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Content		

1. Equations

Equation: An algebraic expression equated to a constant / zero

Example: $x^2 = 4$ or $x^2 - 4 = 0$

Types of Equation

Sl.No	Equation	Highest power of the variable	Example
I	Linear / Simple	1	$8x+17(x-3) = 44(4x-9)+12$
II	Simultaneous Linear Equations	a. Two variables	$x+2y = 1$ & $2x+3y = 2$
		b. Three variables	$2x - y + z = 3$ $x + 3y - 2z = 11$ $3x - 2y + 4z = 1$
III	Quadratic	2	$3x^2 + 5x + 6 = 0$
IV	Cubic	3	$4x^3 + 3x^2 + x - 7 = 0$

Quadratic Equation

General form: $ax^2 + bx + c = 0$ Here x – variable a($\neq 0$), b, c – constants Degree – 2 → The equation will have two roots	**Pure quadratic equation,** b = 0 Example: $x^2 - 4 = 0$ **Affected Quadratic Equation,** b≠0 Example $x^2 - 5x + 6 = 0$

Methods of solving: Solve $x^2 - 5x + 6 = 0$

Factorisation Method	**Discrimination Method (Formula Method)**
$(x-a)(x-b) = 0$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x^2 - 5x + 6 = 0$ → $x^2 - 2x - 3x + 6 = 0$ → $x(x-2) - 3(x-2) = 0$ → $(x-2)(x-3) = 0$ → $x = 2$ or 3	Here, a=1, b=-5, c=6 (Comparing the equation with $ax^2 + bx + c = 0$) $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-5) \pm \sqrt{25-24}}{2} = \frac{6}{2}$ and $\frac{4}{2}$ $\therefore x = 3 \text{ and } 2$

Remarks

1. Let the roots be $\propto and\ \beta$. Then

Sum of the roots = $\propto + \beta = \frac{-b}{a} = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ $\propto + \beta = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{-2b}{2a} = \frac{-b}{a}$	Product of the roots $= \propto\beta = \frac{c}{a} = \frac{constant\ term}{coefficient\ of x^2}$ $\propto\beta = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right) = \frac{c}{a}$

Simple Problems – Set 3

1. If α and β be the roots of $x^2+7x+12$ =0 find the equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$

Solution:

Sum of the roots of the required equation	Product of the roots of the required equation
=$(\alpha+\beta)^2+(\alpha-\beta)^2$ = $(-7)^2+(\alpha+\beta)^2-4\alpha\beta$ = 49 + $(-7)^2-4\times12$ = 50	equation = $(\alpha+\beta)^2(\alpha-\beta)^2$ = 49 (49-48) = 49
The required equation is $x^2-(sum\ of\ the\ roots)x+product\ of\ the\ roots=0$ $x^2-50x+49=0$	

2. If α,β be the roots of $2x^2-4x-1=0$ find the value of $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

Solution: $\alpha+\beta=\frac{-(-4)}{2}=2$, $\alpha\beta=\frac{-1}{2}$

$\therefore \frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}=\frac{2^3-3\left(-\frac{1}{2}\right).2}{\left(-\frac{1}{2}\right)}$ = -22

3. If α and β are the two roots of the equation $x^2-px+q=0$ form the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

Solution: As α,β are the roots of the equation $x^2-px+q=0$

$\alpha+\beta$ = - (-P) = p and $\alpha\beta$ = q.

Now $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}.=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{p^2-2q}{q}$; and $\frac{\alpha}{\beta}.\frac{\beta}{\alpha}=1$

$\therefore$ Required equation is $x^2-\left(\frac{p^2-2q}{q}\right)x+1=0$ → q $x^2-(p^2-2q)x+q=0$

2. Construction of a Quadratic Equation

$x^2-(sum\ of\ the\ roots)x+product\ of\ the\ roots$ =0

Explanation:

$ax^2+bx+c=0$

$\Rightarrow x^2+\frac{b}{a}x+\frac{c}{a}=0$(dividing the full equation by a)

$\Rightarrow x^2-\left(\frac{-b}{a}\right)x+\frac{c}{a}=0$

$\Rightarrow x^2-\ (sum\ of\ the\ roots)x+product\ of\ the\ roots=0)$

3. Nature of the roots: Discriminant , b^2-4ac discriminates between the roots

Sl.no	b^2-4ac	Roots	Example
1	= 0	Real & equal	$x^2-8x+16=0$ [a =1, b= -8, c= 16] $b^2-4ac=(-8)^2-4\times1\times16$ =64 – 64 = 0 The roots are real and equal.

