This result holds in the limiting case where there is no discounting of future payoffs.

[11] Taxing is distortionary and the budget can be used for lump-sum transfers across districts (pork-barrel spending) or to fund a durable public good (or both). In their model, each player represents one district.

a wage in Battaglini and Coate (a payment per unit of labor), while in our main model they receive ownership shares. Both models consider a linear production function, however Battaglini and Coate incorporate a convex disutility of labor which guarantees an interior optimum of labor choice. There is no possibility to avoid paying taxes in their setting, while in our setting a player may receive a positive ownership share and not invest in the common project. Finally, bargaining can continue indefinitely until approval in our game while Battaglini and Coate impose a final period with a default distribution of resources.

The trade-off between productive and appropriative activities has been the object of study in political economy and organizational behavior. For example, Hirschleifer (1991), Skaperdas (1992), and Grossman and Kim (1995) develop models in which players can invest in *swords* as a means to appropriate others' production or invest in *plowshares* that yield a given output. In these settings, too many swords are useless when there is little production, so that in equilibrium the marginal investment in appropriation activities yields the same return as the marginal investment in production. Autocrats face a similar trade-off when determining property rights over their subjects in order to maximize their rents. Oslon (1993) explains how a stable monarchy will have stronger long run incentives to create and respect property rights compared to a "roving bandit". North and Weingast (1989) argue that "it is not always in the ruler's interest to use power arbitrarily or indiscriminately; by striking a bargain with constituents that provides them some security, the state can often increase its revenue (pg. 806)".[12] Our model assumes that property rights will be respected, hence we focus on how rights will be assigned and not the extent to which they will be upheld. Moreover, players may only appropriate what has been contributed to the common project through a preestablished agreement of ownership but may not appropriate other's endowments.

[12] The authors provide a historical account of how the possibility to limit the English monarch's confiscatory power after the Glorious Revolution lead to an increase in capital flows in the eighteenth century.

III. The Model

Let there be a committee with n players (odd) that are endowed with a wealth level normalized to 1. In stage 1, members bargain sequentially with alternating offers and voting on how to split the rents of a potential common fund that they will produce in stage 2. In each bargaining round (denoted by superscript $t \in \{1, 2, ...\}$) within stage 1, a player is randomly called upon (with equal probability) to propose a scheme on how to divide a common fund. Denote by $\mathbf{s}^t = (s_1^t, ...s_n^t)$ a division of the fund such that the sum of percentage shares satisfies $\sum s_j^t = 1$ (which we call the equity constraint) and each share $s_j^t \in [0, 1]$ for every player j. For each proposal on the floor, players vote to accept or reject and q votes are required for approval (including the proposer's vote).[13] In case of rejection, the proposal and voting rounds are repeated. Once an allocation is approved in round τ, players proceed to stage 2 in which they simultaneously choose a contribution $c_i \in [0, 1]$. Each unit contributed is multiplied times $\alpha \in (1, q]$ and becomes part of the common fund.

Let h^t denote the history up to round t in the bargaining period, which includes the list of previous proposers and their proposals, as well as the voting record. We denote by H^t the set of all possible histories. A strategy for a proposer in round t is given by $s : H^t \to [0, 1]^n$ and for voters it is given by $v : (s^t, H^t) \to \{Yes, No\}$.

A contribution strategy is a function $c_j : s^\tau \times H^\tau \to [0, 1]$ where τ denotes the round in which the proposal is approved.[14] Player j's payoff for a given profile of strategies $(\mathbf{s}^\tau, \mathbf{v}^\tau, \mathbf{c})$ is given by

$$u(\mathbf{s}^\tau, \mathbf{v}^\tau, \mathbf{c}) = \delta^{\tau-1} \left(s_j^\tau F - c_j + 1 \right) , \tag{1}$$

where $\delta \in (0, 1)$ is the discount factor and $F = \alpha \sum c_i(s^\tau, H^\tau)$ is the total profit of the committee.

[13] Voting takes place sequentially and in a known preestablished order. This avoids trivial multiplicity of equilibria.

[14] If the proposal is never approved, then by definition $\mathbf{c} = \mathbf{0}$ and payoffs are 0 to everyone.

IV. Equilibrium

We restrict attention to stationary subgame perfect equilibria, meaning that bargaining strategies are history-independent. We assume that a member votes in favor if and only if she is offered a share that yields a payoff that is equal to or greater than the continuation value of the game.[15]

Definition 1 A stationary equilibrium of the equity bargaining game, denoted by the stationary strategy profile $\boldsymbol{\sigma}^* := (\mathbf{s}^*, \mathbf{v}^*, \mathbf{c}^*)$ induces a vector of ex ante values of the game given by $\mathbf{V}^* = (V_1^*, ...V_n^*)$ such that $\boldsymbol{\sigma}^*$ and $\mathbf{V}^*$ satisfy:

1. For every player i we have that $u(\boldsymbol{\sigma}^*) \geq u(\tilde{\sigma}_i, \boldsymbol{\sigma}^*_{-i})$ for any other strategy $\tilde{\sigma}_i$;

2. s is only a function of the state of non-agreement, $v : \mathbf{s} \rightarrow \{Yes, No\}$, and $c : \mathbf{s} \rightarrow [0, 1]$;

3. $v = yes$ if and only if $u(\boldsymbol{\sigma}^*) \geq \delta V_i^*$;

4. If there are multiple strategy profiles $\boldsymbol{\sigma}^*$ that satisfy (1)-(3), we select the one that yields the highest payoff to the proposer.

Since we are interested in comparing proposer power between the standard Baron and Ferejohn game with an exogenous fund, we will focus on proposer-optimal equilibria as indicated in point 4 of the definition above.[16] Any other equilibrium notion which entails a larger fund will reduce the proposer's earnings.

In order to simplify our analysis we make one additional assumption about the parameter space. We require that the discount factor is not *too small*.

Assumption 1 The discount factor satisfies that $\frac{n}{n+\alpha-1} < \delta \leq 1$.[17]

[15]This is a standard assumption in the literature.

[16]In Appendix B we show by example that there can exist other equilibria which are more efficient in terms of production but yield a lower payoff to the proposer. When we relax condition (4) in Definition 1 we are not able to find a closed-form solution, hence we provide a general characterization of the conditions that must be met. See Proposition 4.

[17]Note that this depends on the committee size and productivity parameter. In particular, as the committee size increases, the lower bound on δ tends to 1.

The importance of this assumption will become clear in Lemma 2. It is a sufficient condition that ensures that all members in the proposer's coalition are offered a positive share, and rules out equilibria in which members receiving zero equity would also vote in favor.

