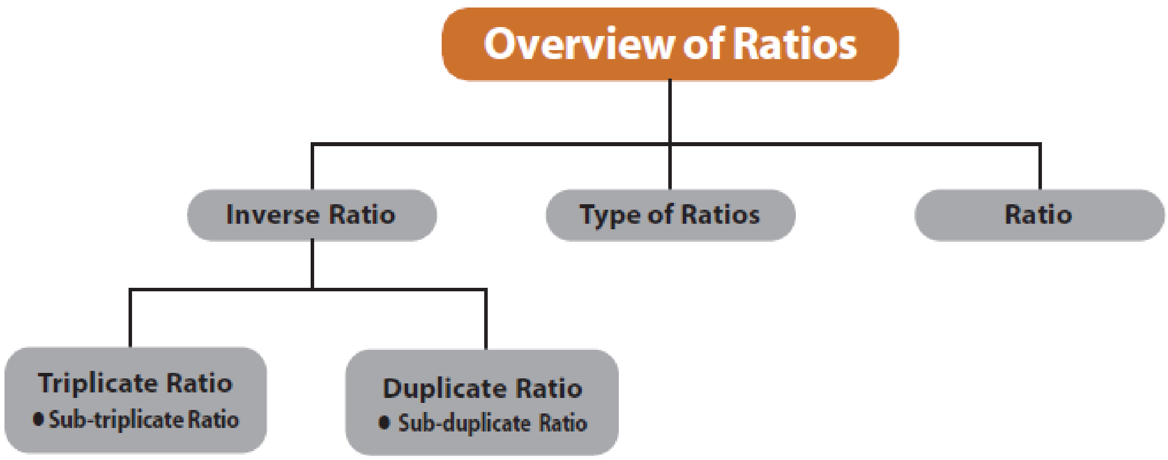
Chapter I: Ratio and Proportion, Indices and Logarithm

Unit I: Ratio



Ratio – A Comparison of the sizes of two or more quantities of the same kind (units by division)

$$Ratio\ of\ a\ to\ b\ is\ represented\ as\ a:b\ or\ \frac{a}{b}$$

Here		a	First term / antecedent
a, b	Term of the ratio	b	Second term / consequent

Remarks

Sl.no	Remarks	Explanation	Expression $a:b$	Example
1	Expressed in lowest terms/simplest form	Both terms can be divided with the same (non-zero) number	$\frac{a}{b}=\frac{2\times a}{2\times b}=\frac{a/2}{b/2}$	$12:16=\frac{12}{16}=\frac{12\div 4}{16\div 4}=\frac{3}{4}=3:4$
2	Importance of order of terms	order of terms should not be changed	$a:b\neq b:a$	$2:3\neq 3:2$
3	Ratio does not exist as the quantities not of same kind	necessity to be in same kind	$a\ (kg):b\ (years)$	Ratio exists between number of students & number of teachers, but not between number of students & salary of teacher

				$2\ kg: 3\ years$
4	Quantities to be compared must be in the same units (expressed in equivalent terms)	quantities to be compared (by division) in the same units in equivalent terms **Note**: it is divided to bring it in the lowest form	$a\ (kg): b\ (gms)$	$2\ kg: 500\ gms$ **Hence it should be** $2{,}000\ grams: 500\ grams$ **And in simplest form** $4: 1$
5	Comparing– fractions	Convert into equivalent fractions		$2\frac{1}{3}: 3\frac{1}{3} = \frac{7}{3}: \frac{10}{3} = 7{:}10 = \frac{7}{10}$
6	Increase / decrease in ratio	$new\ quantity = \frac{b\ \times originalqualitity}{a}$ Here, $\frac{b}{a}$ - factor multipling ratio		Rounaq weighs 56.7kg. If he reduces his weight in the ratio 7:6, find his new weight. Original weight of Rounaq = 56.7kg. He reduces his weight in the ratio 7:6 His new weight = (6× 56.7)/ 7 = 6×8.1 = 48.6kg.

