

Fluency, Word Problems, and Mathematics Intervention



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[@sarahpowellphd](https://www.instagram.com/sarahpowellphd)



Introduce yourself.

Describe your strengths in supporting mathematics.

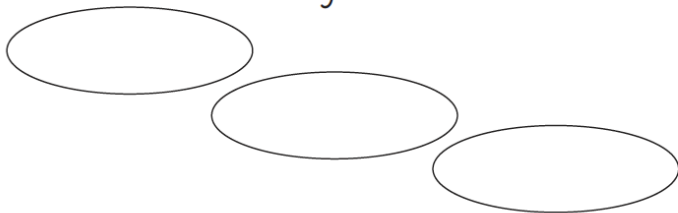
Describe an opportunity for growth.

Fluency, Word Problems, and Mathematics Intervention for Students

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www.sarahpowellphd.com

Instructional Platform

Instructional Delivery



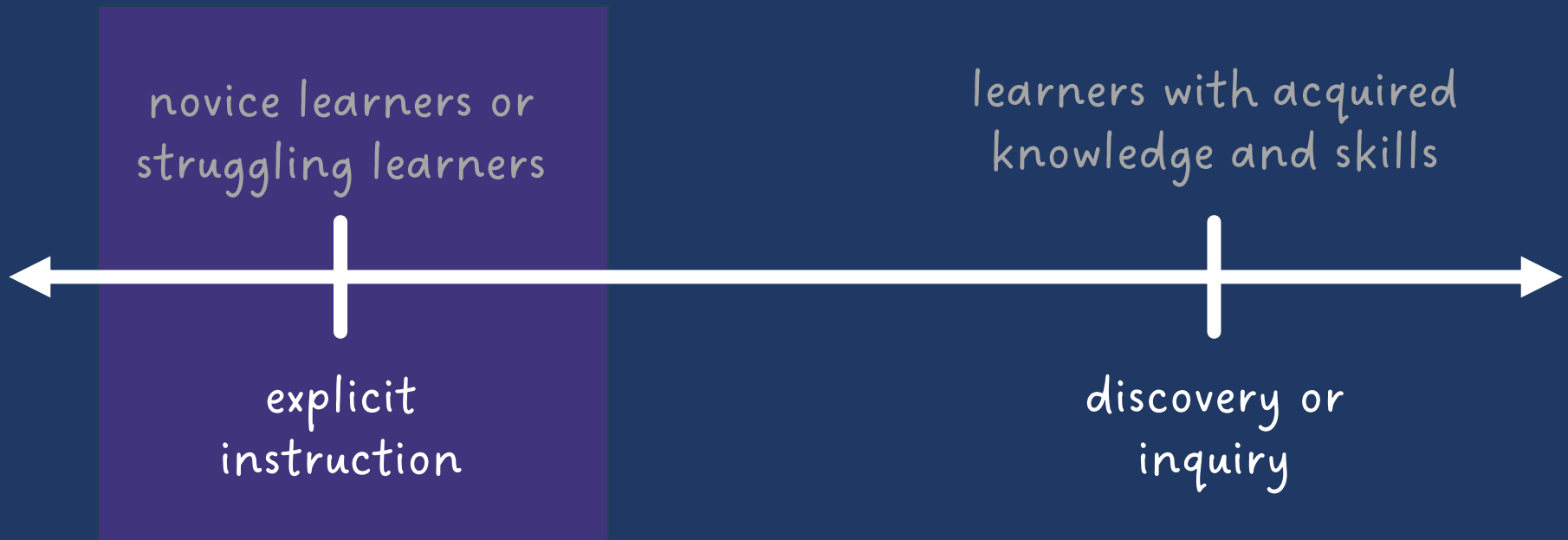
Instructional Strategies



Instructional Platform



What's the continuum of mathematics support?



Anita Archer (2019)



Instructional Platform



evidence-based practice

A practice that
has shown
consistent and
positive results



evidence-based practice



evidence-based intervention

An intervention
(i.e., packaged
program) that
has shown
consistent and
positive results



evidence-based practice

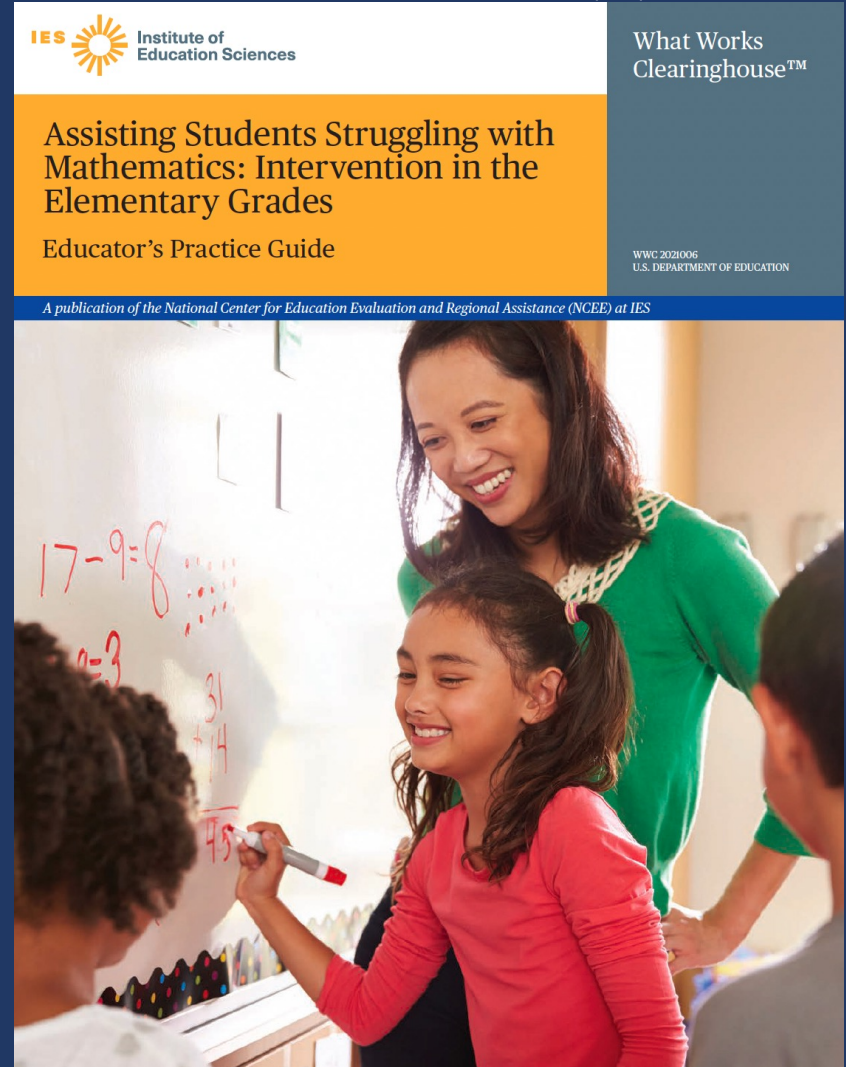


evidence-based intervention

evidence-based strategy

A method or strategy
that has shown
consistent and
positive results

What's the continuum of mathematics support?



Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.



Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and
Practice

INSTRUCTIONAL STRATEGIES

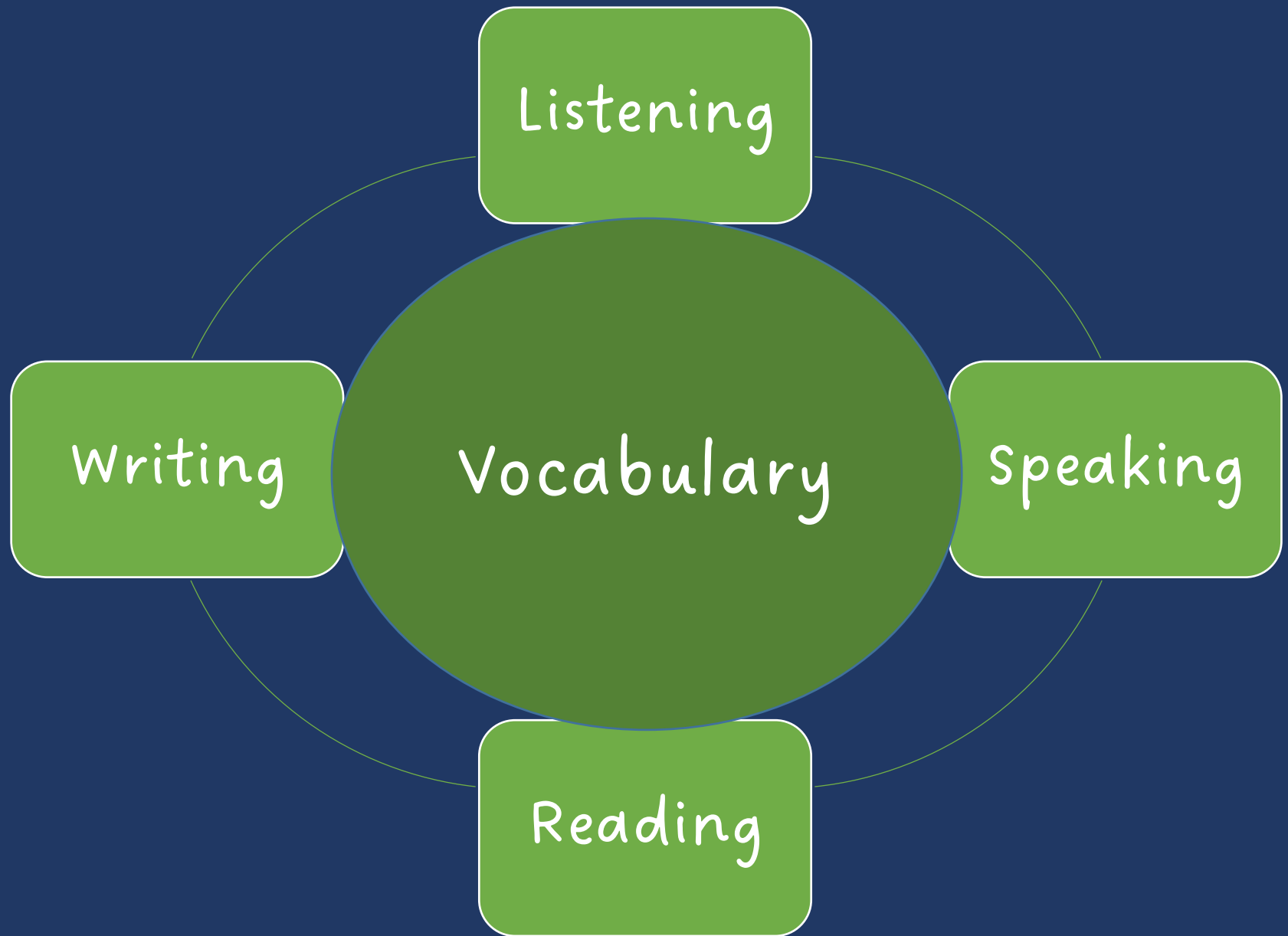
Fluency

Word Problems



Vocabulary





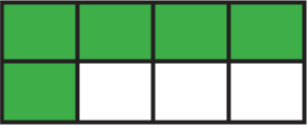



Use formal math language

Use terms precisely



Use semantic maps

Term	Definition	Example	Nonexample
integer	(... -3, -2, -1, 0, 1, 2, 3...)	15	$\frac{1}{3}$
denominator	The equal parts of a whole or set.	$\frac{5}{8}$ <p>8 is the denominator</p> 	$\frac{5}{8}$ 
numerator	The equal parts of a given fraction.	$\frac{5}{8}$ <p>5 is the numerator</p> 	$\frac{5}{8}$ 

(Stevens et al., 2022)



Use word walls

difference

The result of subtracting or the result when comparing two numbers.

$$5 - 4 = 1$$

1 is the **difference**

horizontal line

A straight line that goes from left to right or right to left.



equivalent

Two numbers that have the same value.

$$\frac{2}{4} \text{ is equivalent to } \frac{1}{2}$$

total

The result or sum when adding numbers.

$$5 + 7 = 12$$

(mathspiral.com)



Use flash cards

addend

quotient

divisor

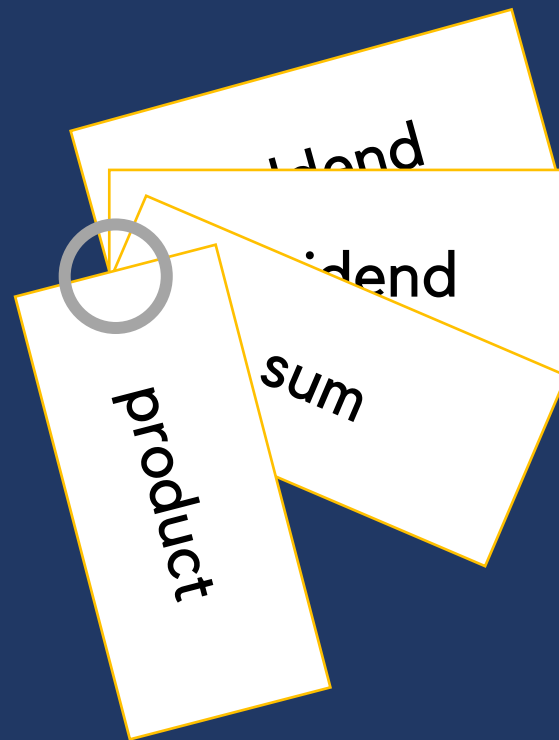
dividend

factor

sum

product

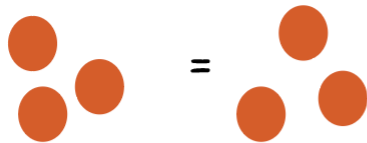
difference



(Petersen-Brown et al., 2019)

Use glossaries

equal - with the same value

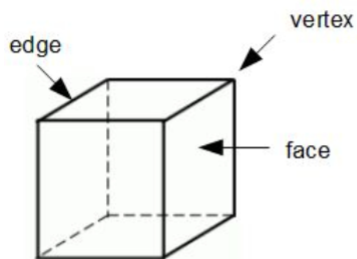


E

equation - expressions with an equal sign

$$5 = 2 + 3$$

edge - line segment between faces



Glosario de Matemáticas

Término	Definición
adición (<i>add</i>)	Juntar o agregar.
algoritmo (<i>algorithm</i>)	Es una serie de pasos organizados que describe el proceso que se debe seguir, para dar solución a un problema específico.
alinear (<i>array</i>)	Un conjunto de objetos, imágenes o números alineados en columnas y filas.
ancho (<i>width</i>)	La medida de un lado de un objeto, generalmente el lado más corto.
ángulo (<i>angle</i>)	Dos semirrectas o segmentos de línea recta que comparten un punto final.
ángulo agudo (<i>acute angle</i>)	Un ángulo que mide menos de 90°.
ángulo obtuso (<i>obtuse angle</i>)	Un ángulo que mide entre 90° y 180°.
ángulo recto (<i>right angle</i>)	Un ángulo que mide exactamente 90°.
área (<i>area</i>)	La cantidad de unidades cuadradas que cubre una figura geométrica cerrada.
balance presupuestario (<i>balance the budget</i>)	Un presupuesto es cuando la cantidad total de dinero gastado, ahorrado y compartido es igual al ingreso total.
báscula (<i>balance scale</i>)	Instrumento de medición que se utiliza para medir el peso o la carga.
base (computación) (<i>base (computation)</i>)	Un número que se multiplica por un exponente.
base (geometría) (<i>base (geometry)</i>)	La forma inferior de una figura tridimensional.