IV.A Stage 2: Investment Subgame

We start in stage two by characterizing the possible subgames after approval. For any approved distribution of shares in round τ of stage 1 ($\mathbf{s}^\tau$), a player's equilibrium strategy in stage 2 is given by

$$c_i^*(\mathbf{s}^\tau) = \begin{cases} 0 & \text{if} \quad \alpha s_i^\tau < 1 \\ 1 & \text{if} \quad \alpha s_i^\tau \geq 1 \end{cases} \tag{2}$$

which simply states that a player finds it optimal to invest if and only if she has a positive return. It is straightforward to show that at most $\|\alpha\| \leq q$ players can be induced to contribute since the sum of shares must be equal to 1. From now on, we say a member is *incentivized* or *productive* if she is given a share such that $c_i^*(\mathbf{s}^\tau) = 1$.

IV.B Stage 1: Bargaining

When players are bargaining over shares, they are implicitly bargaining over the associated payoffs. Hence, we study the *implicit bargaining game* in the payoff space for which there are well-established results in the literature that will be invoked throughout the process.[18]

Before proceeding, is should be noted that any allocation in which there is no production cannot be an equilibrium (see Lemma 3 in Appendix A). We will assume that the proposer is always a producer while solving the model.[19]

We denote the proposer's share by s_{Prop} and the share offered to incentivized members

[18] See Britz, Herings, and Predtetchinski (2013) for a similar approach.

[19] This is technically not an assumption and is only invoked to ease the exposition. As we show, productive members are weakly better off than non-investing members in equilibrium. Hence, the proposer will always be a productive member.

by s_{Cont}. We allow for the possibility that certain members are offered a positive share that does not induce contribution and denote such share by s_{Vote}.[20]

Let k denote the number of productive members excluding the proposer (those to whom s_{Cont} is offered) and let m denote the number of voters to whom s_{Vote} is offered. It follows that the fund is given by

$$F(k) = \alpha(k+1) \quad . \tag{3}$$

(Below we will refer to $F(k)$ simply as F.) We can rewrite the equity constraint as

$$s_{\text{Prop}} + k s_{\text{Cont}} + m s_{\text{Vote}} = 1 \quad . \tag{4}$$

The ex ante value of the game can be defined as

$$V(m,k;q,n,\alpha) := \frac{1}{n} s_{\text{Prop}} F + \frac{k}{n} s_{\text{Cont}} F + \frac{m}{n}(s_{\text{Vote}} F + 1) + \left(1 - \frac{1+k+m}{n}\right) \tag{5}$$

and it is a weighted average of the payoffs that a player receives in each possible role that she might find herself in.[21] The last term, $1 - \frac{1+k+m}{n}$, denotes the expected payoff from not being assigned equity.[22] Using the equity constraint we obtain

$$V(m,k;q,n,\alpha) = \frac{F}{n} + 1 - \frac{1+k}{n} = \frac{(\alpha-1)(k+1)}{n} + 1 \quad . \tag{6}$$

Equation (6) has the intuitive interpretation that the ex ante value of the game, for a given k, is equal to the average fund net of contributions plus the endowment.

[20] Although it seems we are imposing a solution structure in solving the game, all we are doing is assuming that if two members are incentivized, then the offered shares are equal. In order to meet the voting quota, the proposer might require the votes of other non-incentivized partners whom are offered s_{Vote}. Again, we do not impose how many members receive this share.

[21] To calculate this, we are imposing symmetry (what player j offers i is what i offers j). We also assume that proposers randomize over whom to offer s_{Vote} and s_{Cont}. Hence, in k/n times a member is offered s_{Cont} and in m/n she is offerd s_{Vote}.

[22] Notice that $1 - \frac{1+k+m}{n}$ is the probability of being assigned $s_i = 0$ which implies $c_i^* = 0$. This results in a payoff of 1 (keeping the endowment).

In order for the fund to be attainable (i.e. effectively produced), three production consistency conditions must be met:

$$1/\alpha \leq s_{\text{Prop}} \leq 1 \quad , \tag{7}$$

$$1/\alpha \leq s_{\text{Cont}} \leq 1 \quad , \tag{8}$$

$$0 \leq s_{\text{Vote}} < 1/\alpha \; . \tag{9}$$

Conditions (7) - (9) guarantee that there are exactly $k+1$ productive members (including the proposer). Furthermore, we require that members to whom a positive share is offered find it optimal to vote in favor. Hence, the share offered must induce a payoff that is greater than or equal to the continuation value of the game. The voting consistency conditions are:

$$s_{\text{Prop}}F \geq \delta V \quad , \tag{10}$$

$$s_{\text{Cont}}F \geq \delta V \quad , \tag{11}$$

$$s_{\text{Vote}}F + 1 \geq \delta V \quad , \tag{12}$$

$$m + k + 1 \geq q \quad , \tag{13}$$

where the last condition specifies that the proposal receives the necessary amount of votes.[23] We are now ready to present the maximization problem that a proposer faces:

$$\max_{\{k,m,s_{\text{Prop}},s_{\text{Cont}},s_{\text{Vote}}\}} s_{\text{Prop}} \cdot F \tag{14}$$
$$\text{s.t. conditions (4) and (7)-(13)} \quad .$$

A few observations will help us rewrite the problem more concisely in terms of k only. Note that s_{Prop} is decreasing in m and that F is not dependent on m, so that the proposer will restrict the amount of s_{Vote} offers made to exactly meet the voting quota. Also, recall

[23] Since we are working in the parameter space where $\alpha \leq q$ this implies that $k \leq q$.

that $k \leq \|\alpha\| \leq q$.[24] Hence, we have that $m = q - 1 - k$.

LEMMA 1 In equilibrium, $s_{\text{Cont}}(k) = \begin{cases} 1/\alpha & \text{if} \quad k \geq \tilde{k} \\ \delta V/F & \text{otherwise} \end{cases}$ where $\tilde{k} := \frac{\delta n}{n - \delta(\alpha - 1)} - 1$.

PROOF. Conditions (8) and (11) imply that $s_{\text{Cont}} \geq \max\{1/\alpha, \delta V/F\}$ and we can show that

$$\begin{aligned} 1/\alpha &\geq \delta V/F \iff \\ k+1 &\geq \delta\left[\frac{(k+1)(\alpha+1)}{n} + 1\right] \iff \\ k &\geq \frac{\delta n}{n - \delta(\alpha - 1)} - 1 = \tilde{k} \,. \end{aligned}$$

Since the proposer's payoff decreases in s_{Cont}, in equilibrium it must be that constraints (8) or (11) bind (or both at $k = \tilde{k}$). ■

From condition (12) we solve for s_{Vote} to be

$$s_{\text{Vote}}(k) = \max\left\{\frac{\delta V - 1}{F}, 0\right\}$$

and again it is straightforward to verify that this constraint binds. In the following lemma we make use of Assumption 1 to determine $s_{\text{Vote}}(k)$.