Note:

1. $If\ a\ quantity\ increases\ /\ decreases\ in\ ratio\ a: b\ then\ new\ quantity\ = \frac{b}{a} of\ original\ quantity$

2. To compare two ratios, convert them into equivalent like fractions

Other Ratios

	Ratio Forms	Formula	Example	Description
1	Ratio	$a: b$	$2: 3$	**Expressed at its simplest form**
	Duplicate ratio	$a^2: b^2$	$4: 9$	
	Triplicate ratio	$a^3: b^3$	$8: 27$	
	Sub-duplicate ratio	$\sqrt{a}: \sqrt{b}$	$\sqrt{2}: \sqrt{3}$	
	Sub- triplicate ratio	$\sqrt[3]{a}: \sqrt[3]{b}$	$\sqrt[3]{2}: \sqrt[3]{3}$	
	Inverse ratio	$b: a$	$3: 2$	$Ratio \times Inverse\ Ratio = 1$
2	Ratio	$a: b$	$1: 1$	Ratio of equality as $a = b$
			$2: 3$	Ratio of lesser inequality as $a < b$

			$3:2$	Ratio of greater inequality as $a > b$
3	Ratio - Commensurable	$a:b$	$2:3$	a, b are integers (Ratio of integers)
	- In Commensurable		$\sqrt{2}:\sqrt{3}$	a, b are Non - integers (Ratio of non - integers)
4	Ratio	$a:b:c$	$2:3:4$	Continued ratio (relation with 3 or more)
5	Compound Ratio	a:b & c:d ⟹ ac:bd	3:4 & 5:7 $\Rightarrow (3 \times 5):(4 \times 7) = 15:28$	Product of antecedents: Product of consequents Note: Duplicate Ratio is a compound ratio

Simple Problems

1. Mr Singar weighs 60 kg if he reduces his weight in the ratio 12: 11 find his new weight.

Answer:

	Formula	**Calculation**	**Answer**
New Weight	$\frac{b}{a}$ *of original quantity*	$\frac{11}{12} \times 60$	55 kg

2. Find out the greater ratio: $2\frac{1}{3}:3\frac{1}{3}$; $3.6:4.8$

Answer:

	Convert in to like fractions	**Fraction**	**After LCM**
First ratio	$2\frac{1}{3}:3\frac{1}{3} = \frac{7}{3}:\frac{10}{3} = 7:10$	$\frac{7}{10}$	$\frac{7 \times 2}{10 \times 2} = \frac{14}{20}$
Second ratio	$3.6:4.8 = \frac{3.6}{4.8} = \frac{36}{48}$	$\frac{3}{4}$	$\frac{3 \times 5}{4 \times 5} = \frac{15}{20}$
As	$\mathbf{15} > 14$		
Hence	$\frac{15}{20} > \frac{14}{20}$ *or* $3.6:4.8$ *is greater ratio*		

3. Simplify the ratio: $\frac{1}{3}:\frac{1}{8}:\frac{1}{6}$

Answer: L.C.M of 3,8 and 6 is 24.

$\frac{1}{3}:\frac{1}{8}:\frac{1}{6}$	$1 \times \frac{24}{3}:1 \times \frac{24}{8}:1 \times \frac{24}{6}$	$8:3:4$

4. Find out the continued ratio of ₹100, ₹150 and ₹250.

Answer: ₹100:₹150:₹250 = 2:3:5.

5. Anand earns ₹80 in 7 hours and Promote ₹90 in 12 hours. The ratio of their earnings is[1]

(a) 32: 21 (b) 23: 12

(c) 8: 9 (d) None of these

Answer: $\frac{80}{7}$: $\frac{90}{12}$ = 960: 630 = 32: 21

6. P, Q and R are three cities. The ratio of average temperature between P and Q is 11:12 and that between P and R is 9:8. The ratio between the average temperature of Q & R is

(a) 22: 27 (b) 27: 22

© 32: 33 (d) None of these

Answer: P:Q = 11:12 ➔ Q:P = 12:11=108: 99

P:R = 9:8 = 99: 88

Q:P and P:R ➔ Q:R = 108:88 = 27:22

Conceptual Problems

7. The ratio of the number of boys to the number of girls in a school of 720 students is 3: 5. If 18 new girls are admitted in the school, find how many new boys may be admitted so that the ratio of the number of boys to the number of girls may change to 2: 3.

Solution:

The ratio of the no. of boys to the no. of girls = 3:5 Sum of the ratios = 3 + 5 = 8 So, the no.of boys in the school = $\frac{(3\times720)}{8}$= 270 And the no.of girls in the school = $\frac{(5\times720)}{8}$= 450 Let the no. of new boys admitted be x, then the number of boys become (270 + x).	After admitting 18 new girls, the no. of girls become 450 + 18 = 468 According to given description of the problem,$(270 + x)/468$ = 2/3 Or, 3$(270 + x)$ = 2× 468 Or, 810 + 3x = 936 or, 3x = 126 or, x = 42. Hence the number of new boys admitted =42.

