WHAT IS SPACE? WHAT IS TIME?

bj April 2015

"Space is what we measure with a ruler."
"Time is what we measure with a clock."
Hans Mueller, MIT, 1956

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Spacetime vs. Substrate

"Passive" Spacetime

QED
QCD
Electroweak

$$\left.\begin{array}{l}\text{QED}\\ \text{QCD}\\ \text{Electroweak}\end{array}\right\} g_{\mu\nu} = \eta_{\mu\nu}$$

Substrate

$$A_\mu(x) \Rightarrow F_{\mu\nu}$$

"Active" Spacetime

Einstein-Hilbert: $ds^2 = g_{\mu\nu}(x)\,dx^\mu dx^\nu$ $\qquad g_{\mu\nu}(x)$

Einstein-Cartan $\left.\begin{cases} ds^A = e^A_\mu dx^\mu \\ ds^2 = ds^A ds^B \eta_{AB} \end{cases}\right\} \begin{Bmatrix} \omega^{AB}_\mu(x) \Rightarrow R^{AB}_{\mu\nu} \\ e^A_\mu(x) \end{Bmatrix}$

MacDowell-Mansouri: $H ds^{5A} = A^{5A}_\mu dx^\mu$ $\qquad \begin{Bmatrix} A^{5A}_\mu = H e^{5A}_\mu \\ A^{AB}_\mu = \omega^{AB} \end{Bmatrix} \Rightarrow F^{\mu\nu}_{AB}$

The Bottom Line

Substrate: a catalog of spacetime events

Spacetime: an algorithm for correlating them
(The output is a Riemannian manifold.)

An analogy: Google Maps on a touch-screen

Space: what your brain tells you about the information on the screen.

Substrate: The sequence of 0's & 1's that tells the pixels on the screen what to do.

KLEIN GEOMETRY (TOY VERSION)

$O(3)/O(2)$, not $O(4,1)/O(3,1)$

$$A_\mu^{AB} = \begin{array}{c} 13 \\ 23 \\ 12 \\ \\ \end{array} \begin{array}{c}\left(\begin{array}{cc} 1 & 0 \\ 0 & g(x) \\ \hline 0 & -p(x) \end{array} \right) \\ \begin{array}{cc} x & \quad y \end{array}\end{array} \begin{array}{l} \left.\right\}\text{Frame fields } e \\ \left.\right\}\text{spin connection } \omega \end{array}$$

$$F_{\mu\nu}^{AB} = \begin{array}{c} 13 \\ 23 \\ 12 \end{array} \left(\begin{array}{c} 0 \\ g' - p \\ -p' - g \end{array} \right)_{xy}$$

$$ds^2 = dx^2 + g^2 dy^2$$

$$\left[ds^2_{MM} = dx^2 + (g^2 + p^2) dy^2 \right]$$

$$F_{xy}^{13} = \partial_x \cancel{A_y^{13}} - \cancel{\partial_y A_x^{13}} + \cancel{A_x^{12} A_y^{23}} - \cancel{A_y^{12} A_x^{23}} = 0$$

$$F_{xy}^{23} = \partial_x A_y^{23} - \cancel{\partial_y A_x^{23}} + \cancel{A_x^{21} A_y^{13}} - A_y^{21} A_x^{13} = g' - p$$

$$F_{xy}^{12} = \partial_x A_y^{12} - \cancel{\partial_y A_x^{12}} + A_x^{13} A_y^{32} - \cancel{A_y^{13} A_x^{32}} = -p' - g$$