LEMMA 2 If $\delta > \frac{n}{n+\alpha-1}$ then $\frac{\delta V - 1}{F} > 0$ for all k.

PROOF. We have that $\delta > \frac{n}{n+\alpha-1} = \frac{1}{1+(\alpha-1)/n} \geq \frac{1}{1+(k+1)(\alpha-1)/n} \implies \delta\left[1 + (k+1)(\alpha-1)/n\right] = \delta V > 1$ and the result follows. ■

We are now able to write problem 14 as a function of the number of incentivized members (k):

$$\max_{k \in \{0,\ldots,\|\alpha\|\}} \Pi(k) := s_{\text{prop}} \cdot F = \left[1 - k \cdot s_{\text{Cont}}(k) - (q - 1 - k)\left(\frac{\delta V - 1}{F(k)}\right)\right] \cdot F \quad . \tag{15}$$

[24] That fact that $k \leq \|\alpha\|$ follows from equations (7), (8), and (4). If $k > \|\alpha\|$, then $k \cdot \frac{1}{\alpha} + s_{\text{prop}} > 1$ violating (4).

Temporarily, we drop the integrality requirement on k and solve the associated program (where $k \in \mathbb{R}$). By standard arguments, the solution to this problem exists. In Appendix A (Lemma 4) we show that the optimal solution is given by

$$\bar{k} := \frac{1}{2} \frac{n(\alpha+\delta) - \delta q(\alpha-1)}{n - \delta(\alpha-1)} - 1 \quad . \tag{16}$$

We now wish to characterize the optimal integer solution which is denoted by k^*. Given the concavity of the objective function, the proposer's payoff attains its highest value at one of the closest integers to $\bar{k}$ which are $\|\bar{k}\|$ and $\|\bar{k}\| + 1$. However, one needs to verify whether such integers are feasible. We denote by Ω the set of k that satisfy conditions (4) and (7)-(13). It is easy to see that $\|\bar{k}\| \in \Omega$ always and, by example, one can show that $\|\bar{k}\| + 1$ need not be feasible. Hence, we have that

We have that

$$k^* := \begin{cases} \|\bar{k}\| + 1 & \text{if} \quad \|\bar{k}\| + 1 \in \Omega \;\; \text{and } \Pi(\|\bar{k}\| + 1) \geq \Pi(\|\bar{k}\|) \\ \|\bar{k}\| & \text{otherwise} \end{cases} \tag{17}$$

defines the optimal solution to problem (15).

We summarize the equilibrium in the following proposition.

Proposition 1 The equilibrium outcome of the equity bargaining game is as follows:

1. The proposer assigns $s^*_{\text{Cont}} := s_{\text{Cont}}(k^*)$ to k^* members, $q - k^* - 1$ other members receive $s^*_{\text{Vote}} := s_{\text{Vote}}(k^*)$, and the proposer assigns herself $s^*_{\text{Prop}} := 1 - k^* s^*_{\text{Cont}} - (q - k^* - 1) s^*_{\text{Vote}}$. The remaining $n - q$ members do not receive equity shares.

2. All members offered s^*_{Cont} or s^*_{Vote} vote in favor.

3. Only the proposer and those who obtain a share s^*_{Cont} contribute all their endowment.

4. There is no delay in approval.

This equilibrium characterization presents three novel results in the multilateral bargaining literature. The first is that not every member of the coalition obtains the same payoffs. Note that non-productive coalition partners receive a share that yields exactly the continuation value of the game while productive members may receive $\frac{1}{\alpha}$ (depending on the parameter configuration) which yields a higher payoff. This takes us to the second feature: the need to provide productive incentives mitigates the proposer's rent-extraction capacities.

A third difference is that a member's voting decision does not depend only on the share she receives, but on the entire proposal. Recall that a member votes in favor if and only if her share yields a payoff greater than or equal to the discounted continuation value of the game. The total fund and the continuation value are a function of the number of productive members which is determined by the proposal (vector of shares), and not only the individual share. Thus, a member that receives s^*_{Vote} will vote against if the proposer did not offer s^*_{Cont} to k^* other partners.

In general, one can show that k^* is typically below the socially optimal level of production given by $k = \|\alpha\| - 1$. By socially optimal production we mean the level that can be implemented by a social planner who distributes shares without forcing investments. Note that when $\alpha < 2$ the socially optimal production coincides with $k^* = 0$ because there is only enough equity to incentivize one member (the proposer). The socially optimal level can also be achieved in equilibrium for high productivity as α approaches q and q approaches n. A formal argument is presented in the Appendix (see Corollary 8).[25]

Our model can also give rise to fully productive partnerships but this requires a high productivity level and the unanimous voting rule. We now show that there is a range of values of the discount factor for which the proposer-optimal equilibrium coincides with a fully productive equilibrium (i.e. when $k = n - 1$ which means everyone invests).[26]

[25] It is not possible to derive a more concrete characterization of the cases where efficient outcomes coincide with the proposer optimal equilibrium. This is due to the fact that one must verify several feasibility constraints in order to guarantee that the integer solutions match.

[26] Clearly, the fully productive equilibrium is also socially optimal.

Corollary 1 Let $\alpha = q = n$. Then, there exists $\delta(n)$ such that $k^* = n - 1$ for all $\delta \in [\delta(n), 1]$.

Letting $\delta = 1$ and $\alpha = q = n$ one can easily verify that $\bar{k} = n - 1$ which implies that all members are investing. This is because the proposer offers $1/n = 1/\alpha$ to all of them and a reduction of anyone's share would reduce the fund and not lead to approval. As δ decreases, the discounted continuation value of the game falls, hence the proposer can potentially offer a share $s_{\text{vote}} < 1/\alpha$ to some members and obtain their vote while he increases his own equity holdings. However, if δ is very close to 1, the integer solution need not change because the proposer's increase in equity from downgrading a productive member's share is not enough to compensate the loss in total production. In the Appendix we provide a formal proof.

IV.C Comparative Statics

We are now ready to present various comparative statics results regarding the equilibrium size of the fund. We will focus on the equilibrium region where $s_{\text{Cont}} = 1/\alpha$.