8. The monthly incomes of two persons are in the ratio 4:5 and their monthly expenditures are in the ratio 7:9. If each saves ₹50 per month, find their monthly incomes

Solution: Let the monthly income of two persons be ₹4x and ₹5x so that the ratio is ₹4x:5x = 4:5. If each saves ₹50 per month, then the expenditures of two persons are ₹$(Rs.(4x - 50)$ and ₹(5x-50).

$\frac{4x-50}{5x-50}$= $\frac{7}{9}$ or 36x450 = 35x350

Or, 36x - 35x = 450 – 350, or x = 100

Hence, the monthly incomes of the two persons are ₹4×100 and ₹5×100, i.e.₹400 and ₹500.

9. The ratio of the prices of two houses was 16:23. Two years later when the price of the first has increased by 10% and that of the second by ₹477, the ratio of the prices becomes 11:20. Find the original

[1] a

prices of the two houses.

Solution: Let the original prices of two houses be ₹16x and ₹23x respectively. Then by the given conditions,

$\frac{16x + 10\%\ of 16x}{23x + 477} = \frac{11}{20}$

$\frac{16x + 1.6x}{23x + 477} = \frac{11}{20}$

$320x + 32x = 253x + 5247$

$352x - 253x = 5247$

$99x = 5247;$

$\therefore x = 53$

Hence, the original prices of two houses are ₹ 848 and ₹1,219.

10. Find in what ratio will the total wages of the workers of a factory be increased or decreased if there be a reduction in the number of workers in the ratio 15:11 and an increment in their wages in the ratio 22:25.

Answer: Let x be the original number of workers and ₹y the (average) wages per workers.

Then the total wages before changes $= ₹xy.$

After reduction, the number of workers $= \frac{(11x)}{15}$

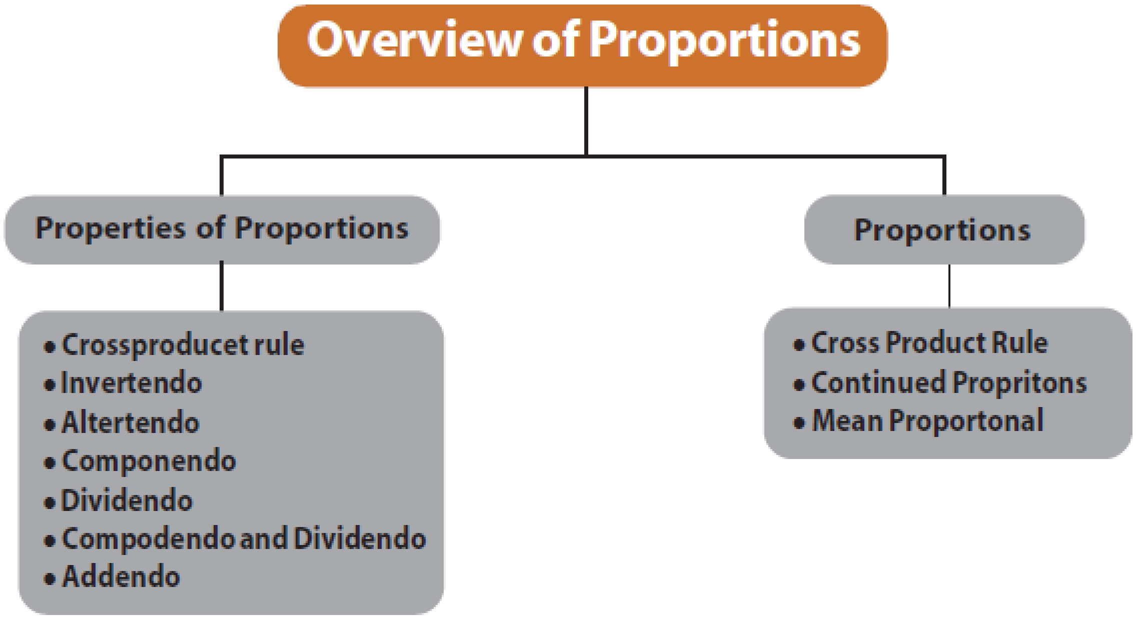
After increment, the (average) wages per workers $= ₹\frac{(25y)}{22}$

∴ The total wages after changes $= \left(\frac{11}{15}x\right) \times \left(₹\frac{25}{22}y\right) = ₹\frac{5xy}{6}$

Thus, the total wages of workers get decreased from $₹xy\ to\ ₹5xy/6$

Hence, the required ratio in which the total wages decrease is $xy{:}\frac{5xy}{6} = 6{:}5$

Unit – II: Proportions



Proportions – An equality of two ratios

Points to Ponder:

Sl. No	
1	a, b, c, d are in proportion – a:b = c:d or a:b : : c:d
2	Cross product rule If $\frac{a}{b} = \frac{c}{d} \Rightarrow$ ad = bc $\Rightarrow$ $product\ of\ the\ extremes = product\ of\ the\ means$
3	a / b / c / d – first / second / third / fourth proportional a / d – extremes and c / d – mean/middle terms
4	Continuous proportion a, b, c of same kind(in same units), if a:b=b:c i.e b^2 = ac, ⇒b =$\sqrt{ac}$ ➔i.e b = Geometric Mean of a and c ⇒ Middle term is the mean proportional between a and c a – first proportional and c – third proportional
5	Continued proportion → $\frac{x}{y} = \frac{y}{z} = \frac{z}{w} = \frac{w}{p} = \frac{p}{q}$
6	Inverse / Reciprocal proportion – A ratio equal to the reciprocal of the other. Example: $\frac{5}{4}$ & $\frac{4}{5}$ are inverse proportion.

Note: In the case of ratio a:b, a & b must be of same kind, whereas in proportions (a:b :: c:d), a & b must be of same kind and c& d must be of same kind(need not be the same kind as a and b)

Properties / Laws

	Properties / Laws	Proof
1	If a:b = c:d, then ad = bc	$\frac{a}{b}=\frac{c}{d}$; ∴ ad = bc (By cross – multiplication)
2	If a:b = c:d, then b:a = d:c (Invertendo) (Inverting the ratios)	$\frac{a}{b}=\frac{c}{d}$ or $\frac{1}{\frac{a}{b}}=\frac{1}{\frac{c}{d}}$, or, $\frac{b}{a}=\frac{d}{c}$ Hence, b : a = d : c.
3	If a:b = c:d, then a:c= b:d (Alternendo) (Alter / Interchange the middle values)	$\frac{a}{b}=\frac{c}{d}$ or, ad =bc Dividing both sides by cd, we get $\frac{ad}{cd}=\frac{bc}{cd}$, or $\frac{a}{c}=\frac{b}{d}$, i.e. a:c = b:d.
4	If a:b = c:d, then a + b : b = c + d : d (Componendo) (Addition – New Nr / Dr = Antecedent + Consequent)	$\frac{a}{b}=\frac{c}{d}$, or, $\frac{a}{b}+1=\frac{c}{d}+1$ or, $\frac{a+b}{b}=\frac{c+d}{d}$, i.e. a + b:b = c + d:d.
5	If a:b = c:d, then a –b :b = c – d :d (Dividendo) (Subtraction – New Nr / Dr= Antecedent + Consequent)	$\frac{a}{b}=\frac{c}{d}$, ∴ $\frac{a}{b}-1=\frac{c}{d}-1$ $\frac{a-b}{b}=\frac{c-d}{d}$, i.e a – b :b = c – d:d.
6	If a:b = c:d, then a + b : a – b = c + d : c – d (Componendo and Dividendo) (New Antecedent= Componendo Rule New Consequent = Dividendo Rule)	$\frac{a}{b}=\frac{c}{d}$, or $\frac{a}{b}+1=\frac{c}{d}+1$, or $\frac{a+b}{b}=\frac{c+d}{d}$ -----------------1 Again $\frac{a}{b}-1$, $\frac{c}{d}-1$, or $\frac{a-b}{b}=\frac{c-d}{d}$ ------------------------------2 Dividing (1) and (2) we get $\frac{a+b}{a-b}=\frac{c+d}{c-d}$, i.e a + b : a –b = c + d : c – d
7	If a : b = c : d = e : f = …………………., then each of these ratios (Addendo) is equal (a + c + e + ……) : (b + d + f + ……) (New Antcedent = Addition of all antecedent values (New Consequent = Addition of all Consequent values) Note: The new ratio is equal to the other ratios	$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ = …………….(say)k, ∴ a = bk, c= dk, e=fk,……… Now a + c + e…… = k (b + d + f) …….. or $\frac{a+c+e......}{b+d+f....}$ = k Hence, (a + c + e + …..): (b + d + f + ……) is equal to each ratio
8	If a: b = c:d = e:f = ….., then each of these ratios (subtrahendo) is equal to (a – c – e – c ……): (b – d – f - …..) (New Antcedent = Subtraction of all antecedent values	$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ = ……………. (say)k, ∴ a = bk, c= dk, e=fk ,……… Now a - c - e…… = k (b - d – f - ...) or $\frac{a-c-e......}{b-d-f....}$ = k

	(New Consequent = Subtraction of all Consequent values) Note: The new ratio is equal to the other ratios	Hence, (a - c - e -…): (b - d - f - ……) is equal to each ratio

Points to Ponder: Applications

1. To find out new income – if the income of a man is increased in the given ratio & if the increase in his income is given
2. To find out the age – if the ages of two men are in the given ratio & if the age of one man is given.