 **SPIRAL**
Specialized Math Intervention to Reach All Learners

(mathspiral.com)



Use anchor charts

Addition Computation

1

17 ← addend

+ 59 ← addend

76 ← sum

Quadrilaterals

Kite



Rhombus



Parallelogram



Square



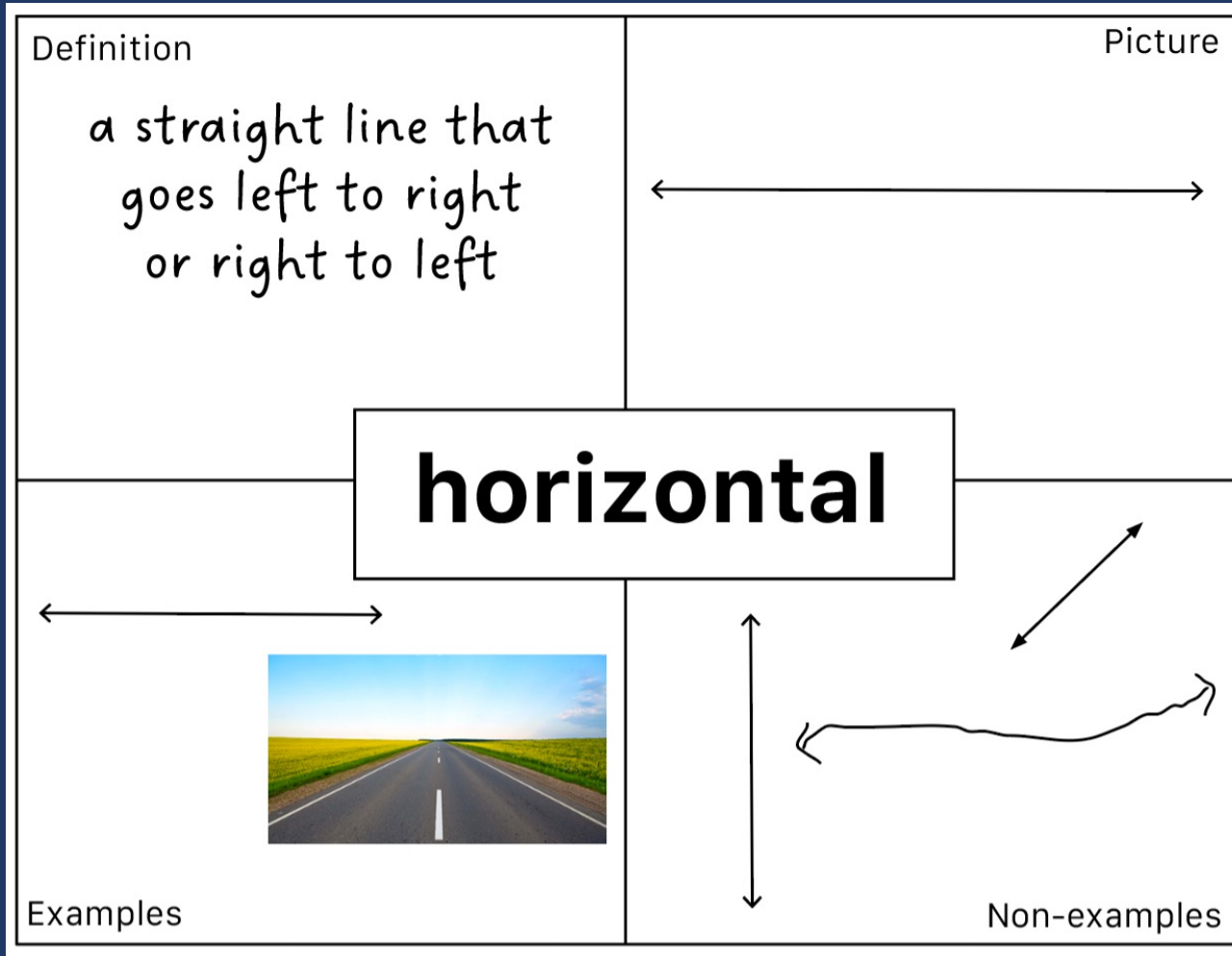
Rectangle



Trapezoid



Use graphic organizers



Use games

Name: _____ Date: _____

3RD GRADE MATH

T W D H P T R I Y T S F T O Y T S P X G U C I C
M S G C E N T I M E T E R S Y S C Y Z D R G D J
I M I E U J R L H F O A W Z X U A L F J O Q F T
U P F N Z L R V C N N P Z D D H L P H F A V J Z
Y R U P M I A C O R D P I V J T E I H Y M F M S
E K X H R T T Y U J E S V X S V A T Z W M S E T
W Z Z W G E Q G M P C V C Z E H E L W L T T A A
N U V V B R J F H Y N E O Z H T K U K F A Z S W
Q Y J P S H A C D X A M Y G C U K M Z L J E U P
T Z W B P N R Z Y F L I Y D N A J J Z M W D R S
O W W U W T G I T I A T F F I S Q G X K K I E Z
E W R Q C S U U C U B J L E Z P B H G B C V M Q
F R A C T I O N S K Z O W M D X B M M C O I E C
N Z C A T L I J B J M U F O X X A D O Y L D N K
D E V P T U C M Y S A H K Z S U O W R S C V T G
M X N L N S O L V E R R I Y I B Z N Q O X C P W
M V W Z C D Q Z Z U G X L U K P Y O B S W I M N
K F L J U R J G J R O Q E M X C U O H N M S F D
S U B T R A C T G U L Y Y A K W U N U S O L H H
E G F D D A I O I A I Y R R J I C A P A C I T Y
I L W B A T R D O X K C J G H T D X E G C S M Z
X J Q Y A H L T B J R A I W J B C K I U S S I U
N U O D N R L V T I T L C U P C L W C B F S Z Q
J F N P R R K P Y M R E L U R Q H M A E Y Y Z H

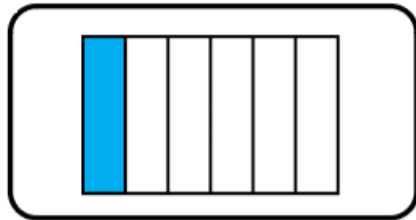
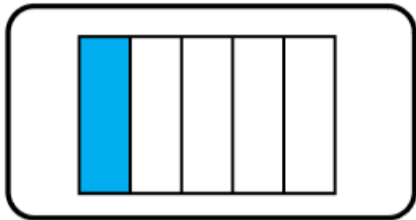
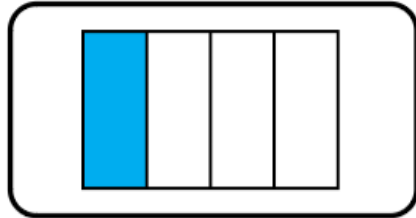
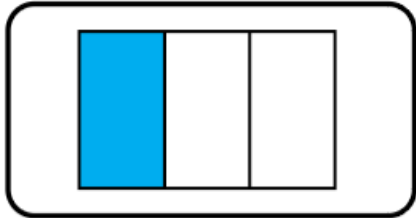
ADD	BALANCED	CAPACITY	CENTIMETERS
CLOCK	DIVIDE	FRACTIONS	GRAM
INCHES	KILO GRAM	LITER	MEASUREMENT
MULTIPLY	NOON	RULER	SCALE
SOLVE	SUBTRACT	TIME	

square	circle	decagon	triangular prism	cone
cylinder	cube	sphere	line	pyramid
parallelogram	octagon	FREE	trapezoid	oval
kite	pentagon	cylinder	rectangle	line segment
hexagon	rhombus	triangle	quadrilateral	rectangular prism

https://wordmint.com/public_puzzles/13055



Use games



one

one-half

one-third

one-fourth

one-fifth

one-sixth

Use discussion



add
change
compare
decrease
difference
increase
part
put together
subtract
total

Use technology



Lessons ▾

Mixes

About ▾



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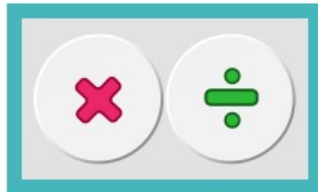
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Math



**Addition &
Subtraction**



**Multiplication &
Division**



**Numbers &
Operations**



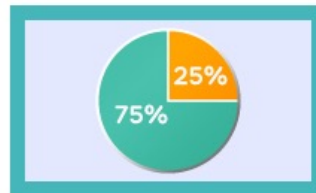
**Expressions &
Equations**



**Geometry &
Measurement**



**Statistics &
Probability**



**Ratios &
Proportional
Relationships**



Use math writing

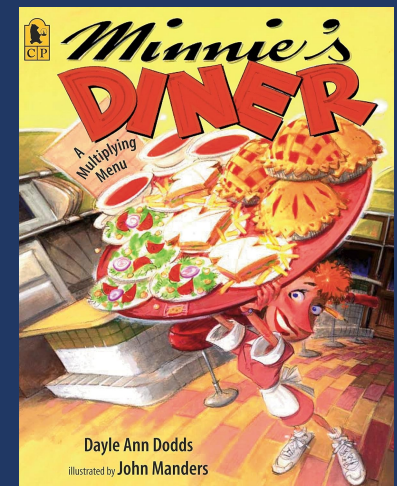
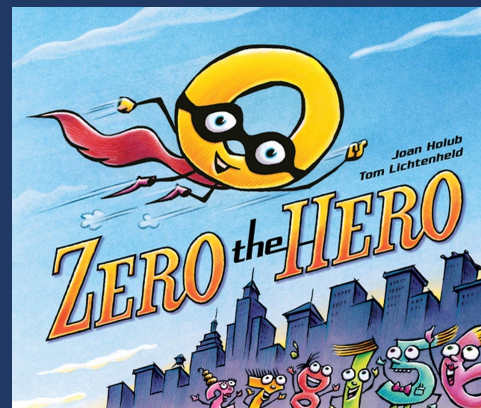
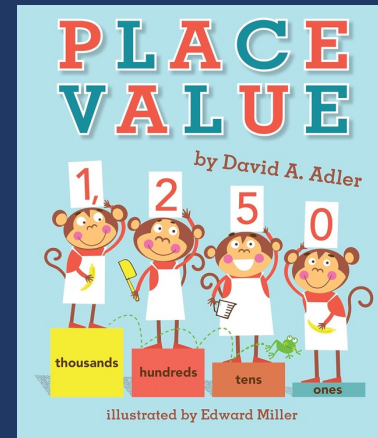
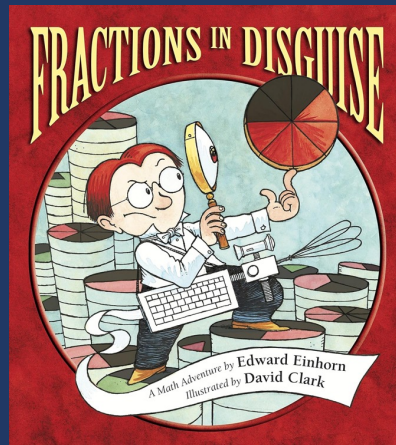
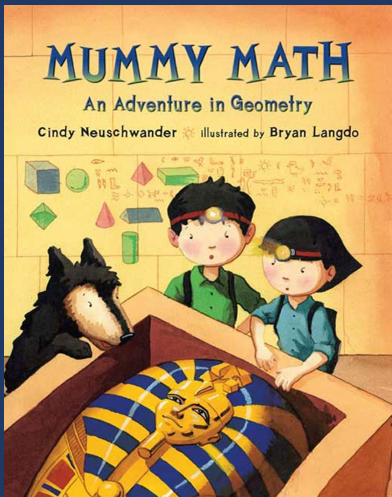
I would help Cole out by not putting them into thirds because he put 1 and $\frac{2}{3}$ but the answer should be $\frac{3}{5}$. So next I would draw the lines to make them into 5 pieces but put next to each other. Then you shade in 3 of the squares and keep the others alone. Then that would be the correct way to do $\frac{3}{5}$.

What Alex did wrong was that it was that he has five rectangles but they are different sizes so first what I would do is draw 5 rectangles side by side then what you would do is shade 3 of them and that would equal $\frac{3}{5}$ so that would be the correct way to solve it.

I would help Bo out by he shaded three rectangles which is correct but he left 5 shaded and there should only be 2 dark rectangles so if you erase 3 rectangles then then answer would be $\frac{2}{5}$ and that is the correct way to solve it.



Use read-alouds



(Purpura et al., 2017)

Model and practice

- Teacher** Let's work on addition. Today, let's think about addition as combining. What does it mean to combine?
- Students** Put together.
- Teacher** When we combine, we put things together. When you cook, you put ingredients together. For example, to make macaroni and cheese, you combine what?
- Students** Macaroni noodles and cheese!
- Teacher** That's right. You combine macaroni and cheese! Now, let's think about combining numbers. Look at this problem.
(Show problem.)

(Powell & Driver, 2015; Stevens et al., 2022)



Make sure students use
mathematics vocabulary
as much as adults do!



SCOPE AND SEQUENCE



Where does VOCABULARY
fit within your scope and
sequence? What are your
plans for your teachers?