Derek Wise, arXiv 0611154

THE "FLAT" SOLUTION

$$F^{AB}_{xy} = 0 \Rightarrow \boxed{\begin{matrix} p = q' \\ q = -p' \end{matrix}}$$

$$q'' + q = 0$$

$$q = \sin x \qquad p = -\cos x$$

$$q^2 + p^2 = 1$$

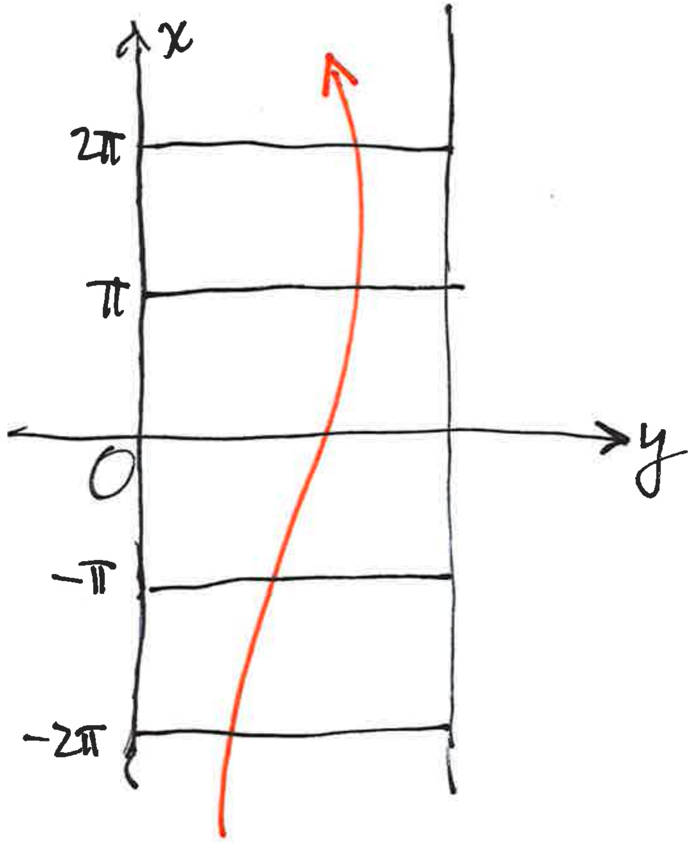
$$\boxed{ds^2 = dx^2 + \sin^2 x \, dy^2} \Rightarrow d\theta^2 + \sin^2\theta \, d\varphi^2$$

Spherical Surface

$$\left[ds^2_{MM} = dx^2 + dy^2 \right]$$

BUCKYBALL SPACE

Suppose $-\infty \leq x \leq +\infty$ $\quad 0 \leq y \leq 2\pi$



A 2-d ~~surface~~ surface of "everywhere" constant curvature.

A. Randono, "Introduction to Gauge Gravity" (~1911 ± 1)

THE MINKOWSKI GENERALIZATION

$$\left.\begin{matrix}\text{Sphere} \\ (x,y)\end{matrix}\right\} \Longrightarrow \left\{\begin{matrix}\text{Hyperboloid} \\ (t,z)\end{matrix}\right\}$$

Three cases: (FRW)

"$k=0$" $\quad a = e^t$

$k = \pm 1 \quad a = \begin{cases} \cosh t \\ \sinh t \end{cases}$

$$ds^2 = dt^2 - a^2(t)\,dz^2$$

$$\left[ds^2_{MM} = \begin{cases} dt^2 & k=0 \\ dt^2 \pm dz^2 & k = \pm 1 \end{cases} \right]$$

FORMFORMS

One-form

a_μ

2-Form

$f_{\mu\nu} = -f_{\nu\mu}$

etc.

One-formform

a_μ^A

2-Formform

$f_{\mu\nu}^{AB} = -f_{\nu\mu}^{AB} = -f_{\mu\nu}^{BA} = +f_{\nu\mu}^{BA}$

etc.

In n-dimensional space,

an $\left\{ \begin{matrix} \text{n-form} \\ \text{n-formform} \end{matrix} \right\}$ is an invariant

The MM action is a sum of 4-formforms:

$$\mathcal{L} \sim \frac{M_{pl}^2}{\Lambda} |FF| + M_{pl}^2 |eeF| + \Lambda |eeee|$$

Formform algebraic properties are user-friendly:

$$|abc| = |bac|$$

$$\frac{d}{dt}|abc| = |\dot{a}bc| + |a\dot{b}c| + |ab\dot{c}|$$

$$|(a+b)cd| = |acd| + |bcd|$$