Simple Problems

Sl. No	Question	Solution
1	Check whether the 2.4, 3.2, 1.5, 2 are in proportion	Here 2.4×2 = 4.8 and 3.2 ×1.5 = 4.8 $product\ of\ the\ extremes = product\ of\ the\ means$ Hence in proportion
2	Find the value of x if 10/3 : x :: 5/2 : 5/4	10/3: x =5/2:5/4 Using cross product rule, x ×5/2 = $(10/3) \times 5/4$ Or, x = $(10/3) \times 5/4 \times (2/5)$ = 5/3
3	Find the fourth proportional to 2/3, 3/7,4	If the fourth proportional be x, then 2/3, 3/7, 4, x are in proportion. Using cross product rule, (2/3) × x = $(3 \times 4)/7$ ➔ x = $(3 \times 4 \times 3)/(7 \times 2)$ = 18/7.
4	Find the third proportion to 2.4kg, 9.6kg.	Let the third proportion to 2.4 kg, 9.6kg be $x$kg. Then 2.4kg, 9.6 kg and $x$kg are in continued proportion since b^2 = ac So, 2.4/9.6 = 9.6/ x or, x = $(9.6 \times 9.6)/2.4$ = 38.4 Hence the third proportional is 38.4 kg.
5	Find the mean proportion between 1.25 and 1.8	Mean proportion between 1.25 and 1.8 is $\sqrt{(1.25 \times 1.8)} = \sqrt{2.25}$ = 1.5.

Conceptual Problems

1. If a:b = c:d = 2.5: 1.5, what are the values of ad : bc and a + c : b + d?

Solution:

We have $\frac{a}{b} = \frac{c}{d} = \frac{2.5}{1.5}$ ……………(1)

From (1) ad = bc, or, $\frac{ad}{bc}$ =1, i.e. ad:bc = 1:1

Again from (1) $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$

$\therefore \frac{a+c}{b+d} = \frac{2.5}{1.5} = \frac{25}{15} = \frac{5}{3}$, i.e. a + c: b + d = 5:3

Hence, the values of ad : bc and a + c : b + d are 1:1 and 5:3 respectively.

2. If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$, then Prove that $\frac{a+b+c}{c} = 2$
   Solution:
   We have $\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = \frac{a+b+c}{3+4+7} = \frac{a+b+c}{14}$
   $\therefore \frac{a+b+c}{14} = \frac{c}{7}$ or $\frac{a+b+c}{c} = \frac{14}{7} = 2$

3. A dealer mixes Tea A costing ₹6.92 per kg with Tea B costing ₹7.77 per kg. and sells the Tea mixture at ₹8.80 per kg. and earns a profit of 17.5 % on his sale price. In what proportion does he mix them?
   Solution:
   Step 1: To find the Cost Price of the Mixture
   Since, the price of Tea A and Tea B is given as Cost Price

Sale Price	₹ 100 (Assumption)	₹ 8.8 (Given)
- Profit	₹ 17.5 (17.5% on Sale Price)	₹ 1.54
Cost Price	₹ 82.5	₹ 7.26

   C.P of the mixture per kg = ₹7.26

   Step 2:
   We have to mix the two kinds in such a ratio that the amount of profit from Tea A must balance the amount of loss from Tea B.
   Difference between the Cost Price of Tea Mixture and Tea A : (₹7.26 – ₹ 6.92) = ₹ 0.34
   Difference between the Cost Price of Tea Mixture and Tea B : (₹7.26 – ₹ 7.77) = (- ₹ 0.51)
   Therefore, 1st Difference : 2^{nd} Difference = 0.34: (-0.51) = 2: (-3) = 3:2

Note:
1. Check:
   3 parts of Tea A costs = 3 * ₹ 6.92 = 20.74
   2 Parts of Tea B costs = 2 * ₹ 7.77 = 15.54
   Therefore, Total Cost = ₹ 20.74 + ₹ 15.54 = ₹ 36.28
   5 Parts (3 parts of Tea A and 2 Parts of Tea B) of Mixture Costs = 5 * 7.26 = 36.3
2. Note: Since, 2: (-3) cannot exists as (-3) of Tea B cannot be mixed, the ratio is inverted to 3:2