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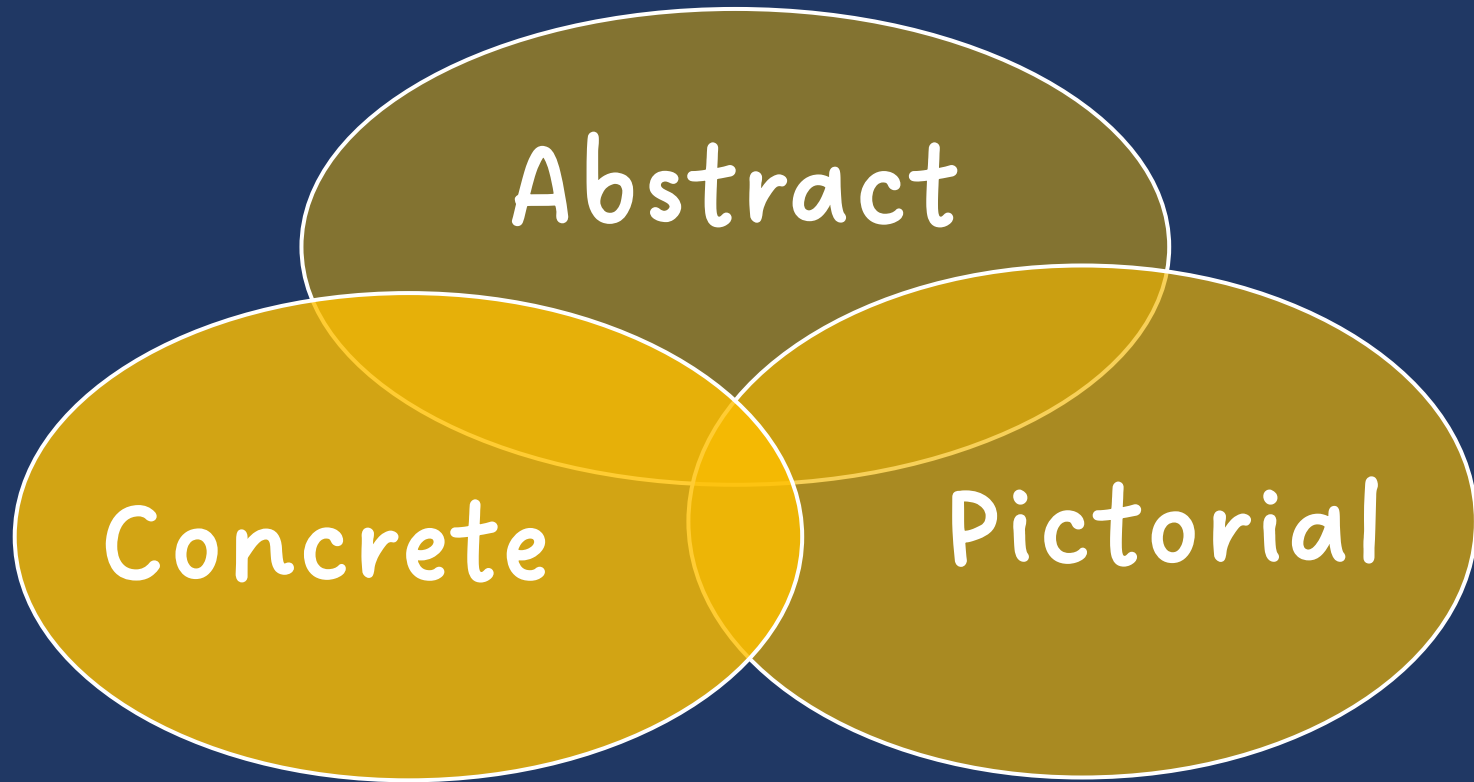
Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.



Representations





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Model and Practice

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



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Fluency



Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and
Practice

INSTRUCTIONAL STRATEGIES

Fluency



FLUENCY

Research and Information

Types of Fluency

Type	Memorization?	
	Yes	No





Mathematics fluency, particularly fluency with facts and computation, is related to overall mathematics performance.

(Bailey et al., 2012; Cirino et al., 2019; Koponen et al., 2007, 2017; Vukovic et al., 2014)

Fact fluency practice improves mathematics fact performance.

(Burns et al., 2010; Coddington et al., 2011; McCallum et al., 2004; Nelson et al., 2013; Poncy et al., 2010; Schutte et al., 2015; Stocker & Kubina, 2017)



Fluency is
doing
mathematics
easily and
accurately.

Fluency in
mathematics
makes
mathematics
easier.

Fluency
provides less
stress on
working
memory.

Fluency helps
students build
confidence
with
mathematics.

With fluency, it is important to emphasize both
conceptual learning and procedural learning.



Addition	Subtraction
Multiplication	Division

Counting

Comparison

Fractions

Geometry

Money

Time

Measurement

Algebra



Ease and
accuracy

Memorization
or automaticity



FLUENCY

Research and Information

Types of Fluency

Type	Memorization?	
	Yes	No



List different types of fluency. Discuss whether they require memorization.



FLUENCY

Fact Fluency

Addition	Subtraction
Multiplication	Division



100 addition facts

Single-digit addends sum to a single- or double-digit number

$$\begin{array}{r} 5 \\ + 4 \\ \hline 9 \end{array}$$

(addend)
(addend)
(sum)



Total

Addition

Count one set, count another set, put sets together, count sum



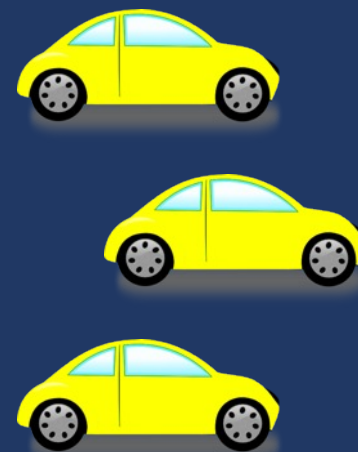
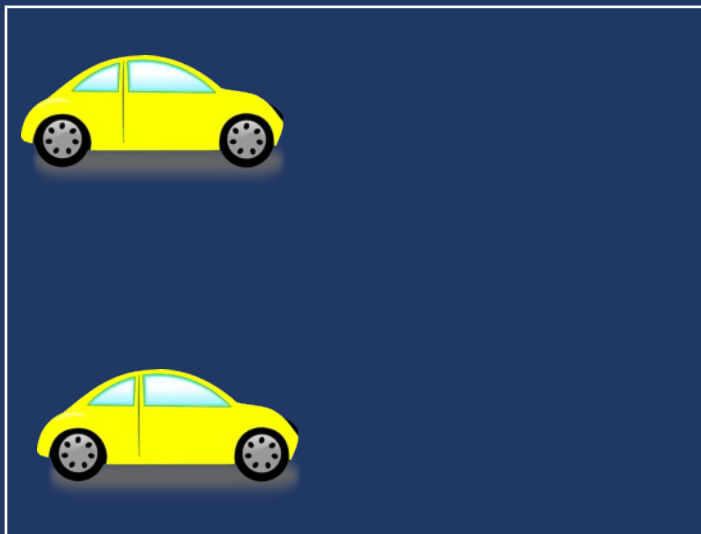
$$2 + 3 = 5$$



Change

Addition

Start with a set, add the other set, count sum



$$2 + 3 = 5$$

Total

Addition

Parts put together into a **total**

Karly saw **4** cardinals and **5** blue jays. How many birds did Karly see?



Change

Addition

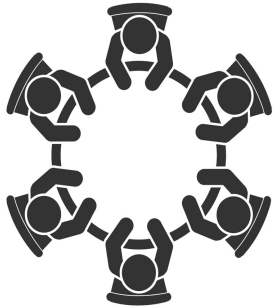
An amount that increases or decreases

Premila had \$4. Then they earned \$5 for cleaning their room.
How much money does Premila have now?



$$3 + 9 = \underline{\quad}$$

Addition



If you teach elementary:
What's a Total story to show
addition?

If you teach secondary:
What's a Change/Join story to
show addition?

100 subtraction facts

Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

$$\begin{array}{r} 16 \\ - 8 \\ \hline 8 \end{array}$$

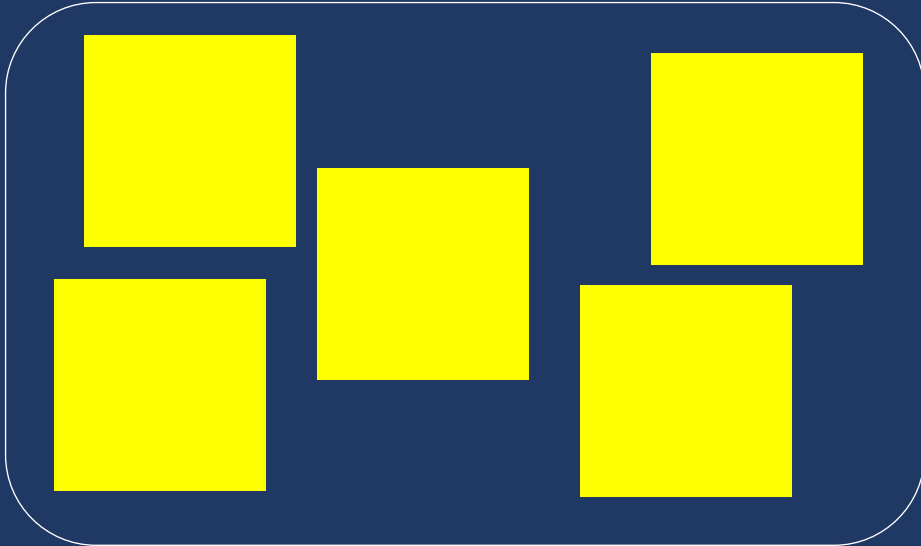
(minuend)
(subtrahend)
(difference)



Change

Subtraction

start with a set, take away from that set, count difference

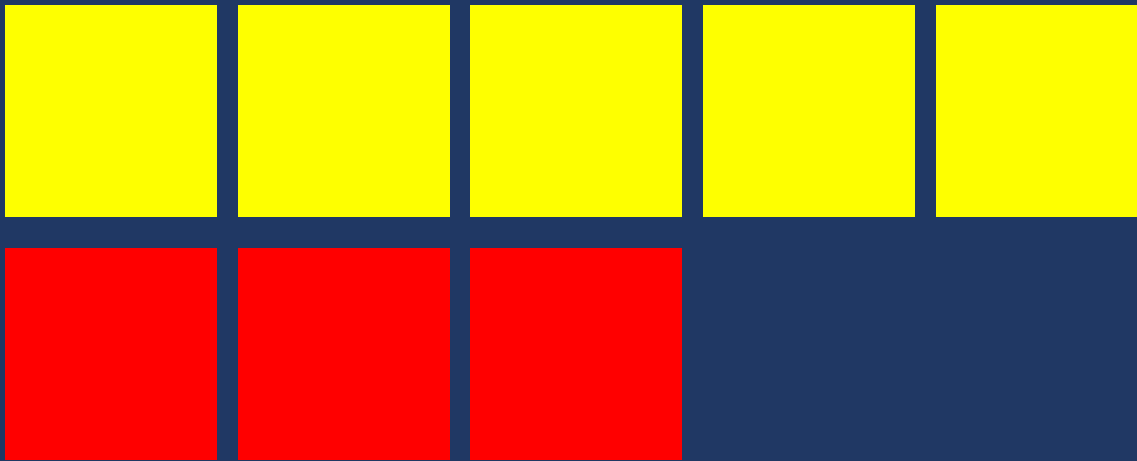


$$5 - 3 = 2$$

Difference

Subtraction

Compare two sets, count difference



$$5 - 3 = 2$$

Change

Subtraction

An amount that increases or decreases

Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?



Difference

Subtraction

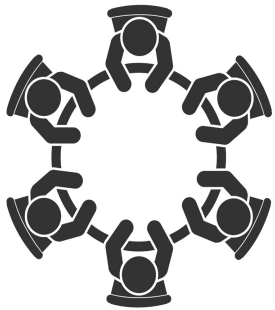
Greater and lesser amounts compared for a difference

Rachel has 9 apples. Jodie has 2 apples. How many more apples does Rachel have? (How many fewer does Jodie have?)



$$9 - 5 = \underline{\quad}$$

Subtraction



If you would chose beaches:
What's a Change/Separate story
to show subtraction?

If you would chose mountains:
What's a Difference story to
show subtraction?

100 multiplication facts

Multiplication of single-digit factors results in a single- or double-digit product

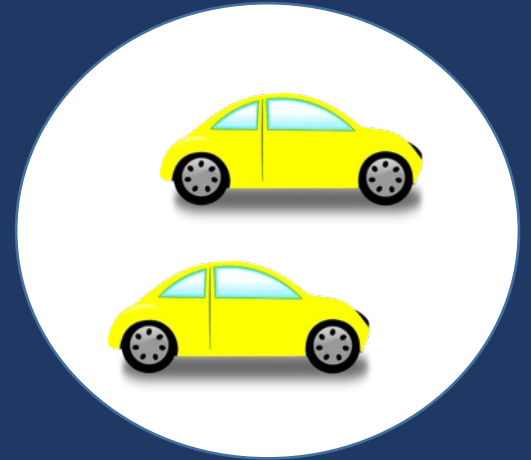
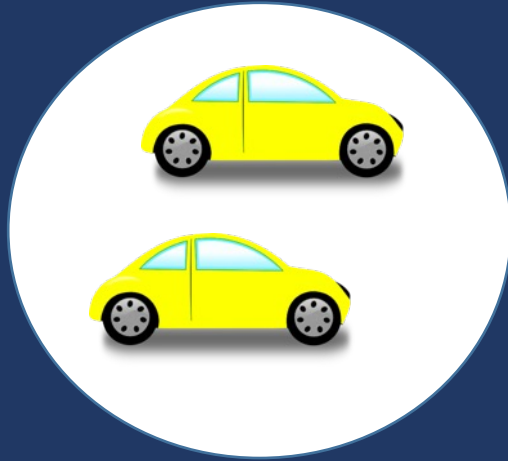
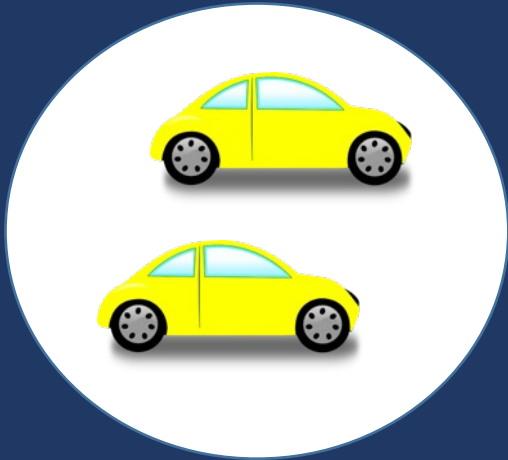
2	(<u>factor</u>)
$\times 3$	(factor)
<hr/>	
6	(<u>product</u>)



Equal Groups

Multiplication

Show the groups, show the amount for each group, count product

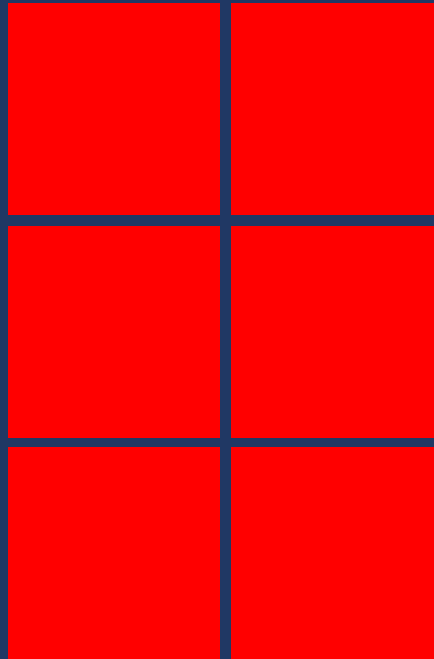


$$3 \times 2 = 6$$

Equal Groups

Multiplication

Show the groups, show the amount for each group, count product

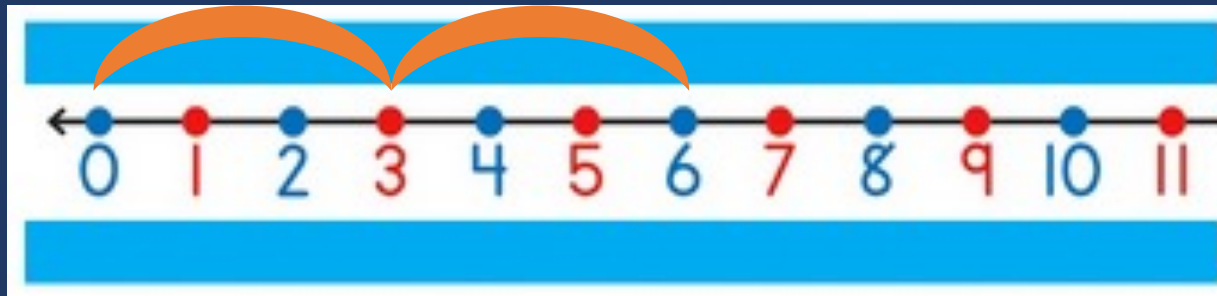


$$3 \times 2 = 6$$

Comparison

Multiplication

Show a set, then multiply the set



$$3 \times 2 = 6$$

Equal Groups

Multiplication

Groups multiplied by **number in each group** for a **product**

Rhiannon has **3** boxes of crayons. There are **8** crayons in each box. How many crayons does Rhiannon have altogether?