Corollary 2 In equilibrium, the following results hold about $F(k^*)$:

1. it is weakly decreasing in q;

2. it is weakly increasing in n;

3. it is weakly decreasing in δ if $n < (q-\alpha)(\alpha-1)$ and weakly increasing if $n \geq (q-\alpha)(\alpha-1)$;

4. it is increasing in α.

Results (1), (2) and (4) of Corollary 2 are straightforward to verify by computing $\partial\bar{k}/\partial q$, $\partial\bar{k}/\partial n$, and $\partial\bar{k}/\partial\alpha$.[27] When more votes are required for approval, the proposer must weigh the benefits of adding an extra productive or voting member and satisfy the feasibility

[27] The comparative statics in (1)-(3) specify weakly monotonic relations is due to the fact that a change in the variable in question might not be enough to induce a discrete change in k^*. We omit the proof because these are simple arithmetic calculations.

constraint. It turns out that as q increases, adding an extra productive member is too expensive in terms of equity and the overall effect is a reduction in the fund.

The positive relationship between the equilibrium fund and the committee size is quite surprising. To see why this is the case, note that when q is fixed, adding more members to the committee does not alter the voting constraint, and that as n gets larger, the ex ante value of the game becomes smaller. This implies that the share offered to a non-productive voter decreases and it can reach a point where the proposer has enough available equity to upgrade a non-productive coalition partner's share so that she becomes productive.

The relationship between the optimal fund and the discount factor is guided by more subtle dynamics. In equilibrium, as δ increases, the share offered to a non-productive coalition member (s_{Vote}) increases (for a fixed k), while s_{Cont} is constant. This means that the proposer must give up own equity if he wants to sustain the same level of production. Alternatively, he can take one of two paths: (1) sacrifice a productive member and replace her by a voting member or (2) replace a voting member by a productive member.[28] Summarizing, the proposer must weigh the payoffs from maintaining the level of output by sacrificing own equity, increasing output and sacrificing equity, and reducing the fund and augmenting his own equity.[29] The optimal choice will depend on the parameter configuration, which we proceed to explain.

Fixing q and n, the inequality $n > (q-\alpha)(\alpha-1)$ is more likely to hold when either α is large (close to q) or small (close to 1), which set us in the region where the fund increases with δ. A large α makes s_{Cont} relatively cheap in terms of equity, thus making it more attractive to enhance a non-productive partner's share to productive levels. When α is small, replacing a voting member by a productive member induces a big loss of equity to the proposer but this loss is less than proportional to the percentage increase in production. Note that for a small α, the total fund is small as well. Thus, adding a productive member generates a large proportional increase in the fund.

[28] Notice that the replacements take place in order to meet the voting quota.

[29] When sacrificing a productive member, the proposer is able to increase his equity because $s_{\text{Vote}} < s_{\text{Cont}}$.

For intermediate values of α, adding a productive member is no longer as cheap as it is for large α, nor does it induce a large proportional change in the size of the fund as it happens for small values of α. Hence, as δ increases, the proposer finds it optimal to replace a productive member by a voting member in this region, a decision that entails a reduction in the total fund.

EXAMPLE 1 Let $n = 15$. For $q = 11$ and $\alpha = 11$ we have that $k^* = 6$ if $\delta = 1$ and $k^* = 5$ if $\delta = 0.5$. Hence, the fund decreases as δ decreases. The opposite effect happens when $n = 21$, $q = 18$ and $\alpha = 15$ because we have that $k^* = 5$ if $\delta = 1$ and $k^* = 6$ when $\delta = 0.5$.

We now examine the effect of committee size on the total fund for the simple majority and unanimity voting rules.[30]

COROLLARY 3 Larger committees yield (weakly) lower efficiency under the unanimity and simple majority voting rules.

A proof can be found in Appendix A. Recall that in Corollary (2) we had fixed the voting quota and considered a change in the size of the committee. Here, the voting quota is pegged to the size of the committee. Therefore, increasing the committee size entails that more votes need to be *bought* in order to obtain approval. Although non-productive coalition partners become cheaper, the decrease in equity that must be disbursed to them is surpassed by the increase in equity disbursed by to non-productive voters as the committee size increases.

V. Bargaining over Contracts

Previously, we considered a setting in which member's compensations could not be conditioned on contribution levels. We now relax this assumption and model a situation in which the proposer can offer members a contract of the form $f_i(c_i) = a_i c_i$ where a_i specifies

[30]Note that for the unanimity rule we have that $q = n$ and for the simple majority rule $q = (n + 1)/2$.

the compensation per unit contributed by player i. A proposal in period t is denoted by $\mathbf{a}^t = (a_2^t, ...a_n^t)$ where the i^{th} entry is player i's contract. Without loss of generality we identify the proposer with the player index $i = 1$. In order to avoid unnecessary theoretical complications, we simply define the proposer as the residual claimant, i.e. his payoff is defined as the amount remaining after paying out contracts based on contributions.

At a terminal bargaining node τ in which the approved proposal is $\mathbf{a}^\tau$, a player's optimal contribution strategy is given by

$$c_i^*(\mathbf{a}^\tau) = \begin{cases} 0 & \text{if} \quad a_i^\tau < 1 \\ 1 & \text{if} \quad a_i^\tau \geq 1 \end{cases} . \tag{18}$$

The total fund is given by $F(k_c) = \alpha(1 + k_c)$ where k_c is the number of productive members excluding the proposer.

Proposition 2 In equilibrium, $n - q$ members receive $a_i = 1$; $q - 1$ members receive $a_i = \max\{\delta\alpha, 1\}$ and $k_c^* = n$. A member votes in favor if and only if $a_i \geq \delta\alpha$. There is no delay in approval.

A sketch of the proof is provided and the details can be found in Appendix A. Note that the smallest a that induces contribution is $a = 1$. For each possible number of productive members, it always pays to incentivize an additional partner because the proposer appropriates a portion of the generated rents. Hence, everyone will obtain a contract in which a is at least 1. In order to meet the voting quota, the proposer must offer a contract such that $q - 1$ members will vote in favor. The minimum winning coalition receives a contract that yields a payoff equal to the continuation value of the game.

Note that when partners are very impatient (such that $\delta\alpha < 1$) every member receives the same contract ($a_i = 1$) and the proposer appropriates all the production net of investments. For comparison with the equity bargaining model, Corollary 4 contains the comparative statics.

Corollary 4 In equilibrium, the following results hold about $F(k_c^*)$:

1. it is constant in q

2. it is increasing in n;

3. it is constant in δ;

4. it is increasing in α.

