Math 2471 – Calc 3

Earlier we ~~saind~~ saw vector functions

$$\bar{r}(t) = \langle f(t), g(t) \rangle$$

these functions are vectors that changed with respect to t (time). Now we consider vectors that change with respect to position.

so ex $\quad \bar{F} = \langle x, y \rangle$

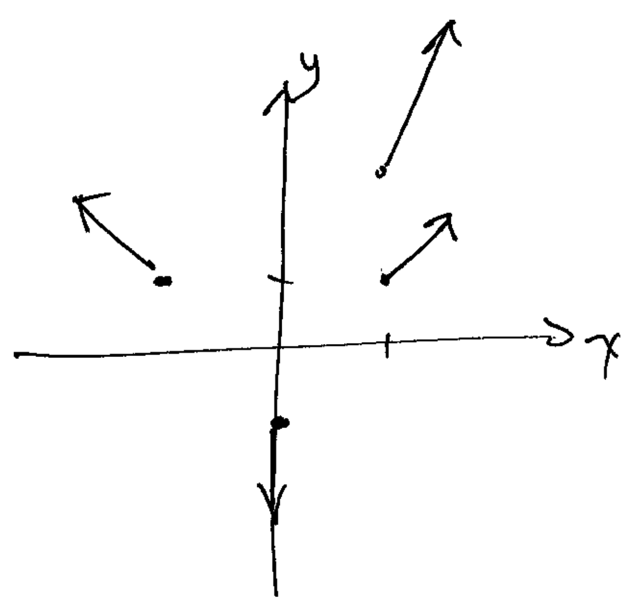
as we move through space these change.

So $\bar{F}(1,1) = \langle 1, 1 \rangle$

$\bar{F}(1,2) = \langle 1, 2 \rangle$

$\bar{F}(0,-1) = \langle 0, -1 \rangle$

$\bar{F}(-1,1) = \langle -1, 1 \rangle$



Ex 2 $\bar{F} = \langle -y, x \rangle$

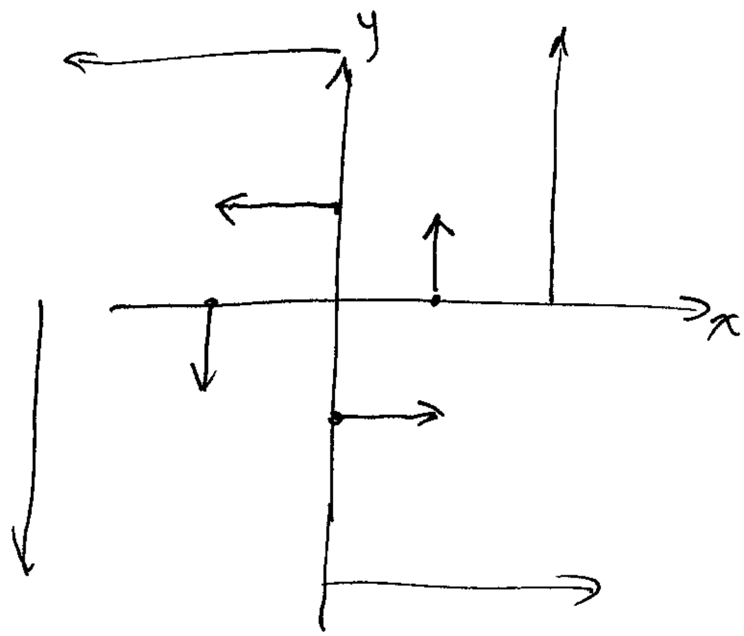
$\bar{F}(1,0) = \langle 0, 1 \rangle$

$\bar{F}(0,1) = \langle -1, 0 \rangle$

$\bar{F}(-1,0) = \langle 0, -1 \rangle$

$\bar{F}(0,-1) = \langle 1, 0 \rangle$

$\bar{F}(2,0) = \langle 0, 2 \rangle$



and what it looks like is a spiral

In general

$$\bar{F}(x,y) = \langle f(x,y), g(x,y) \rangle$$

and these are called "vector fields"