Comparison

Multiplication

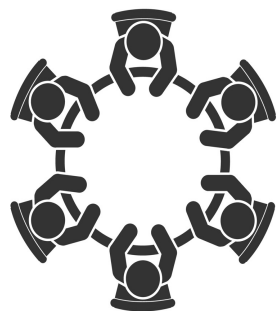
Set multiplied by a number of **times** for a **product**

Vivienne had **8** stickers. Jessica had **3** times as many stickers as Vivienne. How many stickers did Jessica have?



$$2 \times 5 = \underline{\quad}$$

Multiplication



If you wear glasses:

What's an Equal Groups story to show multiplication?

If you don't wear glasses:

What's a Comparison story to show multiplication?

90 division facts

Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

$$8 \div 4 = 2$$

(dividend) (divisor) (quotient)

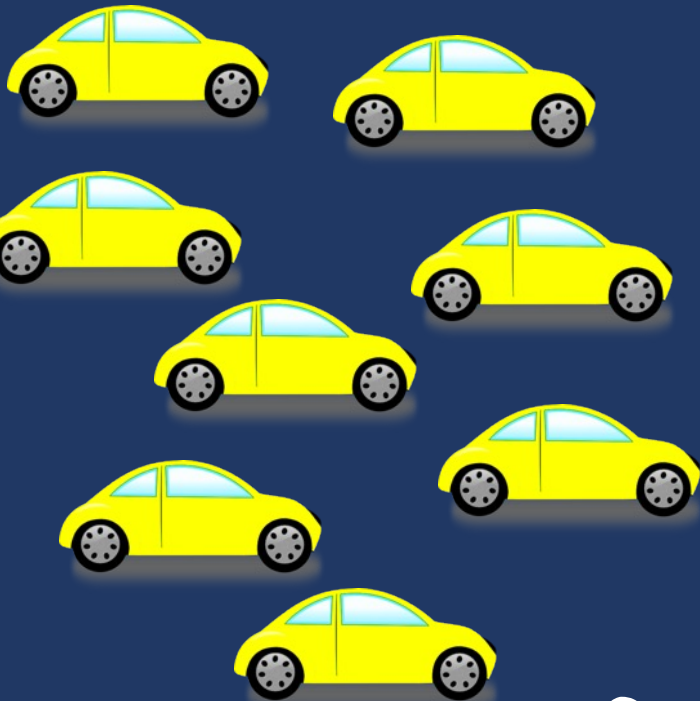


Equal Groups

(Partitive Division)

Division

Show the dividend, divide equally among divisor, count quotient



$$8 \div 2 = 4$$

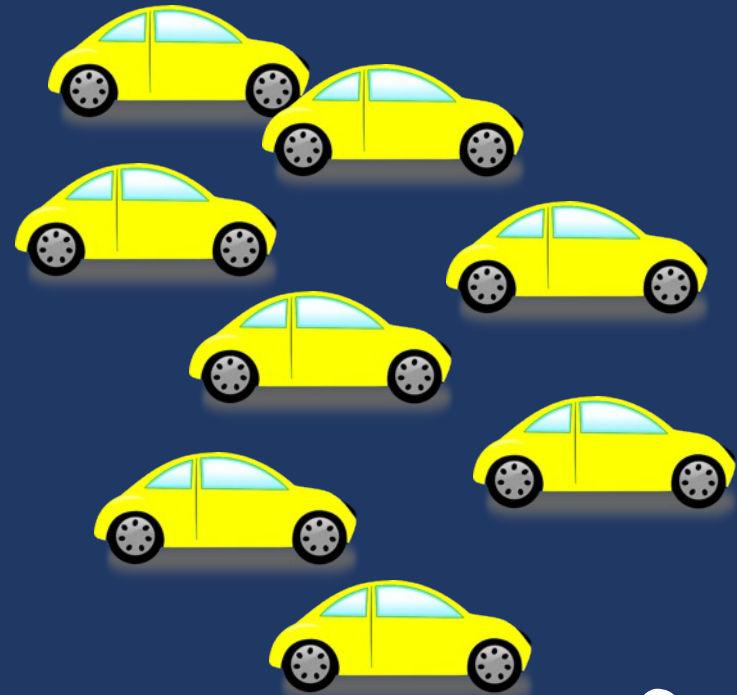


Equal Groups

(Quotative Division)

Division

Show the dividend, make groups of the divisor, count groups



$$8 \div 2 = 4$$



Equal Groups

Division

Groups multiplied by **number in each group** for a **product**

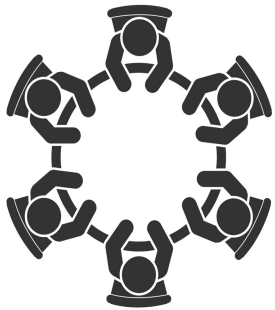
Stefanie has **12** pencils. She wants to share them equally among her **2** friends. How many pencils will each friend receive?

Nicole has **12** pencils. She put them into pencil pockets with **6** pencils each. How many pencil pockets did Nicole use?



$$12 \div 4 = \underline{\quad}$$

Division



If you watch comedies:

What's a Partitive story to show division?

If you watch dramas:

What's a Quotative story to show division?

Addition	Subtraction
Multiplication	Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

$$\begin{array}{r} 5 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ - 4 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 56 \\ \div 8 \\ \hline \end{array}$$

DAILY

BRIEF

Work on small sets of facts

Work on unknown facts
(in combination with known facts)



Dice



Roll the Dice



$$\underline{8} + \underline{7} = \underline{15}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

Beach Ball

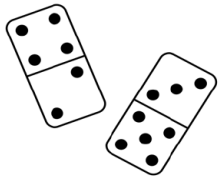


4 plus 6 equals 10.

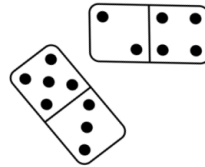
7 plus 6 equals 13.

2 plus 2 equals 4.

Dominoes



Dominoes



$$\underline{4} + \underline{6} = \underline{10}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

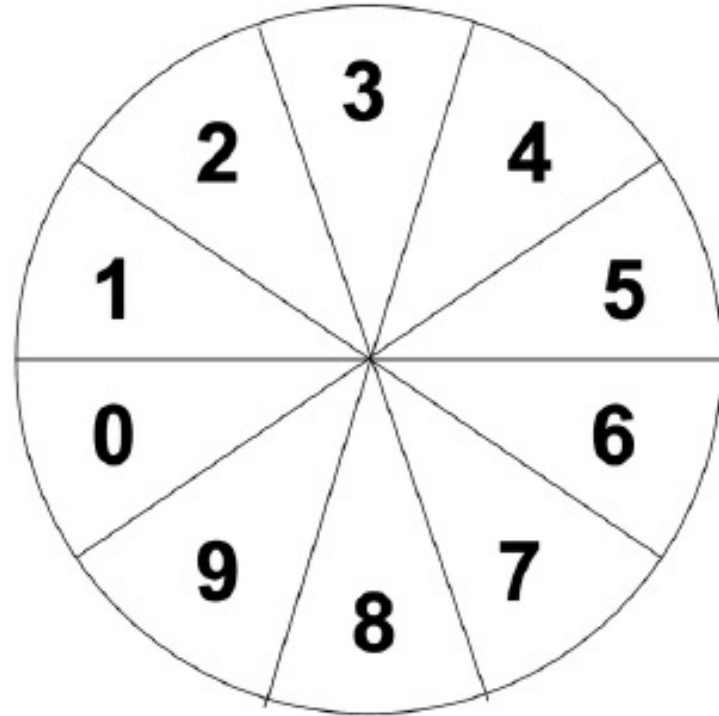


Spinner

2 times 4 equals 8.

6 times 9 equals 54.

7 times 1 equals 7.



Playing Cards

Cards



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{6} - \underline{2} = \underline{4}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

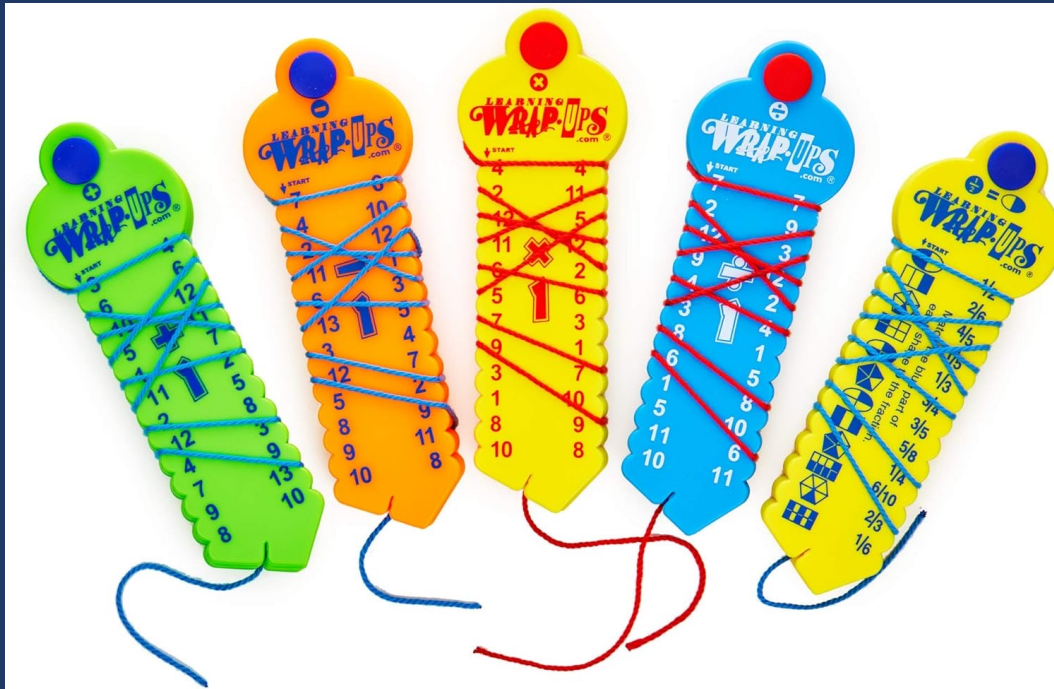
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

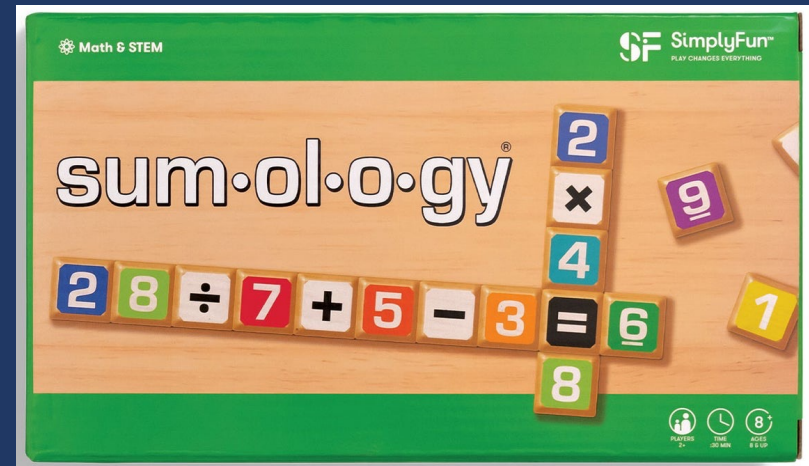
$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$



Wrap-Ups



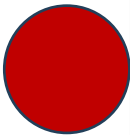
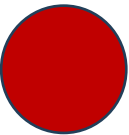
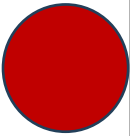
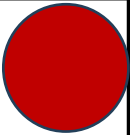
Mobi Math



Bingo

Math Bingo

Your teacher will call out a math problem. Quickly solve the problem. If you have the answer on your bingo card, cover it with a chip. The middle space is free!
The first person to finish the pattern your teacher decides wins!
(5 in a row, 4 corners, etc.)

12		24	100	
15	0	42	16	20
8	35		6	4
	2	40	27	7
50	10	30	48	14

8 times 10 equals...

3 times 1 equals...

2 plus 3 equals...

Magic Squares

Magic Squares Board

- Place the sum or product in the bottom right corner.
- In the bottom row, create a fact with a sum or product of the bottom right corner.
- In the right column, create a fact with a sum or product of the bottom right corner.
- Create two columns with a sum or product of the bottom number.
- Create two rows with a sum or product of the right column number.
- Write the created facts below.