VI. Discussion on Bargaining over Endogenous and Exogenous Funds

In this section we analyze two issues. First we compare the distribution of final payoffs between Baron and Ferejohn game with an exogenous fund and the game of predistribution in order to analyze when they coincide and when they differ. Second, we introduce an exogenous fund into the equity bargaining game in order to analyze how bargaining outcomes and efficiency may be affected. In particular we are able to show that, in the presence of an exogenous fund, rent-seeking incentives can significantly crowd out productive incentives.

VI.A Relationship between Exogenous and Endogenous Fund Bargaining Payoffs

In order to make the bargaining games with an endogenous and exogenous fund comparable, we must transform the standard Baron and Ferejohn game to allow for players to hold an initial endowment and for the surplus to distribute to be defined as the surplus (total fund net of investments) that would be available in equilibrium under the predistributive equity bargaining game.

Proposition 3 (Baron and Ferejohn 1989 Proposition 2) Let there be n players, with a q voting rule and $\delta \in (0,1]$ each endowed with one unit of wealth. Let $(k+1)(\alpha-1)$

with $\alpha > 1$ and k a non-negative integer be the total surplus to distribute. The equilibrium payoffs of the bargaining game including the initial wealth endowment are as follow:

1. The proposer offers $q-1$ members $\frac{\delta(k+1)(\alpha-1)}{n}$ and thus they earn a final payoff of $1+\frac{\delta(k+1)(\alpha-1)}{n}$

2. $n-q$ members are offered 0 and thus they earn a final payoff of 1.

3. The proposer keeps the rest and earns $1+(k+1)(\alpha-1)-(q-1)\frac{\delta(k+1)(\alpha-1)}{n}$.

Corollary 5 The distribution of payoffs resulting from the Baron and Ferejohn game defined in Proposition 3 with $k = k^*$are equivalent to the distribution of payoffs in equity bargaining game if and only if $\delta = 1$ and $k^* \leq \tilde{k}$.

The proof to the corollary relies on the fact that $s_{\text{Cont}} = \frac{\delta V}{F}$ for $k \leq \tilde{k}$ and that non-productive coalition partners are offered a share $s_{\text{Vote}} = \frac{\delta V - 1}{F}$. These conditions imply that all coalition partners (except the proposer) earn a payoff that is equal to the continuation value of the game. It follows that $q-1$ coalition partners (except the proposer) earn $\delta V = \delta\left[\frac{(k+1)(\alpha-1)}{n}+1\right]$ which is equal to $1+\frac{\delta(k+1)(\alpha-1)}{n}$ (the payoff of a coalition member in the BF game with an exogenous fund) if and only if $\delta = 1$.

The equivalence of payoff distribution between the contract bargaining game and the standard BF redistributive game holds for a broader range of parameters than for the equity bargaining game but it still a necessary condition that $\delta = 1$.[31]

Corollary 6 The distribution of payoffs resulting from the Baron and Ferejohn game defined in Proposition 3 with $k = k_c^*$ are equivalent to the distribution of payoffs in contract bargaining game if and only if $\delta = 1$.

[31] If we are only concerned about the distribution of the surplus, namely $\frac{\delta(k+1)(\alpha-1)}{n}$ (i.e. we do not take into accound initial endowments), then the only required condition for equivalence between the endogenous and exogenous fund cases is that $k^* \leq \tilde{k}$.

VI.B Equity Bargaining with a Partially Exogenous Fund [32]

In our equity bargaining model the total fund is determined by the sum of contributions multiplied times the productivity parameter α. Here we consider

$$F = \alpha(k+1) + x \tag{19}$$

where $x \geq 0$ is an exogenous amount that is available to distribute regardless of players' investments and publicly known prior to the start of the bargaining rounds. In this case, the ex ante value of the game for any k is given by

$$V(m, k; q, n, \alpha) = \frac{F}{n} + 1 - \frac{1+k}{n} = \frac{(\alpha-1)(k+1)}{n} + \frac{x}{n} + 1 \tag{20}$$

which is equal to the one derived in equation (6) plus x/n (the average exogenous fund).[33] Solving the problem in the same manner as we did for $x = 0$ we obtain the optimal solution to problem (15) which is given by

$$\bar{k}_x := \bar{k} - \frac{x(n-\delta\alpha)}{2\alpha(n+\delta-\delta\alpha)} \quad . \tag{21}$$

In a similar fashion as before, one can easily specify the integer solution k_x^*.[34]

Corollary 7 In equilibrium we have that:

1. k_x^* is non-increasing in x.

2. $\lim_{x\to 0} k_x^* = k^*$.

[32] I would like to thank an anonymous referee for this suggestion.

[33] Here we will be analyzing the case for which at least the proposer finds it optimal to invest. It should be noted that with a very large exogenous fund the proposer might not be able to retain a share large enough that incentivizes himself to invest. In order to derive a clear comparison with the previous findings we focus on the case in which the proposer produces. The more general case does not add any new insight nor does it change our results in Corollary 7.

[34] This is given by $\|\bar{k}_x\|$ or $\|\bar{k}_x\| + 1$ whichever is feasible and yields the largest payoff to the proposer. Whenever $\bar{k}_x < 0$ then $k_x^* = 0$.

The results are quite intuitive. If x is large enough, the proposer is better off by sacrificing productive members whose contributions to the total fund are not enough to compensate for the extra equity that must be disbursed to them in order to incentivize them to invest. The proposer receives a larger payoff by offering them a lower share which is just enough to obtain their vote. Thus, the existence of an exogenous fund crowds out efficiency by exacerbating the proposer's rent-extraction incentives.

It should be noted that in the particular case in which $q = \alpha = n$ and $\delta = 1$ the total fund is constant in x and $k_x^* = k^*$. The reason is that in equilibrium, regardless of x, each member receives a share equal to $1/n = 1/\alpha$ which implies that everyone invests.

To explain part (2) of the previous corollary, note that $x = 0 \Rightarrow \bar{k}_x = \bar{k}$. However, the integer solution might coincide for small values of x. For this reason we have stated our equivalence result in terms of the limit as x approaches 0.

