Edexcel GCE

Core Mathematics C3

Silver Level S1

(Mark Scheme)

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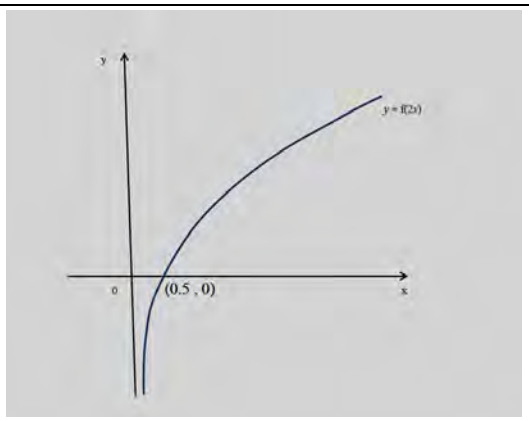
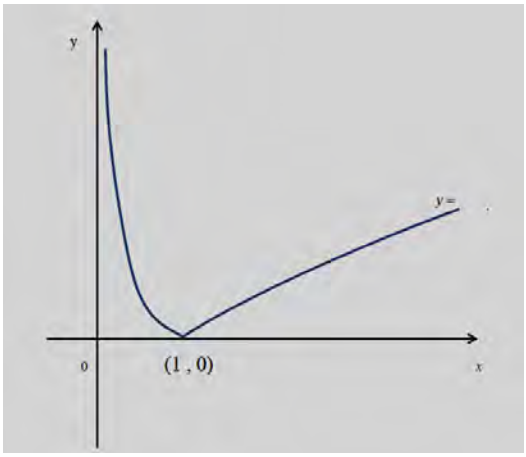
Question Number	Scheme	Marks
1. (a)	$\frac{d}{dx}(\ln[(3x)) \rightarrow \frac{B}{x}$ for any constant B	M1
	Applying $vu' + uv'$, $\ln(3x) \times 2x + x$	M1 A1
		A1
		(4)
(b)	Applying $\frac{vu' - uv'}{v^2}$	M1
	$\frac{x^3 \times 4\cos(4x) - \sin(4x) \times 3x^2}{x^6}$	A1+A1
	$= \frac{4x\cos(4x) - 3\sin(4x)}{x^4}$	A1
		A1 (5)
		[9]
2. (a)	 Shape	B1
	(0.5, 0)	B1
		(2)
(b)	 Shape	B1
	(1,0)	B1
	Cusp at (1,0)	B1
		(3)
		[5]

Question Number	Scheme	Marks
3. (a)	20 (mm^2)	B1 (1)
(b)	$'40' = 20\ e^{1.5t} \rightarrow \quad e^{1.5t} = c$	M1
	$e^{1.5t} = \frac{40}{20} = (2)$	A1
	Correct order $1.5t = \ln' 2' \rightarrow \quad t = \frac{\ln c}{1.5}$	M1
	$t = \frac{\ln 2}{1.5} = (awrt\ 0.46)$	A1
	12.28 or 28 (minutes}	A1 (5)
		[6]
4. (a)	$x^2(3 - x) - 1 = 0$ o.e. (e.g. $x^2(-x + 3) = 1$)	M1
	$x = \sqrt{\frac{1}{3-x}}$ (✱)	A1 (cso)
	Note(✱), answer is given: need to see appropriate working and A1 is cso [Reverse process: Squaring and non-fractional equation M1, form f(x) A1]	(2)
(b)	$x_2 = 0.6455$, $x_3 = 0.6517$, $x_4 = 0.6526$ 1st B1 is for one correct, 2nd B1 for other two correct If all three are to greater accuracy, award B0 B1	B1; B1 (2)
(c)	Choose values in interval (0.6525, 0.6535) or tighter and evaluate both f(0.6525) = –0.0005 (372… f(0.6535) = 0.002 (101… At least one correct "up to bracket", i.e. -0.0005 or 0.002 **Change of sign,** $\therefore$ $x = 0.653$ is a root (correct) to 3 d.p. Requires both correct "up to bracket" and conclusion as above	M1 A1 A1 (3) **[7]**