0	2	2
5	4	9
5	6	11

$$0 + 2 = 2$$

$$2 - 0 = 2$$

$$5 + 4 = 9$$

$$9 - 5 = 4$$

4	5	9
2	0	2
6	5	11

7	3	10
1	0	1
8	3	11

6	1	7
3	2	5
9	3	12

4	4	8
2	2	4
6	6	12

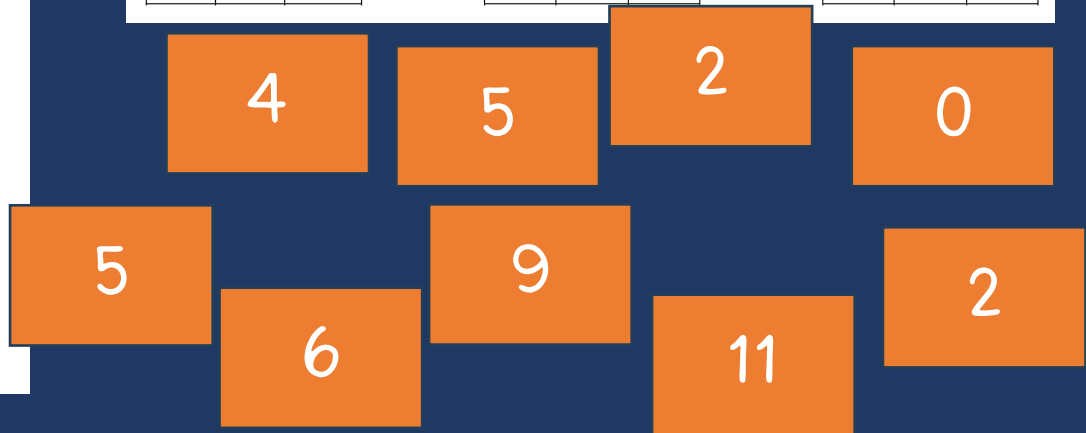
5	1	6
4	3	7
9	4	13

5	1	6
3	4	7
8	5	13

6	3	9
2	3	5
8	6	14

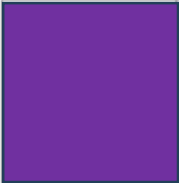
1	5	6
6	2	8
7	7	14

6	2	8
3	4	7
9	6	15



Cover, Copy, Compare

Cover, Copy, Compare

	$\begin{array}{r} 9 \\ \times 6 \\ \hline 54 \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline 48 \end{array}$	
$\begin{array}{r} 7 \\ \times 8 \\ \hline 56 \end{array}$		$\begin{array}{r} 6 \\ \times 5 \\ \hline 30 \end{array}$	
$\begin{array}{r} 9 \\ \times 9 \\ \hline 81 \end{array}$		$\begin{array}{r} 7 \\ \times 9 \\ \hline 63 \end{array}$	
$\begin{array}{r} 6 \\ \times 7 \\ \hline 42 \end{array}$		$\begin{array}{r} 8 \\ \times 5 \\ \hline 40 \end{array}$	
$\begin{array}{r} 8 \\ \times 8 \\ \hline 64 \end{array}$		$\begin{array}{r} 7 \\ \times 7 \\ \hline 49 \end{array}$	

File folder

$6 + 3 = 9$

$1 + 7 = 8$

$6 + 4 = 10$

$7 + 3 =$

$2 + 7 =$

$5 + 6 =$

$4 + 7 =$

$7 + 8 =$

$6 + 7 =$

$7 + 9 =$

$7 + 6 =$

$8 + 7 =$

$7 + 0 =$

$9 + 6 =$

$6 + 0 =$

$6 + 8 =$

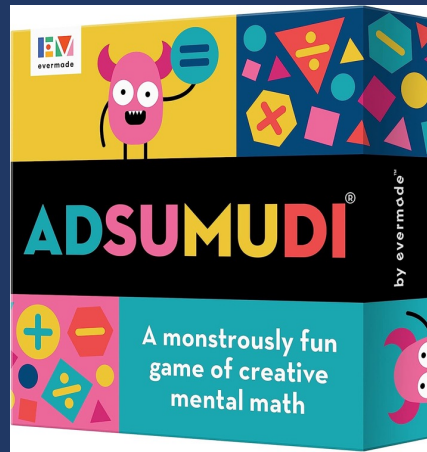
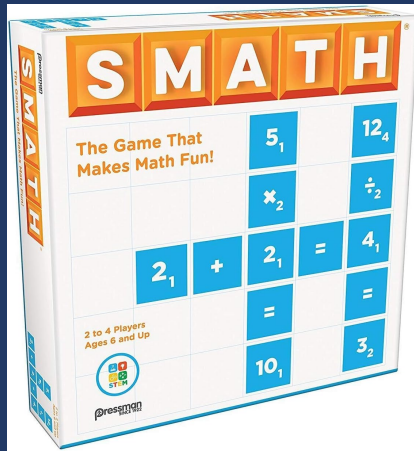


Taped Problems

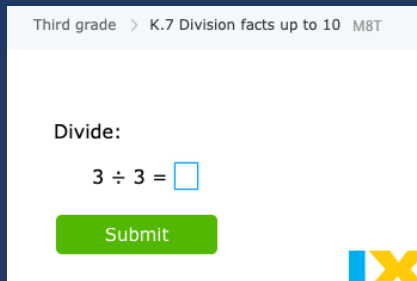
Taped Problems

$\begin{array}{r} 8 \\ \times 8 \\ \hline 64 \end{array}$	$\begin{array}{r} 7 \\ \times 7 \\ \hline 49 \end{array}$	$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 8 \\ \hline \end{array}$
$\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 6 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 6 \\ \hline \end{array}$
$\begin{array}{r} 7 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 5 \\ \hline \end{array}$

Games



Technology



Reflex

Get your free 30-day trial

Help your students attain math fact fluency success whether in-person, remote, or through hybrid learning

Game-based system to improve math fact fluency for grades 2-6 in less than 30 days!

A video player showing a scene from "Crabby's Fact Fair". The scene depicts a fairground with a sign that says "CRABBY'S FACT FAIR". There are colorful lights and a Ferris wheel in the background.

FACT MONSTER

Games / Flashcard

Flashcard

subtraction Level 3 1:51

13
- 6



DAILY

BRIEF

Work on small sets of facts

Work on unknown facts
(in combination with known facts)



FLUENCY

Computational Fluency

Addition

Subtraction

Multiplication

Division



Build fluency with
whole-number computation

Addition

Subtraction

Multiplication

Division

$$\begin{array}{r} 15 \\ + 28 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 1009 \\ - 724 \\ \hline \end{array}$$

$$\begin{array}{r} 7250 \\ \div 15 \\ \hline \end{array}$$

Build fluency with
rational-number computation

Addition

Subtraction

Multiplication

Division

$$\begin{array}{r} 1.4 \\ + 3.9 \\ \hline \end{array}$$

$$\begin{array}{r} \underline{2} \\ 3 \end{array} \times \begin{array}{r} \underline{3} \\ 4 \end{array}$$

$$\begin{array}{r} \underline{9} \\ 4 \end{array} - \begin{array}{r} \underline{3} \\ 8 \end{array}$$

$$\begin{array}{r} 7.892 \\ \div \underline{0.14} \end{array}$$

Build fluency with
integer computation

Addition	Subtraction
Multiplication	Division

$$\begin{array}{r} 1.4 \\ + -3.9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times -12 \\ \hline \end{array}$$

$$-14 - (-7) =$$

$$-135 \div 2 =$$

Partial Sums

A.

$$\begin{array}{r} 74 \\ + 18 \\ \hline 80 \\ + 12 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \\ + 365 \\ \hline 1,000 \\ 80 \\ + 10 \\ \hline 1,090 \end{array}$$



$$\begin{array}{r} 227 \\ + 185 \\ \hline \end{array}$$

Opposite Change

A.

$$\begin{array}{r} 74 \\ + 18 \\ \hline \end{array} \xrightarrow{+4} \begin{array}{r} 70 \\ + 22 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \\ + 365 \\ \hline \end{array} \xrightarrow{-5} \begin{array}{r} 730 \\ + 360 \\ \hline 1,090 \end{array}$$



$$\begin{array}{r} 227 \\ + 185 \\ \hline \end{array}$$

Partial Differences

A.

$$\begin{array}{r} 62 \\ - 17 \\ \hline + 50 \\ - 5 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ - 96 \\ \hline + 300 \\ - 90 \\ - 1 \\ \hline 209 \end{array}$$



$$\begin{array}{r} 232 \\ - 164 \\ \hline \end{array}$$

Same Change

A.

$$\begin{array}{r} 62 \\ - 17 \\ \hline \end{array} \xrightarrow{+3} \begin{array}{r} 65 \\ - 20 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ - 96 \\ \hline \end{array} \xrightarrow{+4} \begin{array}{r} 309 \\ - 100 \\ \hline 209 \end{array}$$



$$\begin{array}{r} 232 \\ - 164 \\ \hline \end{array}$$

Add Up

A.

$$\begin{array}{r} 62 \\ - 17 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ 20 \\ 60 \\ \hline 62 \end{array} \quad \begin{array}{r} 3 \\ 40 \\ 2 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ - 96 \\ \hline \end{array}$$

$$\begin{array}{r} 96 \\ 100 \\ 300 \\ \hline 305 \end{array} \quad \begin{array}{r} 4 \\ 200 \\ 5 \\ \hline 209 \end{array}$$



$$\begin{array}{r} 232 \\ - 164 \\ \hline \end{array}$$

Partial Products

A.

$$\begin{array}{r} 24 \\ \times 43 \\ \hline 800 \\ 160 \\ 60 \\ + 12 \\ \hline 1,032 \end{array}$$

B.

$$\begin{array}{r} 132 \\ \times 53 \\ \hline 5000 \\ 15000 \\ 1000 \\ 3000 \\ 90 \\ + 6 \\ \hline 6996 \end{array}$$



$$\begin{array}{r} 13 \\ \times 47 \\ \hline \end{array}$$

Area (Array)

A.

$$\begin{array}{r} 24 \\ \times 43 \\ \hline 800 \\ 160 \\ 60 \\ + 12 \\ \hline 1,032 \end{array}$$

20	4	
800	160	40
60	12	3

B.

$$\begin{array}{r} 132 \\ \times 53 \\ \hline 5000 \\ 1500 \\ 100 \\ 300 \\ 90 \\ + 6 \\ \hline 6996 \end{array}$$

100	30	2	
5000	1500	100	50
300	90	6	3



$$\begin{array}{r} 13 \\ \times 47 \\ \hline \end{array}$$

Lattice

A.

$$\begin{array}{r} 24 \\ \times 43 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 132 \\ \times 53 \\ \hline \end{array}$$



$$\begin{array}{r} 13 \\ \times 47 \\ \hline \end{array}$$

Partial Quotients

$$\begin{array}{r} \text{A. } 12 \overline{) 158} \\ \underline{-120} \quad 10 \\ 38 \\ \underline{-36} \quad + 3 \\ 2 \quad \underline{+ 3} \\ \quad 13 \text{ R } 2 \end{array}$$

$$\begin{array}{r} \text{B. } 34 \overline{) 1170} \\ \underline{-680} \quad 20 \\ 290 \\ \underline{-170} \quad 5 \\ 120 \\ \underline{102} \quad + 3 \\ 18 \quad \underline{+ 3} \\ \quad 21 \text{ R } 18 \end{array}$$



$$\begin{array}{r} 804 \\ \div \\ \hline 12 \end{array}$$

Lattice

A. $12 \overline{)158} \quad 13 \text{ R } 2$

12 $\begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline \end{array} \begin{array}{|c|} \hline 3 \\ \hline \end{array} \begin{array}{|c|} \hline 8 \\ \hline \end{array}$

B. $34 \overline{)970} \quad 28 \text{ R } 18$

34 $\begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline 9 \\ \hline \end{array} \begin{array}{|c|} \hline 2 \\ \hline \end{array} \begin{array}{|c|} \hline 7 \\ \hline \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array}$



$$\begin{array}{r} 804 \\ \div \quad 12 \\ \hline \end{array}$$

FLUENCY



What are your strengths?



What are your opportunities for growth?

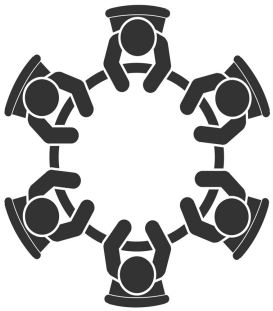


What are your plans for next Monday?
Next month?
Next year?

WORD PROBLEMS

Research and Information





What are your strengths with fluency?

What are your opportunities for growth?

What are your plans for next Monday?

Next month?

Next year?



SCOPE AND SEQUENCE



Where does FLUENCY fit within your scope and sequence? What are your plans for your teachers?



Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.



Word Problems



Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and
Practice

INSTRUCTIONAL STRATEGIES

Fluency

Word Problems



FLUENCY



What are your strengths?



What are your opportunities for growth?



What are your plans for next Monday?
Next month?
Next year?

WORD PROBLEMS

Research and Information





Key words tied to operations is an ineffective word-problem strategy.

(Karp et al., 2019; Powell et al., 2022)

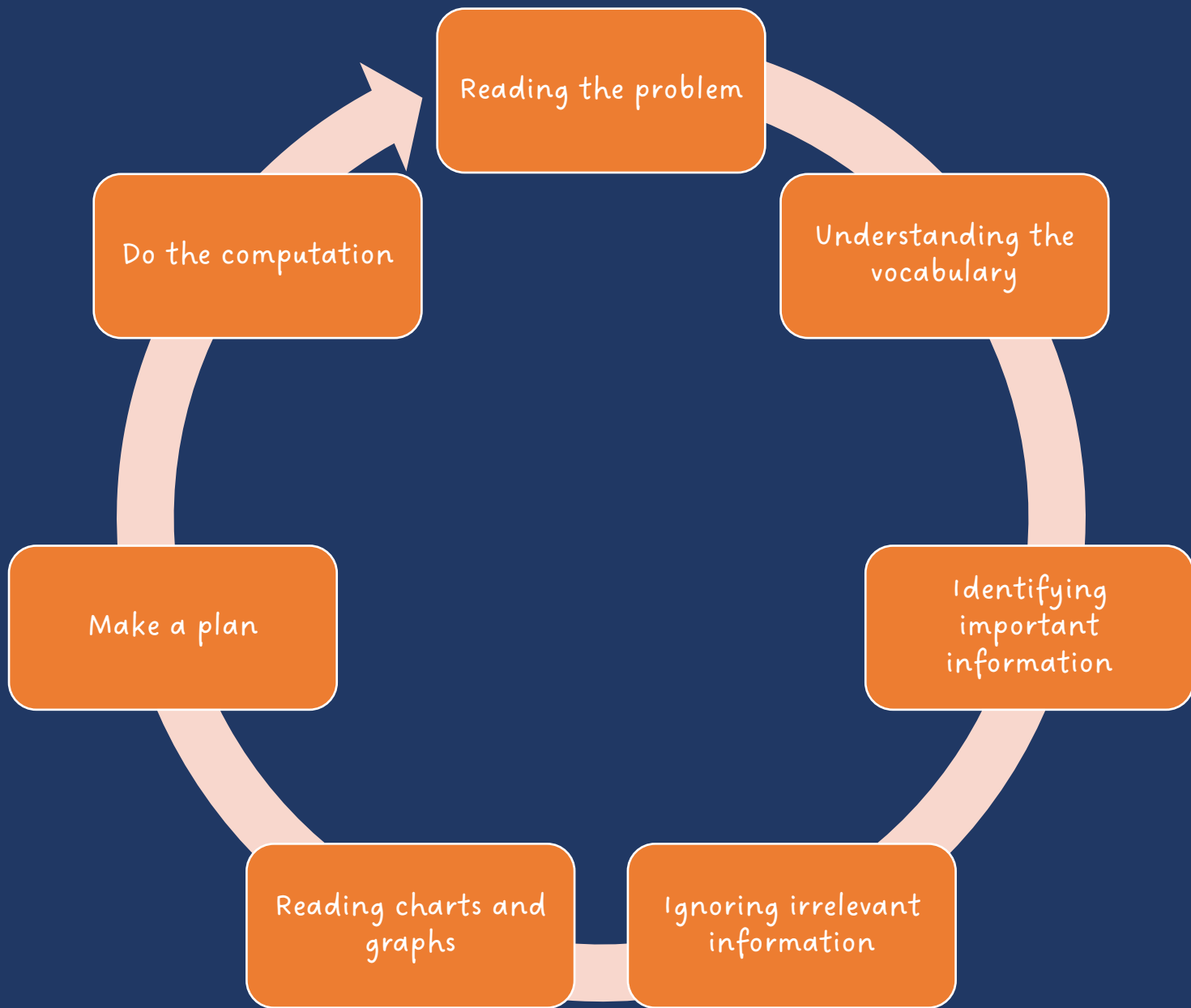
Using a meta-cognitive strategy improves word-problem performance.

