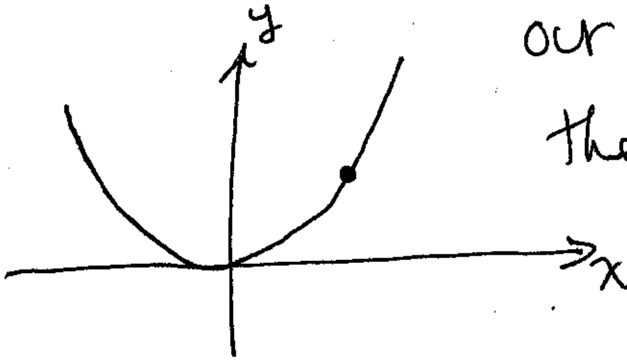
Math 1496 - Calc I

Calculus Preview

There are 2 problems which are fundamental and that is the tangent problem & area problem.

Tangent Problem

Consider $f(x) = x^2$ and the point $(1,1)$.



our goal is to find the eq^n of the tangent at this point. Recall the tangent (line) is a line that touches the curve at only 1 pt. If $y = mx + b$ & we know that $(1,1)$ is on the line then $1 = m + b$ so $b = 1 - m$ and the eq^n becomes

$$y = mx - m + 1$$

Now we find the intersection pt of the line and parabola so

$$x^2 = mx - m + 1$$

or $$x^2 - mx + m - 1 = 0$$

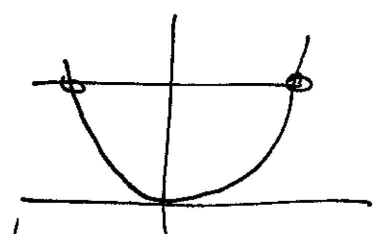
Solving the quadratic eqⁿ gives

$$x = \frac{m \pm \sqrt{m^2 - 4(m-1)}}{2} \leftarrow \begin{matrix} m^2 - 4m + 4 \\ (m-2)^2 \end{matrix}$$

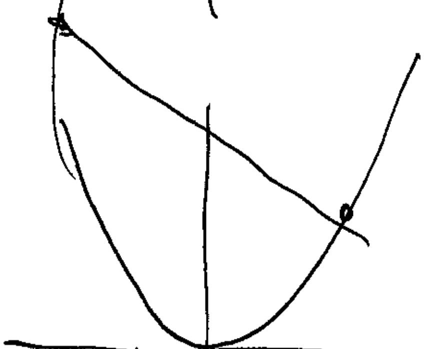
$$= \frac{m \pm (m-2)}{2} = \frac{m-(m-2)}{2}, \frac{m+m-2}{2}$$

$$= 1, \frac{2m-2}{2}$$

if $m = 0$ $x = 1, -1$



if $m = -1$ $x = 1, -2$

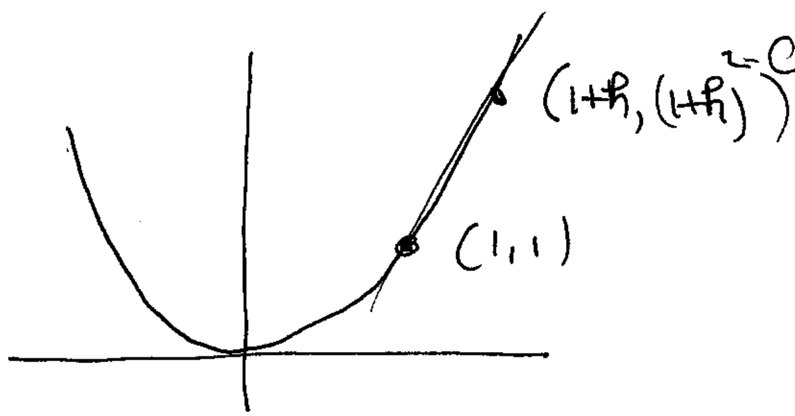


in both cases there are 2 intersection pts - we only want 1 so choose $m = 2$

and so the slope $m = 2$ and the eq^n of the tangent is

$$y = 2x - 1$$

ca - let's reconsider this problem and another pt close to $(1,1)$



Now the slope of this line (secant to curve)

is $\frac{\Delta y}{\Delta x} = \frac{(1+h)^2 - 1}{\cancel{1}+h-\cancel{1}} = \frac{(1+h)^2 - 1}{h} = \frac{\cancel{1}+2h+h^2-\cancel{1}}{h}$