Question Number	Scheme		Marks
5. (a)	Crosses x-axis $\Rightarrow \mathrm{f}(x) = 0 \Rightarrow (8-x)\ln x = 0$		
	Either $(8-x)=0$ or $\ln x = 0 \Rightarrow x = 8, 1$	Either one of {x}=1 OR x={8}	B1
	Coordinates are $A(1, 0)$ and $B(8, 0)$.	Both $A(1,\{0\})$ and $B(8,\{0\})$	B1 (2)
(b)	Apply product rule: $\left\{\begin{array}{ll} u=(8-x) & v=\ln x \\ \frac{\mathrm{d}u}{\mathrm{d}x}=-1 & \frac{\mathrm{d}v}{\mathrm{d}x}=\frac{1}{x} \end{array}\right\}$	$vu' + uv'$	M1
	$\mathrm{f}'(x) = -\ln x + \frac{8-x}{x}$	Any one term correct	A1
		Both terms correct	A1 (3)
(c)	$\mathrm{f}'(3.5) = 0.032951317...$ $\mathrm{f}'(3.6) = -0.058711623...$ Sign change (and as $\mathrm{f}'(x)$ is continuous) therefore the x-coordinate of Q lies between 3.5 and 3.6.	Attempts to evaluate **both** $\mathrm{f}'(3.5)$ and $\mathrm{f}'(3.6)$	M1
		both values correct to at least 1 sf, sign change and conclusion	A1 (2)
(d)	At Q, $\mathrm{f}'(x) = 0 \Rightarrow -\ln x + \frac{8-x}{x} = 0$	Setting $\mathrm{f}'(x) = 0$.	M1
	$\Rightarrow -\ln x + \frac{8}{x} - 1 = 0$	Splitting up the numerator and proceeding to x=	M1
	$\Rightarrow \frac{8}{x} = \ln x + 1 \Rightarrow 8 = x(\ln x + 1)$		
	$\Rightarrow x = \frac{8}{\ln x + 1}$ (as required)	For correct proof. No errors seen in working.	A1 (3)

Question Number	Scheme	Marks
(e)	Iterative formula: $x_{n+1} = \dfrac{8}{\ln x_n + 1}$	
	$x_1 = \dfrac{8}{\ln(3.55) + 1}$ An attempt to substitute $x_0 = 3.55$ into the iterative formula. Can be implied by $x_1 = 3.528(97)...$	M1
	$x_1 = 3.528974374...$ $x_2 = 3.538246011...$ $x_3 = 3.534144722...$ Both $x_1 =$ awrt 3.529 and $x_2 =$ awrt 3.538	A1
	$x_1 = 3.529,\ x_2 = 3.538,\ x_3 = 3.534,$ to 3 dp. x_1, x_2, x_3 all stated correctly to 3 dp	A1
		(3) **[13]**

Question Number	Scheme	Marks
6. (a)	$A = B \Rightarrow \cos(A + A) = \cos 2A = \underline{\cos A \cos A - \sin A \sin A}$	M1
	$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives	
	$\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required)	A1 (2)
(b)	$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$	M1
	$3\sin 2x = 4\left(\frac{1 - \cos 2x}{2}\right) - 2\cos 2x$	M1
	$3\sin 2x = 2(1 - \cos 2x) - 2\cos 2x$	
	$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$	
	$3\sin 2x + 4\cos 2x = 2$	A1 (3)
(c)	$3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$	
	$3\sin 2x + 4\cos 2x = R\cos 2x\cos\alpha + R\sin 2x\sin\alpha$	
	Equate $\sin 2x$: $3 = R\sin\alpha$ Equate $\cos 2x$: $4 = R\cos\alpha$	
	$R = \sqrt{3^2 + 4^2}$; $= \sqrt{25} = 5$	B1
	$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765...^\circ$	M1 A1
	Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$	A1 (3)
(d)	$3\sin 2x + 4\cos 2x = 2$	
	$5\cos(2x - 36.87) = 2$	
	$\cos(2x - 36.87) = \frac{2}{5}$	M1
	$(2x - 36.87) = 66.42182...^\circ$	A1
	$(2x - 36.87) = 360 - 66.42182...^\circ$	
	Hence, $x = 51.64591...^\circ$, $165.22409...^\circ$	A1 A1 (4)
		[12]