2	< 0	Imaginary	$5x^2 - 4x + 2 = 0$ [a = 5, b = -4, c = 2] $b^2 4ac$ = $(-4)^2 - 4 \times 5 \times 2$ = 16 – 40 = -24 <0 The roots are imaginary and unequal
3	>0 & Perfect square	Real, Rational & Unequal (distinct)	$3x^2 - 8x + 4 = 0$ [a = 3, b = -8, c = 4] $b^2 - 4ac$ = $(-8)^2 - 4 \times 3 \times 4 = 64 - 48$ = 16> 0 and a perfect square The roots are real, rational and unequal
4	>0 & not a perfect square	Real, Irrational & unequal	$2x^2 - 6x - 3$ = 0 $b^2 - 4ac$ = $-(6)^2 - 4 \times 2 \times (-3)$ = 60 >0 The roots are real and unequal

Points to Ponder

1. Irrational roots occurs in conjugate pairs : $(m + \sqrt{n})$&$(m - \sqrt{n})$ are the roots.

   Example: If one root of the equation is 2 - $\sqrt{3}$ from the equation given that the roots are irrational

   Solution: other roots is 2 + $\sqrt{3}$ ∴ sum of two roots =2 – $\sqrt{3}$ + 2 + $\sqrt{3}$ =4

   Product of roots = $(2 - \sqrt{3})(2 + \sqrt{3})$ = 4 - 3 = 1

   ∴ Required equation is: $x^2 - (sum\ of\ roots)x + (product\ of\ roots)$ = 0 → $x^2 - 4x$+1 = 0.

2. Reciprocal roots - $\propto \beta = 1 \Rightarrow \frac{c}{a} = 1$ c=a

3. Roots are equal but opposite in sign, ∝ = - β

   Then $\propto + \beta = 0$, $\frac{-b}{a} = 0 \Rightarrow$ b = 0.

Conceptual Problems

1	**Solve x: $4^x - 3.2^{x+2} + 2^5 = 0$**	
	$4^x - 3.2^{x+2} + 2^5 = 0$ $(2^x)^2 - 3.2^x.2^2 + 32 = 0$ $(2^x)^2 - 12.2^x + 32 = 0$ $y^2 - 12y + 32 = 0$ (taking $y = 2^x$) $y^2 - 8y - 4y + 32 = 0$	$y(y-8) - 4(y-8) = 0$ ∴ $(y-8)(y-4) = 0$ Either $y - 8 = 0$ or $y - 4 = 0$ ∴ $y = 8$ or $y = 4$ $\Rightarrow 2^x = 8 = 2^3$ or $2^x = 4 = 2^2$ Therefore $x = 3$ *or* $x = 2$.
2	Solve: $\left(x - \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) = 7\frac{1}{4}$	
	$\left(x - \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) = \frac{29}{4}$ $\left(x + \frac{1}{x}\right)^2 - 4 + 2\left(x + \frac{1}{x}\right) = \frac{29}{4}$ (as $(a-b)^2 = (a+b)^2 - 4ab$) $p^2 + 2p - \frac{45}{4} = 0$ Taking p = $x + \frac{1}{x}$ $4p^2 + 8p - 45 = 0$	$4p^2 + 18p - 10p - 45 = 0$ $2p(2p+9) - 5(2p+9) = 0$ $(2p-9)(2p+9)$ = 0 ∴ Either 2p + 9 =0 or 2p – 5 = 0 $\Rightarrow$ p = $-\frac{9}{2}$ or p = $\frac{5}{2}$

