CAP-5993 Homework 2
Deadline: 3:30PM on 2/21 (Tuesday)

1. Exercise 4.2 from textbook. (10 pts)
2. Exercise 4.7 from textbook. (10 pts)
3. Exercise 4.10 from textbook. (10 pts)
4. Exercise 4.12 from textbook. (10 pts)
5. Exercise 4.14 from textbook. (10 pts)
6. Exercise 4.23 from textbook. (10 pts)
7. Exercise 4.35 from textbook. (10 pts)
8. Exercise 4.40 from textbook. (10 pts)
9. Write an algorithm similar to the one we saw in lecture (Lecture 9 Slide 9), that determines whether mixed strategy s_i is weakly dominated by any pure strategy. (10 pts)
10. We saw one optimization formulation for determining whether a mixed strategy is strictly dominated by another mixed strategy that is not a linear program due to having a strict inequality constraint (Lecture 9 Slide 11). We then saw a new formulation that is a linear program (top of Lecture 9 Slide 12). Prove that these two formulations are equivalent. That is, prove that any solution to one is also a solution to the other, and vice versa. (10 pts)
11. **Three-card Poker (55 pts)**

    In three-card poker, two players are both dealt a card from a deck consisting of a king (K), a queen (Q) and a jack (J) (the final card is not dealt). Both players ante \$1, and player 1 (P1) is first to act. He has two choices: he can bet \$1 (B) or check (C). If P1 bets, player 2 (P2) can either call or fold. If P1 bets and P2 calls, then whoever has the higher card (K > Q > J) wins the entire pot of \$4. If P1 bets and P2 folds, then P1 wins the pot of \$2. If P1 checks, then P2 can either bet or check. If P1 checks and P2 checks, then whoever has the higher card wins \$2. If P1 checks and P2 bets, then P1 can call or fold. If P1 calls, then whoever has the higher card wins \$4. If P1 folds, then P2 wins \$2.

    (a) Draw the extensive-form game tree of this game. (10 pts)
    (b) How large would the equivalent normal-form representation of this game be? Note that you do not need to explicitly write down the normal-form representation of this game. (5 pts)
    (c) Say that an action a for player i at information set s in an extensive-form game is weakly dominated if there exists another action b at the same information set, such that for all leaf nodes n_x reachable by playing action x at s, we have $u_i(n_b) \geq u_i(n_a)$. Based on your extensive-form representation, perform iterated elimination of weakly dominated actions and list all actions that end up being eliminated for each player. (10 pts)
    (d) Now reconstruct the extensive-form representations of the game from part a with the dominated actions eliminated. (5 pts)
    (e) Write down the equivalent normal-form representation of the game you just constructed in part d. (5 pts)
    (f) What is one equilibrium of this game? What is the value of the game to player 1? You can use any method you want (we recommend solving it manually from the extensive-form representation), but must show your work and justify your answer. (10 pts)
    (g) What are all of the equilibria of this game? You can use any method you want (again we recommend solving it manually from the extensive-form representation), but must show your work and justify your answer. (10 pts)

    Hint: There are infinitely many equilibria, so you should provide as detailed a description as possible of the set of equilibria. For example, you could give a parameterized strategy for each player, and say what sets the parameters can range over. Of course, your parameterized strategies should include your answer to 2(f) as a special case.