Question Number	Scheme	Marks
7. (a)	$\mathrm{f}(x) = 1 - \dfrac{2}{(x+4)} + \dfrac{x-8}{(x-2)(x+4)} \quad x \in \mathbb{R},\ x \neq -4, x \neq 2.$	
	$\mathrm{f}(x) = \dfrac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$	M1 A1
	$= \dfrac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$	
	$= \dfrac{x^2 + x - 12}{[(x+4)(x-2)]}$	A1
	$= \dfrac{(x+4)(x-3)}{[(x+4)(x-2)]}$	M1
	$= \dfrac{(x-3)}{(x-2)}$	A1 cso (5)
(b)	$\mathrm{g}(x) = \dfrac{\mathrm{e}^x - 3}{\mathrm{e}^x - 2} \quad x \in \mathbb{R},\ x \neq \ln 2.$	
	Apply quotient rule: $\left\{ \begin{array}{ll} u = \mathrm{e}^x - 3 & v = \mathrm{e}^x - 2 \\ \dfrac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x & \dfrac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^x \end{array} \right\}$	
	$\mathrm{g}'(x) = \dfrac{\mathrm{e}^x(\mathrm{e}^x - 2) - \mathrm{e}^x(\mathrm{e}^x - 3)}{(\mathrm{e}^x - 2)^2}$	M1 A1
	$= \dfrac{\mathrm{e}^x}{(\mathrm{e}^x - 2)^2}$	A1 cso (3)
(c)	$\mathrm{g}'(x) = 1 \Rightarrow \dfrac{\mathrm{e}^x}{(\mathrm{e}^x - 2)^2} = 1$	
	$\mathrm{e}^x = (\mathrm{e}^x - 2)^2$	M1
	$\mathrm{e}^x = \mathrm{e}^{2x} - 2\mathrm{e}^x - 2\mathrm{e}^x + 4$	
	$\underline{\mathrm{e}^{2x} - 5\mathrm{e}^x + 4 = 0}$	A1
	$(\mathrm{e}^x - 4)(\mathrm{e}^x - 1) = 0$	M1
	$\mathrm{e}^x = 4 \text{ or } \mathrm{e}^x = 1$	
	$x = \ln 4 \text{ or } x = 0$	A1 (4)
		[12]

Question Number	Scheme	Marks
8. (a)	$f'(x) = 3e^x + 3xe^x$	M1 A1
	$3e^x + 3xe^x = 3e^x(1+x) = 0$	
	$x = -1$	M1 A1
	$f(-1) = -3e^{-1} - 1$	B1 (5)
(b)	$x_1 = 0.2596$	B1
	$x_2 = 0.2571$	B1
	$x_3 = 0.2578$	B1 (3)
(c)	Choosing $(0.257\,55, 0.257\,65)$ or an appropriate tighter interval.	M1
	$f(0.257\,55) = -0.000\,379\ldots$	
	$f(0.257\,65) = 0.000\,109\ \ldots$	A1
	Change of sign (and continuity) $\Rightarrow$ root $\in (0.257\,55, 0.257\,65)$ ✱ cso	A1
	($\Rightarrow x = 0.2576$, is correct to 4 decimal places)	(3) **[11]**
	Note: $x = 0.257\,627\,65\ \ldots$ is accurate	

Statistics for C3 Practice Paper Silver Level S1

				Mean score for students achieving grade:							
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	9		81	7.25	8.84	8.46	7.84	7.46	6.31	5.91	2.97
2	5		93	4.67	4.90	4.83	4.69	4.63	4.21	4.17	2.95
3	6		88	5.29	5.87	5.69	5.44	5.13	4.69	4.22	2.49
4	7		74	5.17		6.12	5.47	4.90	4.27	3.52	2.41
5	13		74	9.59	12.78	11.87	10.45	8.81	6.84	5.34	3.24
6	12		72	8.69		11.10	9.59	7.82	5.71	3.71	1.69
7	12		78	9.41		11.22	10.12	8.88	7.31	5.51	3.15
8	11		69	7.59		9.88	8.25	7.01	5.50	3.66	2.08
	75		**77**	**57.66**		**69.17**	**61.85**	**54.64**	**44.84**	**36.04**	**20.98**