(Freeman-Green et al., 2015; Krawec et al., 2012; Montague et al., 2011; Swanson et al., 2014)

A focus on schemas improves word-problem performance.

(Alghamdi et al., 2020; Cook et al., 2020; Flores et al., 2016; Fuchs et al., 2021; Griffin et al., 2019; Jitendra et al., 2013; Lein et al., 2020; Peltier et al., 2020; Powell et al., 2022; Xin & Xhang, 2009; Zheng et al., 2013)





Ineffective Strategies





1. Keywords tied to operations

Carmelita had 8 pencils fewer than Jenny. If Jenny had 18 pencils, how many pencils did Carmelita have?

Carmelita had 8 pencils fewer than Jenny. If Carmelita had 18 pencils, how many pencils did Jenny have?

Key Words Used in Math Word Problems

Addition Words	Subtraction Words
<ul style="list-style-type: none"> add all together or altogether and both combined how many in all how much in all increased by plus sum together total 	<ul style="list-style-type: none"> change decreased by difference fewer or fewer than how many are left (or have left) how many did not have how many how much taller, heavier, less or less lost minus need to reduce remain subtract take away
<ul style="list-style-type: none"> x by (dimension) double each group every factor of increased by multiplied by x of x product x times x triple 	<ul style="list-style-type: none"> Divis as much cut up each group equal share half (or at) how many parts per percent quotient ratio of separated share som

Division

Taking a total and sharing it

(on)

Addition

Putting two or more things/amounts together.

Keywords

Total
Altogether
In all
Sum
More than
Combined

Problem Solving Key Words

Addition	Subtraction
<ul style="list-style-type: none"> add together 	<ul style="list-style-type: none"> are not decrease difference fewer, larger, shorter left less than minus remain take away

key words

combined

addition: sum, both

in all: together, perimeter

total: plus, add

more than

triple

factor

product

multiply

each

per

in all

multiple

area

double

times

average

division: equal groups, half

split

quotient

divide

shared

equally

each

distribute

OPERATION cue words

ADDITION	SUBTRACTION
<ul style="list-style-type: none"> and total join more than in all altogether sum increased 	<ul style="list-style-type: none"> less than decreased remaining left fewer take away difference minus
<ul style="list-style-type: none"> product times each as many as by of equal groups 	<ul style="list-style-type: none"> quotient each broken into per distributed evenly parts

Key Words for All Operations

Addition	Subtraction
<ul style="list-style-type: none"> Sum Total Plus In all And Join Altogether Perimeter Together 	<ul style="list-style-type: none"> Fewer Less Exceed Remain Are not Minus Difference How many more Take away Left over

When they say... They mean...

Math Key Words

Addition	Subtraction	Multiplication	Division
<ul style="list-style-type: none"> added to plus join more than add altogether sum combined 	<ul style="list-style-type: none"> decrease subtract share less 	<ul style="list-style-type: none"> double times each per of product triple 	<ul style="list-style-type: none"> goes into quotient

Math Operation - Key Words

Addition	Subtraction	Multiplication	Division
<ul style="list-style-type: none"> add altogether and both in all sum total increase 	<ul style="list-style-type: none"> difference fewer than gave/take away decreased by how many more show much longer/smaller/shorter minus remaining 	<ul style="list-style-type: none"> area product Each by - of - per Times double, twice, triple total increase 	<ul style="list-style-type: none"> quotient divide into equal parts/share equally per amount of each

KEY WORDS

ADDITION	MULTIPLICATION	SUBTRACTION	DIVISION
<ul style="list-style-type: none"> -sum -total -more than -plus 	<ul style="list-style-type: none"> -product -double -area -times -per -every -each -by 	<ul style="list-style-type: none"> -difference -remain -left -less than -minus -how many more -decrease -give away -reduce -discount -how many more 	<ul style="list-style-type: none"> -quotient -divide by -into -split -out of -shared -per -every -each -evenly -equal groups -half

Math Key Words

Addition	Subtraction	Multiplication	Division
<ul style="list-style-type: none"> plus sum add total all together increase more combine 	<ul style="list-style-type: none"> subtract minus difference left left over decrease take away fewer 	<ul style="list-style-type: none"> times product factor double groups each area rows 	<ul style="list-style-type: none"> quotient split share divide separate each average equal groups



Math Words Poster Set

★★★★★ (4.1)

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Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords ^a		Multiple keywords ^a		Keyword(s) led to correct solution ^a	
	n	%	n	%	n	%	n	%	n	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

^aWhen a problem featured a keyword.





Description of Multi-Step Word Problems (n = 84)

Schema	Occurrence of schema ^a		Any keyword		Keyword(s) led to correct solution ^b	
	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

^aSum across schemas does not equal 100 because each word problem featured more than one schema.

^bWhen a problem featured a keyword.



Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords ("What does *more than* tell you about?")



But, do not tie a keyword to a specific operation!



2. Presenting problems by operation



Name: _____

Date: _____

Addition Word Problems

Solve the word problems. Show your work.

1. Noah had 12 books. He got 5 more books. How many books did Noah have in all?
2. Bonnie found 8 rocks on her front yard and 7 rocks in her backyard. How many rocks did she find in all?
3. Edward had 5 toy cars. He got 7 more toy cars. How many toy cars did Edward have in all?
4. Mariela collected 11 feathers. She found 3 more feathers. How many feathers did she have in all?
5. LaMonte made 14 cookies. He made 12 more cookies. How many cookies did LaMonte have in all?

Division Word Problems

1. Zookeeper Al wants to give 567 bananas to 9 monkeys at the zoo an equal number of bananas to each monkey in the zoo and 567 bananas. How many bananas will each monkey get? And how many are left over?
2. Betty has 427 oranges. She wants to pack them up equally in 23 boxes. How many oranges will go in each box and how much does she have left over?
3. Mr. King has 1376 pages of paper. He wants to give 32 pages to each student. She wants to give 32 pages to each student. How many extra pages will he have left over?
4. Mr. King has 1376 pages of paper. He instead wants to give 32 pages to each student. Will there be enough paper for 40 students. How much more scrap paper does he need?

WORD PROBLEMS

Attack Strategies

SOLVE

- Study the problem
- Organize the information
- Line up a plan
- Verify the plan
- Examine the answer

R-CUBES

- Read the problem
- Circle key numbers
- Underline the question
- Box action words
- Evaluate steps
- Solve and check

UPS Check

- Understand
- Plan
- Solve
- Check



Effective Strategies



Teach an attack strategy

Teach about schemas



RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RICE

Read and record the problem.

Illustrate your thinking.

Compute.

Explain your thinking.

RIDGES

Read the problem.

I know statement.

Draw a picture.

Goal statement.

Equation development.

Solve the equation.



SUPER

Slowly read the story problem twice.

Underline the question and circle the numbers you need.

Picture it. Draw the scenario to show what is happening.

Explain the problem with a number sentence.

Rewrite the answer in a sentence.

SHINES

Slowly and carefully read the problem.

Highlight or underline key information.

Identify the question by drawing a circle around it.

Now solve the problem. Show your work.

Examine your work for precision, accuracy, and clarity.

Share your answer by writing a sentence.

STAR

Stop and read the problem carefully.

Think about your plan and the strategy you will use.

Act. Follow your plan and solve the problem.

Review your answer.



SOLVE

Study the problem.

Organize the information.

Line up the plan.

Verify the plan.

Examine the answer.

R-CUBES

Read the problem.

Circle key numbers.

Underline the question.

Box action words.

Evaluate steps.

Solve and check.

UPS✓

UNDERSTAND

Read and explain.

PLAN

How will you solve the problem?

SOLVE

Set up and do the math!

✓CHECK

Does your answer make sense?

Created by: Sarah Powell (srpowell@austin.utexas.edu)





Share your favorite attack strategy.

Teach an attack strategy

Teach about schemas



Total

Difference

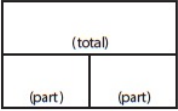
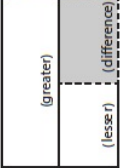
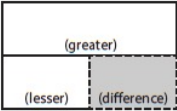

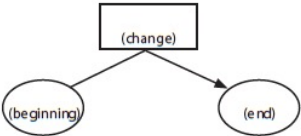
Change

Equal Groups

Comparison

Ratios/Proportions



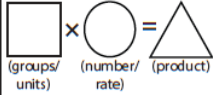
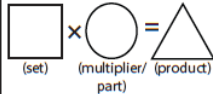
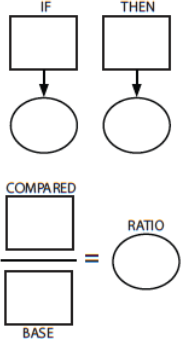
Schema and Definition	Equations and Graphic Organizers	Examples	Variations
Total (Combine; Part-part-whole) Parts combined for a sum	$P1 + P2 = T$ (part + part = total) 	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether? Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?	More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	$B - s = D$ (bigger - smaller = difference)  $G - L = D$ (greater - less = difference) 	Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha? Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write? Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	$ST +/- C = E$ (start +/- change = end)  	End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now? Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting? Start (increase) unknown: Jorge has some money, and then he earned \$16 for babysitting. Now, Jorge has \$68. How much money did he have to start with?	Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?
		End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now? Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark? Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?	



Powell & Fuchs (2018).

Material collected from: Griffin & Jitendra, 2009; Fuchs et al., 2014; Fuchs, Seethaler, et al., 2008; Fuchs et al., 2010; Jitendra, 2002; Kintsch & Greeno, 1985; Van de Walle, Karp, & Bay-Williams, 2013.



Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units	$GR \times N = P$ 	Product unknown: Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	Groups unknown: Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	Number unknown: Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	With rate: Maria bought 5 cartons of eggs. Each carton cost \$2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set	$S \times T = P$ 	Product unknown: Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	Set unknown: Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	Times unknown: Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	With fraction: Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Ratios/Proportions (Percentages; Unit Rate) Relationships among quantities Ratio		Subject unknown: Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	Object unknown: Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?	Ratio unknown: Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	With percentage: Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly? With unit rate: Paula bought 5 boxes of markers. She spent \$9.75. What is the price of one box of markers?

Material collected from: Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra & Star, 2011; Jitendra et al., 2009; Van de Walle et al., 2013; Xin, Jitendra, & Deatline-Buchman, 2005; Xin & Zhang, 2009.



Total

Difference

Change

Additive Schemas



Total

Part-part-whole
Combine

Parts put together into a **total**

Mandy saw **3** canoes and **8** kayaks. How many boats did Mandy see?

Mandy saw **11** boats. If **3** of the boats were canoes, how many were kayaks?

Mandy saw **11** boats. **8** of the boats were kayaks, how many were canoes?

Total

Part

Part



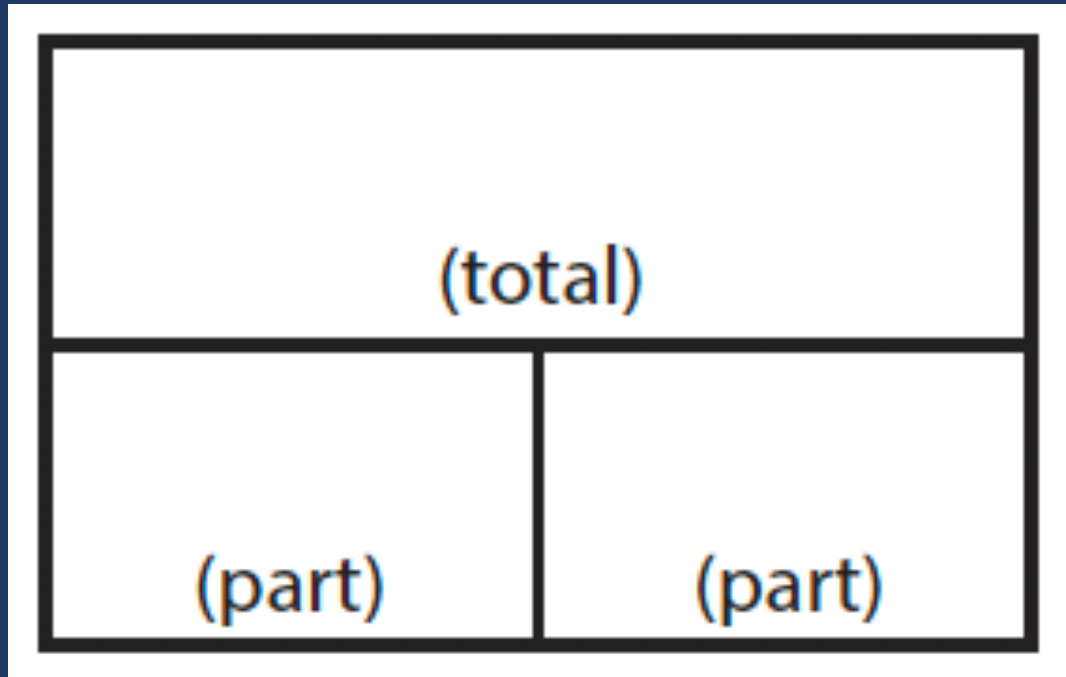
Total

“Are parts put together for a total?”



Total

$$P1 + P2 = T$$



Difference

Compare

Greater and **lesser** amounts compared for a difference

L'Tanya has **10** pencils. Vickie has **4** pencils. How many more pencils does L'Tanya have?