VII. Conclusion

One application of our model can be found in the formation of international alliances, a topic which is mainly studied in the field of international relations. Several studies have attempted to answer why members of an alliance bear different burdens, especially in military alliances as measured by investments in military operations (see Olson and Zeckhauser 1966, Thies 1987, Sandler 1993, Sandler and Hartley 2001). Some authors have argued that an alliance creates a public good and differences in valuations for the public good will generate differences in investments or expenditures between members. However, not all authors agree that alliances create pure public goods, and instead attribute the differences in burdens to the individual and private incentives that allies might have.[35] Our model takes the latter stance without imposing any asymmetry between potential allies. Instead, every member is ex ante

[35] See the seminal paper by Olson and Zeckhauser (1966) for the pure public goods nature of alliances. See Loehr (1973) and Sandler and Cauley (1975) for models challenging the Olson and Zeckhauser (1966) setting. See Sandler (1993) and Sandler and Hartley (2001) for comprehensive reviews of the literature on alliance formation and burden sharing.

equal. Ex post asymmetries resulting from the bargaining outcomes assign property rights over the benefits of the alliance which induce differences in investment levels or burdens. In such settings, the voting quota can be understood as the minimum required consenting parties for the alliance to form and effectively implement its objective, i.e. not as a political requirement but as a productive technology constraint.

Although the bargaining protocol and the production technology are quite stylized and abstract from relevant factors at play in *real* business partnerships, it has been well documented that certain firms opt for a *retrospective* profit redistribution mechanism (or *a posteriori)* as in Baranski (2016), while others implement a *prospective* (or *a priori*) profit redistribution scheme as in the current model.[36] Our theoretical results support the claim that prospective mechanisms are better from an efficiency standpoint in the context of multilateral negotiations. However, the experiments in Baranski (2016) revealed that ex post bargaining can be used to reward high contributors and punish low contributors, thus inducing almost fully efficient outcomes. It remains an open experimental question of whether or not pre-distributive bargaining will also enhance efficiency beyond the theoretical predictions and outperform the efficiency levels observed in the investment games with ex post bargaining.

In our analysis we have employed a linear and symmetric production technology. The symmetry assumption has been useful in solving the equilibrium in the outcome space without having to explicitly characterize bargaining strategies. As such, we have benefitted from well established results about the stationary value of the game in a symmetric setting. If one were to consider asymmetric partners, the equilibrium strategies must be explicitly solved for (i.e., the probability of each player of being included or not in a winning coalition and the

[36]According to an weblog post of the New York Bar Association (Rose, 2011), a key question about the partner compensation system is "Will the distribution be prospective (distribution percentages or units of participation determined in advance of the year) or retrospective (distribution percentages or units of participation determined when year-end results are known)?" Multiple management consulting firms offer advice on how to design an optimal partner compensation plan in order to induce effort and maximize profits. See the reports on partner compensation systems in law firms by Edge International (Anderson, 2001), Altman and Weil (Cotterman, 2014), and surveys reports by Edge International (Wessman and Kerr, 2015) and Major, Lindsey, and Africa (Lowe, 2014).

resulting stationary value of the game for each player as a function of her characteristics). For example, a partner with very high productivity is more valuable when properly incentivized to produce compared to a low-productivity partner. The highly productive partner is likely to ask for a higher portion of the pie, but such an increase in the demand of surplus might decrease his probability of inclusion in the winning coalition. Recall that in our setting, a partner can be productive or non-productive (and payoffs resulting from each role are generally different), hence the problem is much more complex. We conjecture that partners with higher productivities or endowments are more likely to be invited to the coalition as productive than as non-productive partners.

The linearity assumption implies that there are no synergies in production. If we consider an alternative extreme case in which the member with the lowest investment choice determines the aggregate production (as in a weakest-link type production function), then we would not expect the allocations of equity shares to be minimal winning coalitions. This is because an excluded member would have no incentive to exert effort or invest and the total surplus would be zero. Hence, we would expect synergies to induce more inclusive ownership agreements and the voting rule to have less of an impact in the provision of productive incentives.

We also confirm a common intuition that committees with higher voting requirements are less productive. In our model, this trade-off takes place because much of the available equity must be used for buying votes instead of fostering investments, which evidences how the bargaining process can take a toll at efficiency.[37]

Pre-distributive bargaining has been widely neglected in the literature where the focus has been on the *divide-the-dollar* paradigm. We have shown that negotiating incentives to *create the dollar* poses new strategic dynamics that are absent in the classical setting with an exogenously given fund. Future research should focus on varying the production technology or bargaining protocol to better understand the trade-off between rent-generation and rent-

[37] However, this result relies on the fact that partner productivity is not affected by the number of agreeing partners which again highlights the relevance of productive synergies.

extraction in the multilateral bargaining context, a setting that resembles many economic and political activities.

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Appendix A. Proofs

Positive production in equilibrium

LEMMA 3 In equilibrium, at least one member will produce.

PROOF. Suppose that $\mathbf{s}$ is an equilibrium allocation of shares such that $\alpha_i s_i < 1$ for all i. Then, the equilibrium payoffs of the game are 1 for each player. A proposer can unilaterally deviate by giving ϵ to $q-1$ members where ϵ is small and keeping $1-q\epsilon > 1/\alpha$. The proposal will be approved and it yields a higher payoff to some members because the proposer is incentivized to produce. ■

Derivation of the Optimal k.

LEMMA 4 For $k \in [0, \|\alpha\| - 1] \subset \mathbb{R}$ the function

$$\Pi(k) := \begin{cases} \left[1 - k \cdot \frac{1}{\alpha} - (q-1-k)\left(\frac{\delta V - 1}{F(k)}\right)\right] \cdot F(k) & \text{if} \quad k \in [\tilde{k}, \|\alpha\|] \\ \left[1 - k \cdot \frac{\delta V}{F(k)} - (q-1-k)\left(\frac{\delta V - 1}{F(k)}\right)\right] \cdot F(k) & \text{if} \quad k \in [0, \tilde{k}] \end{cases} \tag{22}$$

has the following properties:

1. It is linear and increasing in $k \in [0, \tilde{k}]$.

2. It is quadratic in $k \in [\tilde{k}, \|\alpha\|]$with a maximum at $\bar{k}$.

3. It is continuous at $k = \tilde{k}$.

4. $\tilde{k} \leq \bar{k}$ with equality if and only if $\delta = 1$ and $q = n$.

PROOF. The proofs for (1)-(3) are simple arithmetic computations. For (4) we have that

$$\begin{aligned} \tilde{k} &\leq \bar{k} \iff \\ \frac{\delta n}{n - \delta(\alpha - 1)} - 1 &\leq \frac{1}{2}\frac{n(\alpha + \delta) - \delta q(\alpha - 1)}{n - \delta(\alpha - 1)} - 1 \iff \\ \delta q(\alpha - 1) &\leq n(\alpha - \delta) \end{aligned}$$

and the result follows because $\delta q \leq n$ and $\alpha - 1 \leq \alpha - \delta$. Equalities only hold when $\delta = 1$ and $q = n$. ■

Proof of Corollary 3

Let $\bar{k}^M$ and $\bar{k}^U$ denote $\bar{k}$ given by (16) under the simple majority and unanimity rules. For the majority rule we are evaluating at $q = (n+1)/2$ and for the unanimity at $q = n$. We have that

$$\frac{\partial \bar{k}^M}{\partial n} = -\frac{(\alpha - 1)[\alpha(2 - \delta) + 3\delta - 1)]}{4(n + \delta - \alpha\delta)^2}$$

and this is always negative because $\alpha(2-\delta)+3\delta-1>0$ given that $\alpha>1$ and $2-\delta>1$ thus $\alpha(2-\delta)>1$.