Easily extend to 3-D

$$\bar{F}(x,y,z) = \langle f(x,y,z), g(x,y,z), h(x,y,z) \rangle$$

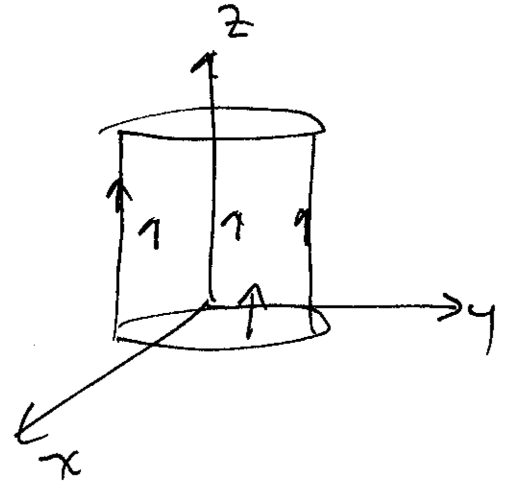
so for example

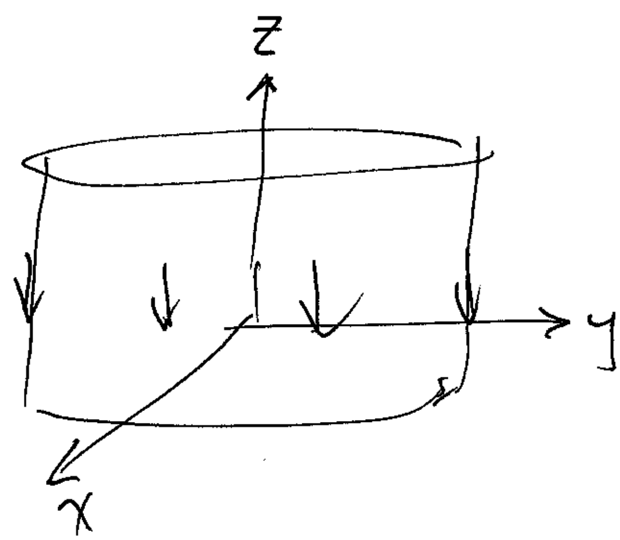
$$\bar{F}(x,y,z) = \langle 0, 0, 1-x^2-y^2 \rangle \qquad \text{bodk}$$

so on the cylinder $x^2+y^2 = 1$ $\quad \bar{F}(x,y,z) = \vec{0}$

on $x^2+y^2 = \frac{1}{2}$ $\quad \bar{F}(x,y,z) = \langle 0, 0, \frac{1}{2} \rangle$

while on $x^2+y^2 = 3/2$ $\quad \bar{F}(x,y,z) = \langle 0, 0, -1/2 \rangle$





Gradient Vector Fields

Recall the gradient

$$\bar{\nabla} f = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle$$

so if $f = x^2 + y^2$

$\bar{F} = \bar{\nabla} f = \langle 2x, 2y \rangle$

so in general

if $f = xy$

$\bar{F} = \bar{\nabla} f$

then $\bar{F} = \langle y, x \rangle$

is a gradient field

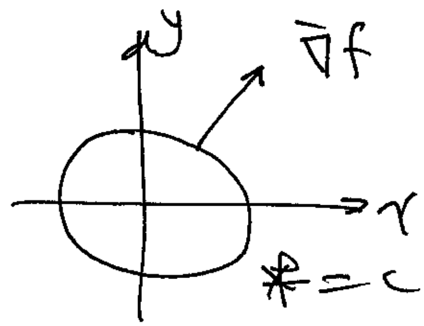
f — potential function.

interesting to note that $\bar{\nabla} f$ is

perpendicular the the level curves of $f = c$

Ex $f = x^2 + y^2 = c$ circles

$\bar{\nabla} f = \langle 2x, 2y \rangle$



Ex $\quad f = \sqrt{x^2+y^2+z^2}$

$$\nabla f = \left\langle \frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}} \right\rangle$$

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6, 7, 9, 13, 29, 31, 33