L'Tanya has **6** more pencils than Vickie. If Vickie has **4** pencils, how many does L'Tanya have?

Vickie has **6** fewer pencils than L'Tanya. L'Tanya has **10** pencils. How many pencils does Vickie have?

Difference

Greater
amount

Lesser
amount



Total

“Are parts put together for a total?”

Difference

“Are amounts compared for a difference?”



Difference

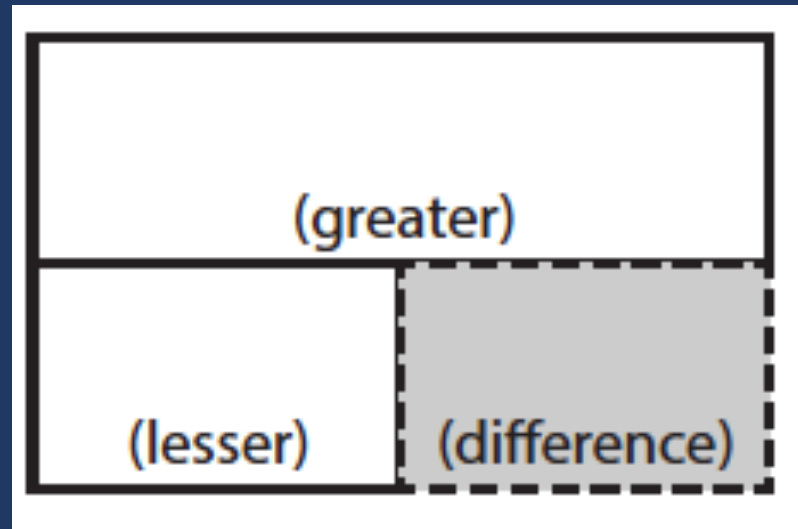
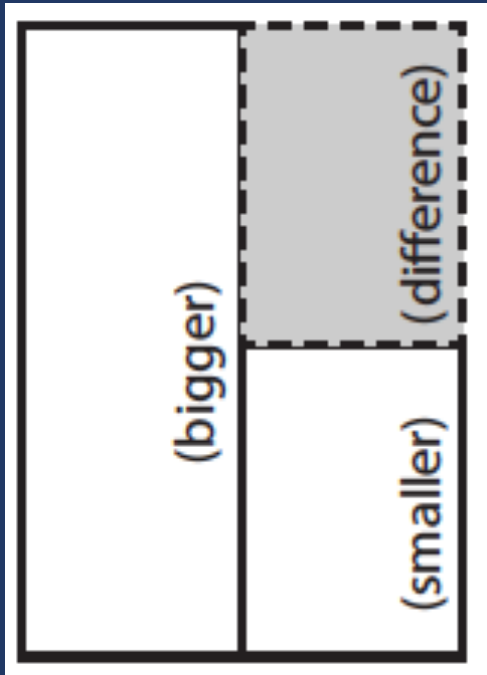
G

-

L

=

D



Change

Join

An amount that increases or decreases

Annette had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Annette have now?

End amount

Annette had 6 notebooks. Then, she bought a few more notebooks. Now, Annette has 9 notebooks. How many notebooks did she buy?

Change amount

Annette had some notebooks. Then, she bought 3 notebooks. Now, Annette has 9 notebooks. How many notebooks did she have to start with?

Start amount



Change

Separate

An amount that increases or decreases

Jenny baked 9 cookies. Then, she ate 3 of the cookies.
How many cookies does Jenny have now?

End amount

Jenny baked 9 cookies. Then, she ate some of the cookies.
Now, she has 6 cookies. How many cookies did Jenny eat?

Change
amount

Jenny baked some cookies. She ate 3 of the cookies and
has 6 cookies left. How many cookies did Jenny bake?

Start amount



Total

“Are parts put together for a total?”

Difference

“Are amounts compared for a difference?”

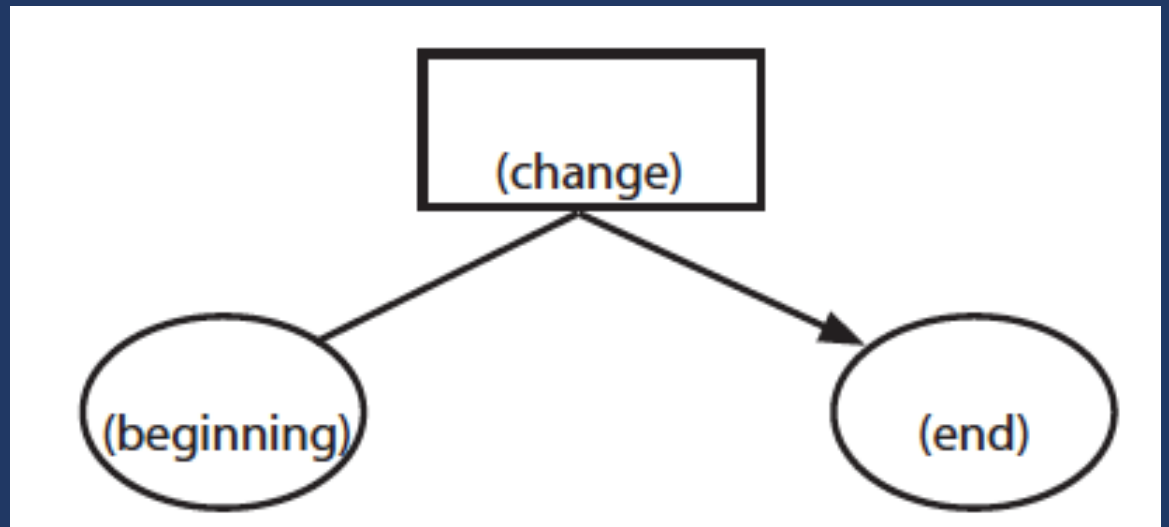
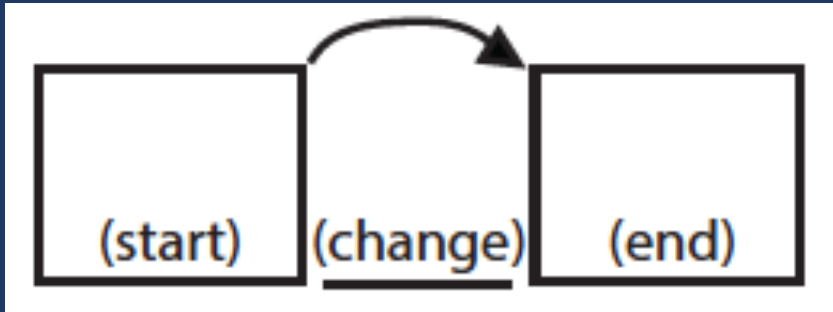
Change

“Does an amount increase or decrease?”



Change

$$ST + / - C = E$$



Total

Difference

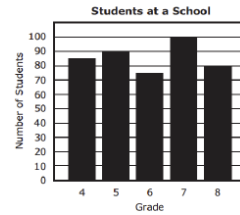
Change

WORD PROBLEMS

Additive Word Problems

A.

The bar graph shows the number of students in each grade at a school. How many more students are in grade 7 than are in grade 4?

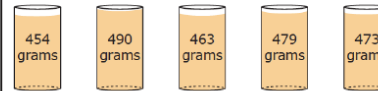


B.

A package of bread has a mass of 623 grams. One slice of bread is removed from the package. The slice of bread has a mass of 55 grams. What is the mass, in grams, of the package of bread after the slice of bread is removed?

C.

A student measures the mass of 2 jars of sand. The total mass of the 2 jars of sand is 963 grams. Which two jars of sand have a total mass of 963 grams?



D.

A photographer has a picture album that holds 100 pictures. The photographer fills $\frac{57}{100}$ of the album with pictures of trees. She fills $\frac{30}{100}$ of the album with pictures of animals. What fraction of the album is filled with either pictures of trees or animals?



Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions



Multiplicative Schemas

Equal Groups

Comparison

Ratios/Proportions



Equal Groups

Array
Vary

Groups multiplied by **number in each group**
for a **product**

Evan has **2** boxes. There are **6** muffins in each box. How many muffins does Evan have?

Product

Evan has **12** muffins. They want to place them equally into **2** boxes. How many muffins will Evan place in each box?

Number in
each group

Evan has **12** muffins. They put them into boxes with **6** muffins each. How many boxes did Evan use?

Groups



Equal Groups

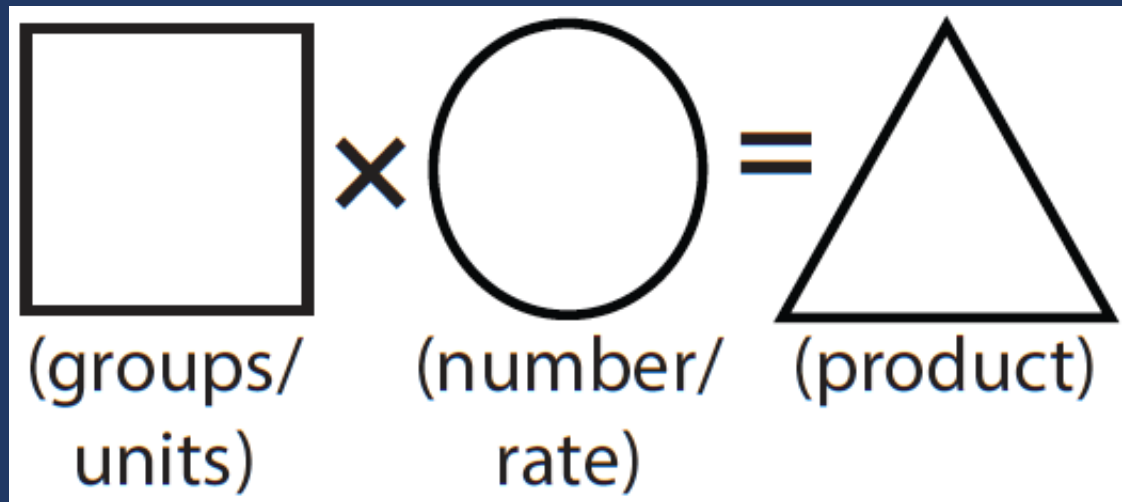
“Are there groups with an equal number in each group?”



Equal Groups

$$GR \times N = P$$

$$GR \times E = P$$



Comparison

Set multiplied by a number of times for a product

Joan ran 6 minutes. L'Tanya ran 4 times longer than Joan. How many minutes did L'Tanya run?

Set

Number of
times

Product



Equal Groups

“Are there groups with an equal number in each group?”

Comparison

“Is a set compared a number of times?”



Comparison

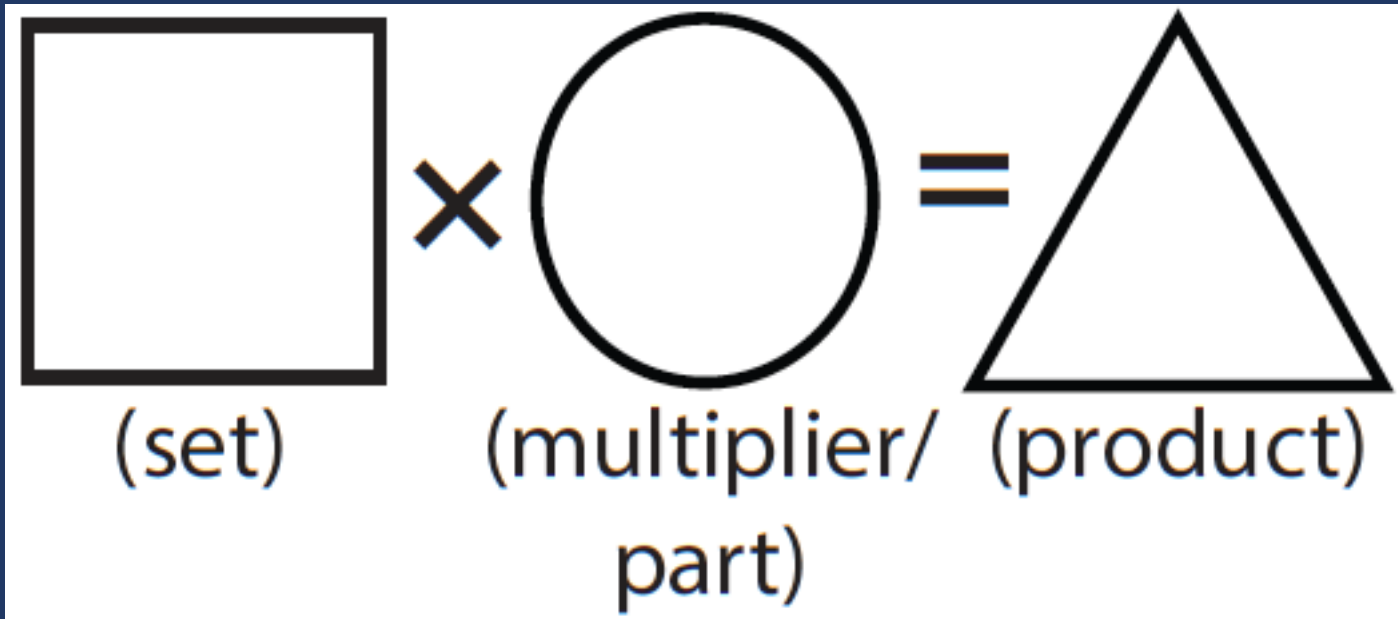
S

X

T

=

P



Ratio/Proportion

Description of **relationships** among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was **3:5**. If she baked **25** brownies, how many cookies did she bake?

Emma typed **56** words in **2** minutes. At this rate, how many words could Emma type in **7** minutes?



Equal Groups

“Are there groups with an equal number in each group?”