3	If the roots of the equation $p(q-r)x^2+q(r-p)x+r(p-q)=0$ are equal show that $\frac{2}{q}=\frac{1}{p}+\frac{1}{r}$ = 0	
	Since the roots of the given equation are equal the discriminant must be zero $q^2(r-p)^2-4.p(q-r)r(p-q)=0$ $q^2\,r^2+q^2p^2-2q^2rp-4pr(pq-pr-q^2+qr)=0$ $p^2q^2+q^2r^2+4p^2r^2+2q^2pr-4p^2qr-4pqr^2=0$	$(pq+qr-2rp)^2=$ $pq+qr=2pr$ $\frac{pq+qr}{2pr}=1$ $\frac{q}{2}.\frac{(p+r)}{pr}=1$ $\frac{1}{r}+\frac{1}{p}=\frac{2}{q}$
4	Solve for x, y and z: $\frac{xy}{x+y}=70$ $\frac{yz}{y+z}=140$	
	we can write as $\frac{x+y}{xy}=\frac{1}{70}$ or $\frac{1}{x}+\frac{1}{y}=\frac{1}{70}$ ....(i) $\frac{x+z}{xz}=\frac{1}{84}$ or $\frac{1}{z}+\frac{1}{x}=\frac{1}{84}$ ....(ii) $\frac{y+z}{yz}=\frac{1}{140}$ or $\frac{1}{y}+\frac{1}{z}=\frac{1}{140}$ ......(iii) By (i) + (ii) + (iii), we get $2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=\frac{1}{70}+\frac{1}{84}+\frac{1}{140}=\frac{14}{420}$ $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{7}{420}=\frac{1}{60}$ .... (iv)	, By (iv) – (iii), $\frac{1}{x}=\frac{1}{60}-\frac{1}{140}=\frac{4}{420}$ → $x=105$ By (iv) –(ii), $\frac{1}{y}=\frac{1}{60}-\frac{1}{84}=\frac{2}{420}$ → y = 210 By (iv) – (i) $\frac{1}{z}=\frac{1}{60}-\frac{1}{70}$ → z = 420 Thus $x=105, y=210$ $z=420$
Statement Problems		
4	Difference between a number and its positive square root is 12. Find the numbers.	
	Let the number be x. Then $x-\sqrt{x}=12$.........(i) $x-\sqrt{x}-12=0$ Taking $y=\sqrt{x}$, $y^2-y-12=0$ $(y-4)(y+3)=0$	∴ Either y = 4 or y = -3 Either $\sqrt{x}=4$ or $\sqrt{x}=-3$ If $\sqrt{x}=-3$ → $x=9$ It does not satisfy equation (i) so $\sqrt{x}=4$ or $x=16$
5	A piece of iron rod costs ₹60. If the rod was 2 metre shorter and each metre costs ₹1.00 more, the cost would remain unchanged. What is the length of the rod?	
	Let the length of the rod be x metres. The rate per meter is ₹ $\frac{60}{x}$ As given $\frac{60}{x-2}=\frac{60}{x}+1$ → $\frac{60}{x-2}-\frac{60}{x}=1$ → $\frac{120}{x(x-2)}=1$ → $x^2-2x=200$	$x^2-2x-120=0$ $(x-12)(x+10)=0$ Either $x=12$ or x = -10 (not possible) ∴ Hence the required length = 12m
6	Divide 25 into two parts so that sum of their reciprocals is $\frac{1}{6}$	
	Let the parts be x and 25 – x By the question $\frac{1}{x}+\frac{1}{25-x}=\frac{1}{6}$ $\frac{25-x+x}{x(25-x)}=\frac{1}{6}$ $150=25x-x^2$ → $x^2-25x+150=0$	$x^2-15x-10x+150=0$ $x(x-15)-10(x-15)=0$ $(x-15)(x-10)=0$ $x=10$ and $x=15$ So the parts of 25 are 10 and 15