Similarly for the unanimity rule we have that

$$\frac{\partial \bar{k}^U}{\partial n}=-\frac{(\alpha-1)[\alpha(1-\delta)+2\delta)]}{2(n+\delta-\alpha\delta)^2}$$

and this equation is always negative.

Corollary

Corollary 8 We have that $\alpha-1\geq\bar{k}$.

Proof. By Corollary 2 $\bar{k}$ can be increasing or decreasing in δ, thus we examine the lowest and highest possible values of $\bar{k}$. We define $\bar{k}_1:=\bar{k}\big|_{\delta=1}$ and $\bar{k}_2:=\bar{k}\big|_{\delta=\frac{n}{n+\alpha-1}}$. Now we compute

$$\begin{aligned} g_1 &: =\alpha-1-\bar{k}_1=\frac{(\alpha-1)(n+q-2\alpha)}{2(n-\alpha+1)} \\ g_2 &: =\alpha-1-\bar{k}_2=\frac{(\alpha-1)(q+n-\alpha)}{2n} \end{aligned}$$

and conclude that when $\alpha\rightarrow 1$ both g_1 and g_2 approach 0 meaning that efficient production will coincide with the proposer optimal solution. When $\delta=1$ we have that the proposer optimal solution will coincide with the socially optimal production for cases where α is close to q and q is close to n. ■

Proof of Corollary 1

Let $k=n-1$. It follows that the proposer's payoff is $\pi_{\text{Prop}}(k=n-1)=n$. Now let $k=n-2$, one member is offered $s_{\text{vote}}<1/n$. The proposer's payoff is given by $\pi_{\text{Prop}}(k=$

$n-2) = (2n^2 - n - d + dn - dn^2)/n$. We have that

$$\pi_{\text{Prop}}(k = n-1) \geq \pi_{\text{Prop}}(k = n-2) \iff (-n^2 + n + d - dn + dn^2)/n > 0$$

if and only if $\delta \geq (n-1)n/(1-n+n^2) = \delta(n)$. One can easily show that $\delta(n)$ increases in n and is bounded from above by 1 and from below by 6/7 (because $n \geq 3$).

Proof of Proposition 2

We will solve the model where the proposer's (assigned player index 1) contract is defined as a transfer irrespective of his contribution. Thus, we have that the proposer's payoff is given by $\sum_{i=1}^{n} \alpha_i c_i - \sum_{i=2}^{n} a_i c_i$. This simplifies the analysis because it guarantees that the partnership is always solvent to honor the contracts offered (we allow for negative earnings). Define $\Lambda := \{i > 1 \text{ s.t. } a_i \geq \delta V\}$, which represents the set of those who will vote in favor and $\Omega := \{i > 1 \text{ s.t. } a_i \geq 1\}$ represents the set of incentivized members besides the proposer. Clearly, $\Lambda \subset \Omega$ because $\delta V > 1$ (Assumption 1). As before, we are characterizing equilibria in which the proposer produces. Let $k_c := |\Omega|$ and $F(k_c) = \alpha(k_c + 1)$ denote the size of the fund.

The voting approval constraint is given by

$$|\Lambda| \geq q - 1 \tag{23}$$

and the proposer's maximization problem is given by

$$\max_{k_c, \{a_i\}_{i=2}^{n}} F(k_c) - \sum_{i \in \Omega} a_i \quad \text{s.t. (23)} . \tag{24}$$

In the problem above we have imposed equilibrium behavior in the subgame in the sense that $c_i = 1$ if $i \in \Omega$.

It is clear that condition (23) binds because the proposer's payoff decreases in a_i. Hence, we have that there will be $q-1$ members receiving $\bar{a}_i(k_c) = \max\left\{\delta\left[\frac{(\alpha-1)(k_c+1)}{n}+1\right],1\right\}$. It is useful to note that $F(k_c) - \sum_{i\in\Omega} a_i$ is increasing in k_c as long as $a_i \leq \alpha$ (i.e. the contract offers a compensation lower than the member's productivity). It follows that $k_c^* = n-1$. Given that the proposer already has the necessary votes and that his payoff decreases in a_i, he chooses $a_i = 1$ for the remaining $n-q$ partners.

In equilibrium $\bar{a}_i^* := a_i(k_c^*) = \max\{\delta\alpha, 1\}$ and the proposer's payoff if given by $\alpha n - (q-1)\max\{\delta\alpha,1\} - (n-q-1)$. It is straightforward to show that his payoff increases in n and decreases in q (constant in q when $\delta\alpha < 1$).

Appendix B. Other Stationary Equilibria

Consider the following example with a five person committee. Let $\alpha = q = 3$ and $\delta = 1$. Plugging these values into (17) we obtain that $k^* = 1$. This implies that $F(k^*) = 6$, $s_{\text{Prop}} = 8/15$, $s_{\text{Vote}} = 2/15$, and $s_{\text{cont}} = 1/3$, and the ex ante value of the game given by $V = 9/5$. The proposer's payoff is $6 \times 8/15 = 48/15$.

Now I show that $k = 2$, s_{cont} =1/3, and $s_{\text{Prop}} = 1/3$ is a stationary equilibrium as well, but it yields a lower payoff to the proposer. When $k = 2$, the fund is equal to 9 and the stationary value of the game is $V = 11/5$. The payoff to each member of the coalition, including the proposer, is 3. Note that if the proposer deviates and assigns himself a larger share, one productive member must be sacrificed because of the binding equity constraint. We focus on the most profitable deviation when the non-productive coalition member is taken against the continuation value, then he must be offered $s_{\text{vote}} = 1/5$. Recall that a single deviation does not change the continuation value of the game due to stationarity. The proposer keeps 7/15 shares and obtains a payoff of $42/15 < 3$. This implies that the deviation was not profitable. In the equilibrium characterized by $k^* = 1$ the proposer earns a payoff of $48/15 > 3$.