$= \frac{\cancel{h}(2+h)}{\cancel{h}}$ (as long as $h \neq 0$)

$= 2+h$

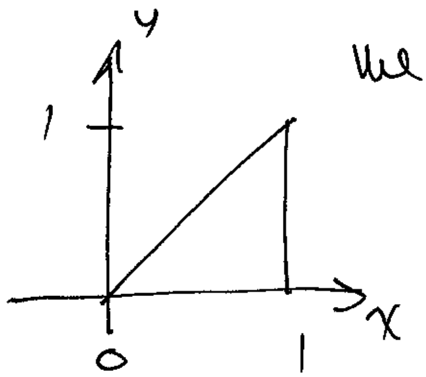
Now if we let the pt slide down the curve and approach the point (1,1) which is letting h get smaller and smaller so

$$\frac{\Delta y}{\Delta x} \to 2 \quad \text{the value we found earlier.}$$

So the idea of letting h get smaller & smaller is know as the limit problem.

Area Problem

Consider $f(x) = x$ on the interval $[0, 1]$



we can easily find the area under this curve on $[0, 1]$

$A = \frac{1}{2}(1)(1) = \frac{1}{2}$ (area of a triangle)

Suppose instead we wanted to approximate the area with some rectangles and here we will assume that base of the rectangles are the same so let us consider 2 first

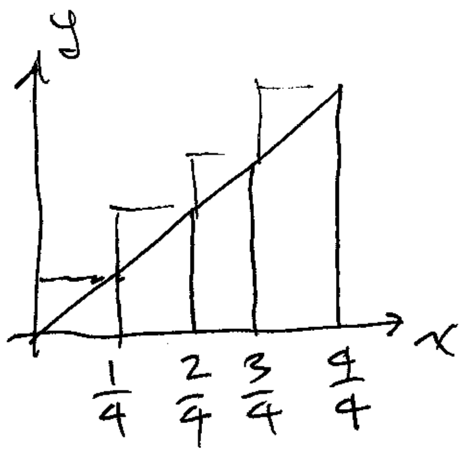
$A_1 = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$

$A_2 = \frac{1}{2}(1) = \frac{1}{2}$

$A \simeq \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

too much

Next we try 4 rectangles



$A_1 = \frac{1}{4}\left(\frac{1}{4}\right) = \frac{1}{16}$

$A_2 = \frac{1}{4}\left(\frac{2}{4}\right) = \frac{2}{16}$

$A_3 = \frac{1}{4}\left(\frac{3}{4}\right) = \frac{3}{16}$

$A_4 = \frac{1}{4}\left(\frac{4}{4}\right) = \frac{4}{16}$

$A \simeq \frac{1+2+3+4}{16} = \frac{10}{16} = \frac{5}{8}$ better but still too much

$= \frac{1}{2} + \frac{1}{8}$

Next try 8

$A_1 = \frac{1}{8}\left(\frac{1}{8}\right) \quad A_2 = \frac{1}{8}\left(\frac{2}{8}\right) \; \cdots\cdots \quad A_8 = \frac{1}{8}\left(\frac{8}{8}\right)$

$= \frac{1+2+3+4+5+6+7+8}{64} = \frac{36}{64} = \frac{9}{16} = \frac{1}{2} + \frac{1}{16}$

so continuing in this fashion using n rectangles

n	A
2	$\frac{1}{2}+\frac{1}{4}$
4	$\frac{1}{2}+\frac{1}{8}$
8	$\frac{1}{2}+\frac{1}{16}$
16	$\frac{1}{2}+\frac{1}{32}$
32	$\frac{1}{2}+\frac{1}{64}$
64	$\frac{1}{2}+\frac{1}{128}$

and we see as we increase the # rectangles the base is getting small and in the "limit" the area approches the value of $\frac{1}{2}$

So again we see the "limit process".

so now we want to introduce limits.

This we do the next few class.