However, if we consider a committee with $n = 7$, $q = \alpha = 4$ then it is not true that maximum efficiency can be attained in equilibrium. The reason is that if $k = 3$, s_{cont} =1/4,and $s_{\text{Prop}} = 1/4$, is part of a stationary equilibrium, the proposer can deviate to $s_{\text{Prop}} = 5/14$, $k = 2$, and $s_{\text{Vote}} = 1/7$ and receive a higher payoff while still obtaining the required votes.

We are not able to provide a closed-form solution to the problem of finding the most efficient stationary equilibrium but in the following proposition we provide a characterization of equilibrium strategies.

PROPOSITION 4 Let $\hat{\boldsymbol{\sigma}} = (\hat{\boldsymbol{\sigma}}_i, \hat{\boldsymbol{\sigma}}_{-i})$ be a stationary strategy profile where $\hat{k}$ members are incentivized and the stationary values of the game are given by $\hat{V} := V(\hat{k})$. Then $\hat{\boldsymbol{\sigma}}$ is the most efficient equilibrium if the equilibrium strategies satisfy following conditions:

1. $s_{\text{Prop}}(\hat{k},\hat{V}) = 1 - \hat{k} \cdot s_{\text{Cont}}(\hat{k},\hat{V}) - m s_{\text{Vote}}(\hat{k},\hat{V})$, $s_{\text{Cont}}(\hat{k},\hat{V}) = \max\left\{\frac{1}{\alpha}, \frac{\delta\hat{V}}{\hat{k}}\right\}$, $s_{\text{Vote}}(\hat{k},\hat{V}) = \frac{\delta\hat{V}-1}{F(\hat{k})}$.

2. A member votes in favor if and only if $s_{\text{Cont}} F(\hat{k}) \geq \delta\hat{V}$, $s_{\text{Vote}}(\hat{k},\hat{V})F(\hat{k}) + 1 \geq \delta\hat{V}$, or $s_{\text{Prop}}(\hat{k},\hat{V})F(\hat{k}) \geq \delta\hat{V}$ and the proposal receives q votes ($q = m + \hat{k} + 1$).

3. Given $\hat{\boldsymbol{\sigma}}_{-i}$, $\hat{k} = \arg\max_{k\in\{0,\ldots,\|\alpha\|-1\}} s_{\text{Prop}}(k,\hat{V})F(k)$ subject to conditions (1) and (2) above.

4. $\hat{k} = \arg\max_{k\in\{0,\ldots,\|\alpha\|-1\}} F(k)$ subject to conditions (1), (2), and (3) above.

PROOF. Conditions (1) and (2) state that no resources should be wasted, otherwise, this could result in lower efficiency or the proposer could improve his position (for a given $\hat{V}$) without failing to obtain the majority vote. Hence, productive members are offered the smallest share that induces contribution and non-productive coalition partners are offered a share that yields exactly the continuation value. Condition (3) states that the proposer cannot deviate to any other equity scheme and earn a higher profit while still obtaining the majority vote. Finally, condition (4) states that we choose the highest amount of productive members for which the previous conditions hold. Existence is guaranteed because the equilibrium characterized in Proposition 1 satisfies (1)-(3), thus we know that $\hat{k} \geq k^*$. ■

Appendix C. An Application to Partnerships: Endogenous Membership

Our models so far have assumed that membership is exogenously given and that all players are endowed with voting rights, while in reality it is hard to imagine that business partnerships or alliances operate in such way. In this section we will use the insights obtained from solution to the equity bargaining model and we will endogenize the partnership size. Our main goal is to show that the exogenous membership assumption is not detrimental to our previous analysis.

Another assumption that we will dispose of is that the productivity parameter is fixed. Instead, the productivity parameter $\alpha(n)$ is determined where $\alpha : \mathbb{N} \to \mathbb{R}^+$ and $1 < \alpha(n) \leq n$. The integer requirement on α is to simplify the exposition and is inconsequential for the economic relevance of the results.

We now explain the timing of the game which has three stages. In the fist stage, which we call the partnership formation stage, members will decide whether to join or not the partnership. We assume an exogenous network structure in the following sense: a randomly chosen member of the population (with I players) decides whether to form or not a partnership. If she does not form the partnership, then she exits the population, receives a payoff of 1, and another member has the chance to form a partnership. When a member decides to start the partnership, she can invite one more member into the firm. If the newly invited member declines the invitation, then he exits the population, and the initial partner may invite another member. However, if the new member accepts, the current partners can decide to invite a third partner. It does not matter if they have to agree unanimously or not, because in our model either all partners want to invite a new member or all wish to stop growing. The invitation process continues until partners decide to no longer invite additional members or all players have received an invitation. Given that we are interested in multilateral bargaining, we impose a minimal requirement in that the partnership should have at least 3 members. Hence, if at the end of the formation process there are two or less members, the partnership dissolves and the game ends with payoffs of 1 to all.

Once the partnership formation stage ends and there are $n \in \{3, 4, ...I\}$ members, partners proceed to stage 2 in which they engage in the equity bargaining game with the unanimity voting rule. (The voting rule is not relevant for our analysis, only for the restiction on α. We have focused on the unanimity rule where α can be quite large.) After an agreement is reached, partners proceed to stage 3 in which they either invest or not.

COROLLARY 9 The optimal partnership size is given by n^* which solves

$$\arg\max_{n \in \{3,\ldots,I\}} \frac{(\alpha(n) - 1)(k^* + 1)}{n} + 1 \quad . \tag{25}$$

PROOF. Consider any subgame following the partnership formation stage in which there are n members. The ex ante value of the equity bargaining game is given by $\frac{(\alpha(n)-1)(k^*+1)}{n} + 1$ which was derived in equation (6), the difference being that α is a function of n. ■

It should also be pointed out that if we consider an equity bargaining game in which part of the fund is exogenous ($x > 0$) then the partnership size is smaller (weakly) compared to the case of no exogenous fund. The reason is that as additional members join, the per capita exogenous fund shrinks, thus reducing the ex ante value of the bargaining game.

Our previous result in Corollary 9 depends strongly on the partnership formation process modelled via invitations. If one allows for unrestricted entry, then all the population would join the partnership because not joining yields a payoff of 1, which is lower than the ex ante value of the bargaining game. It is straightforward to incorporate heterogeneity in the outside options of players such that not all want to join as long as the outside option does not alter bargaining strategies once they are partners. See Miller, Montero, and Vanberg (2015) for the effect of heterogeneity in outside options in the equilibrium to Baron and Ferejohn game.

Our purpose in this appendix has been to show that the exogenous membership assumption employed throughout the body of the paper does not invalidate the results obtained in the equity bargaining game.