Comparison

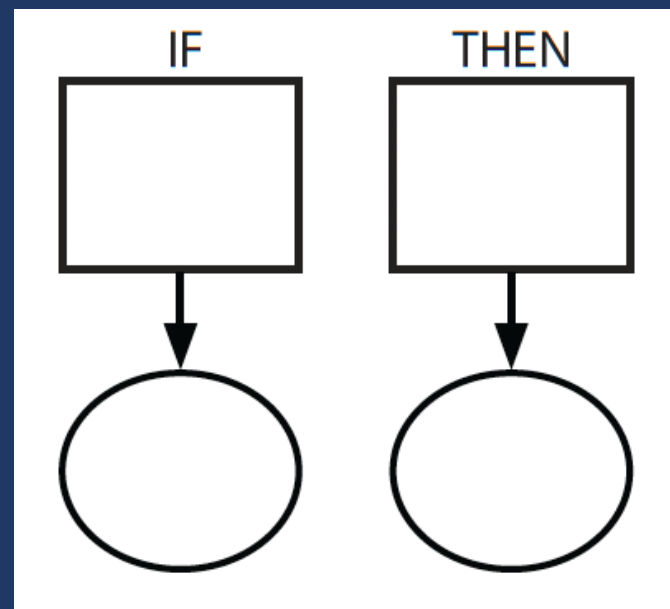
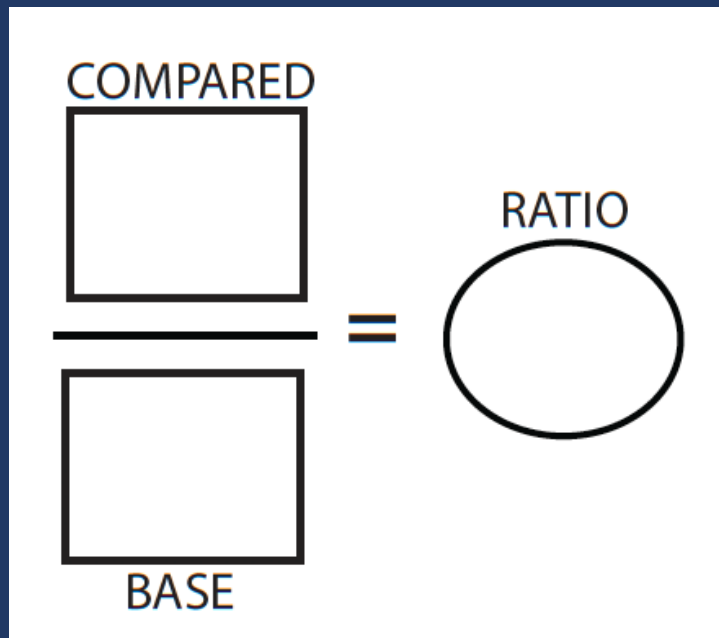
“Is a set compared a number of times?”

Ratio/Proportion

“Are there relationships among quantities - if this, then this?”



Ratio/Proportion



WORD PROBLEMS

Multiplicative Word Problems

E.
There are 8 people. They each have 4 oranges.
Which expression shows how many oranges the
people have altogether?

- A. $8 + 4$
- B. $8 - 4$
- C. 8×4
- D. $8 \div 4$

F.
A farmer plants 4 rows of trees. Each row has an
equal number of trees. There are 1,580 trees on the
farm. What is the number of trees in each row?

G.
A store has two lamps for sale. Lamp A costs \$9.
Lamp B costs 6 times more than Lamp A. How much
is Lamp B ?

H.
There are 3,726 students spending the summer at
a camp. The students are divided equally into 9
groups. How many students are in each group?



Equal Groups

Comparison

Ratio/
Proportion



Total

Difference

Change

Equal Groups

Comparison

Ratio/
Proportion



Identify 1 of
each schema.

WORD PROBLEMS

Total

Difference

Change

Equal Groups

Comparison

Ratios and Proportions



Multi-Step Problems

WORD PROBLEMS

Multi-Step Word Problems

i.
A frog wants to reach a pond that is 10 feet away. The frog hops 5 times. Each hop is 18 inches. How many more inches does the frog need to travel to reach the pond?

j.
Three people play a video game.
• Person A scores 3,793 points.
• Person B scores 4,286 points.
• Person C scores 5,941 points.
How many more points do Person A and Person C have together than Person B?

k.
A store has a parking lot. There are 6 rows of parking spaces in the parking lot. There are 8 parking spaces in each row. There are 19 cars parked in the parking lot. How many parking spaces in the parking lot are empty?

l.
An egg farm packages 264 total cartons of eggs each month. The farm has 3 different sizes of cartons.
• The small carton hold 8 eggs, and $\frac{1}{6}$ of the total cartons are small.
• The medium carton holds 12 eggs, and $\frac{2}{3}$ of the total cartons are medium.
• The large carton holds 18 eggs, and the rest of the total cartons are large.
Determine how many each size of carton is needed each month. Then determine how many eggs are needed to fill the 264 cartons.



Total

Difference

Change

Equal Groups

Comparison

Ratio/
Proportion



Find or write
your own.
Have a
partner solve.

WORD PROBLEMS

Multi-Step Word Problems

M.

N.



Directive Problems

UPS✓

UNDERSTAND

Read and explain.

PLAN

How will you solve the problem?

SOLVE

Set up and do the math!

✓CHECK

Does your answer make sense?

Created by Sarah Powell (spowell@gausim.utexas.edu)

WORD PROBLEMS

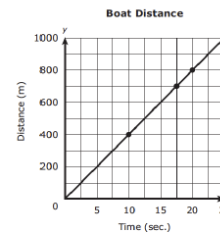
Directive Word Problems

O.
What fraction, when added to $\frac{3}{8}$, would make one whole?

P.
Three shapes are listed in the table. Place a check mark to show what is true for each shape. Select one or more than one box per row.

Shape	Is a Quadrilateral	Has More Than 5 Sides
rectangle	<input type="checkbox"/>	<input type="checkbox"/>
hexagon	<input type="checkbox"/>	<input type="checkbox"/>
square	<input type="checkbox"/>	<input type="checkbox"/>

Q.
Which coordinate pairs represent points on the graph? Select the two coordinate pairs.



R.
A fish tank is in the shape of a right rectangular prism. The fish tank has a length of 6 feet, a width of 2 feet, and a height of 3 feet. What is the volume, in cubic feet, of the fish tank?



WORD PROBLEMS



What are your strengths?



What are your opportunities for growth?



What are your plans for next Monday?

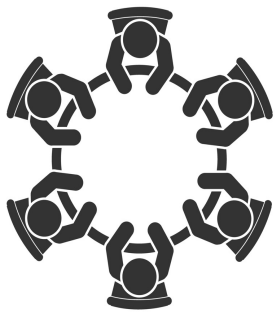
Next month?

Next year?

INSTRUCTIONAL PLATFORM

Instructional Platform





What are your strengths with problem solving?

What are your opportunities for growth?

What are your plans for next Monday?

Next month?

Next year?



Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.



Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and
Practice

INSTRUCTIONAL STRATEGIES

Fluency

Word Problems



WORD PROBLEMS



What are your strengths?



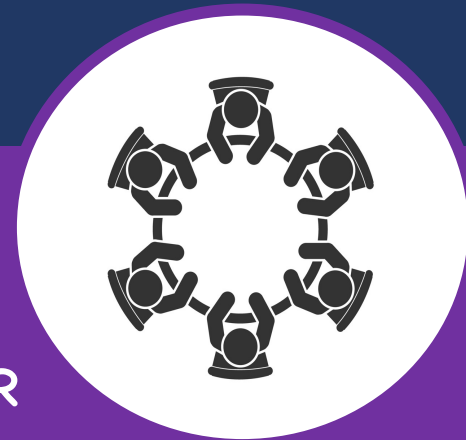
What are your opportunities for growth?



What are your plans for next Monday?
Next month?
Next year?

INSTRUCTIONAL PLATFORM

Instructional Platform



What's YOUR
instructional platform?
What are your plans for
your teachers?



SCOPE AND SEQUENCE



Where does WORD-
PROBLEM SOLVING fit
within your scope and
sequence? What are your
plans for your teachers?



Resources





Pirate Math Equation Quest

About

Research

Individual

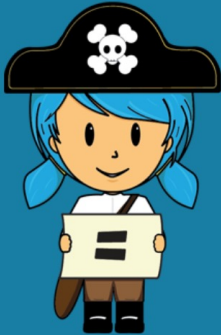
Small Group

STAAR

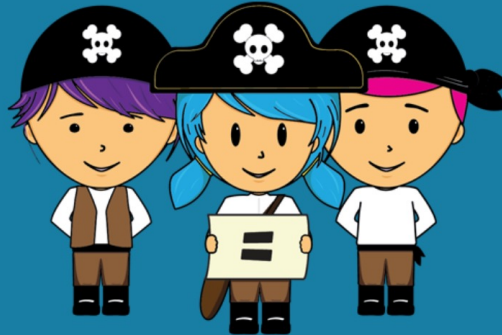
Videos

Welcome to Pirate Math Equation Quest!

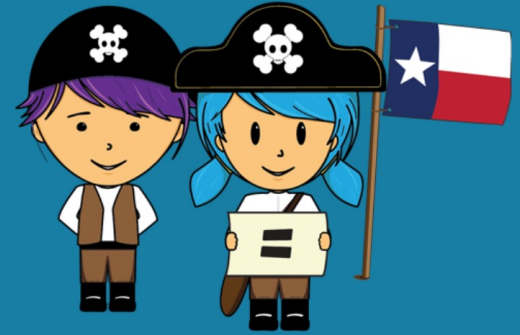
Individual Word-Problem Intervention



Small-Group Word-Problem Intervention



Small-Group Word-Problem Intervention for STAAR



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<https://intensiveintervention.org>

National Center on
INTENSIVE INTERVENTION

at American Institutes for Research

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Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

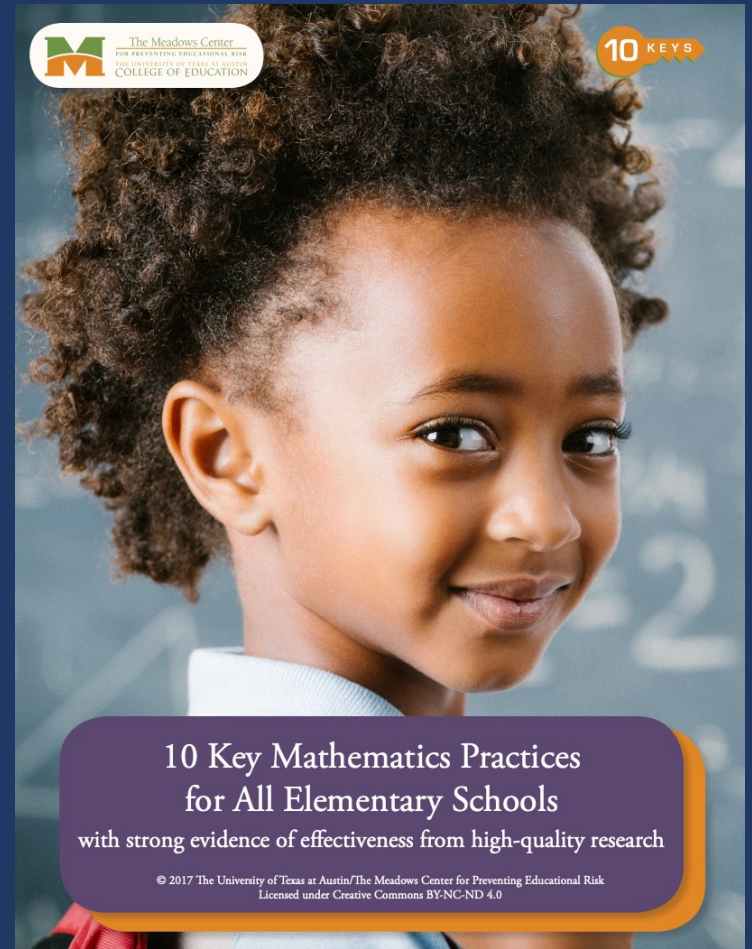
Intensive instruction was recently identified as a [high-leverage practice in special education](#), and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the [National Center on Leadership in Intensive Intervention](#) and with support from the [CEEDAR Center](#), developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the [Features of Explicit Instruction Course](#) and so we suggest that users complete both courses.



**MODULE 5: INTENSIVE
MATHEMATICS INTERVENTION:
INSTRUCTIONAL STRATEGIES**





<https://meadowscenter.org/resource/10-key-mathematics-practices-for-all-elementary-schools-with-strong-evidence-of-effectiveness-from-high-quality-research/>





10 KEY MATH PRACTICES for All Middle and High Schools

with strong evidence of effectiveness from high-quality research

All middle and high school students can become proficient in mathematics if:

1. Teachers help students to solve mathematics problems by using manipulatives and tools to bridge concrete to abstract and symbolic understandings of mathematics.
2. Students are asked to make their mathematics thinking transparent by talking about their solution process, drawing a picture, or making a graph and using mathematically correct language (for example, using the terms "numerator" and "denominator" rather than "top number" and "bottom number" for fractions, telling how many groups of a divisor are in the dividend for division rather than saying 5 "goes into" 20 four times, or using the term "zero pairs" rather than "canceling out").
3. Students are asked to read and critique one another's written responses to problems.
4. Teachers present "real-life" word problems for students to solve daily.
5. Students are expected to solve multiplication and division facts regularly as a basis for working on rational numbers and algebraic problems.
6. Students are expected to master the properties of operations (order of operations; commutative, associative, and distributive properties; multiplicative identity property; multiplicative inverse property).
7. Students are given solved problems (correctly solved and incorrectly solved using common misconceptions) to analyze and discuss how the problems were solved and where the solution strategy broke down for incorrectly solved problems.
8. Teachers differentiate mathematics instruction for diverse learners (for example, struggling learners, English language learners, gifted students, and average achievers).
9. Teachers verbalize (think aloud, describe steps for a strategy) explanations of concepts and steps for solving problems.
10. Teachers collect data regularly to determine whether their students are benefiting from instruction and use the data to make informed instructional decisions for subsequent lessons.

10 KEYS

The Meadows Center
FOR PREVENTING EDUCATIONAL RISK
THE UNIVERSITY OF TEXAS AT AUSTIN
COLLEGE OF EDUCATION

—www.meadowscenter.org—

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<https://meadowscenter.org/resource/10-key-math-practices-for-all-middle-and-high-schools-with-strong-evidence-of-effectiveness-from-high-quality-research/>





Module 9: Subtraction of Rational Numbers


Problem Sets

- A. [Proper fractions with like denominators and sums <1 \(20\)](#)
- B. [Improper fractions with like denominators and sums >1 \(10\)](#)
- C. [Mixed numbers with like denominators and sums >1 \(10\)](#)
- D. [Proper fractions with unlike denominator and sums <1 \(20\)](#)
- E. [Improper fractions with unlike denominator and sums >1 \(10\)](#)
- F. [Mixed numbers with unlike denominator and sums >1 \(10\)](#)
- G. [Decimals with tenths; no regrouping \(20\)](#)
- H. [Decimals with tenths; regrouping \(20\)](#)
- I. [Decimals with hundredths; no regrouping \(20\)](#)
- J. [Decimals with hundredths; regrouping \(20\)](#)
- K. [Decimals with tenths and hundredths; mix of regrouping \(20\)](#)



Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.



TEA
Texas Education Agency

numerator

The term in a fraction that tells how many parts of a fraction.

$2/3$ $\frac{2}{3}$ In these fractions, **2** is the numerator.

ones

The digit representing **1**.

In the number **4.23**, **4** is in the ones place.

<https://spedsupportstage.tea.texas.gov/resource-library/instructional-routines-mathematics-intervention>





Myths That Undermine Maths Teaching

Sarah R. Powell, Elizabeth M. Hughes, and Corey Peltier



Analysis Paper 38
August 2022



Susan
McKinnon
Foundation

CIS Education
Program

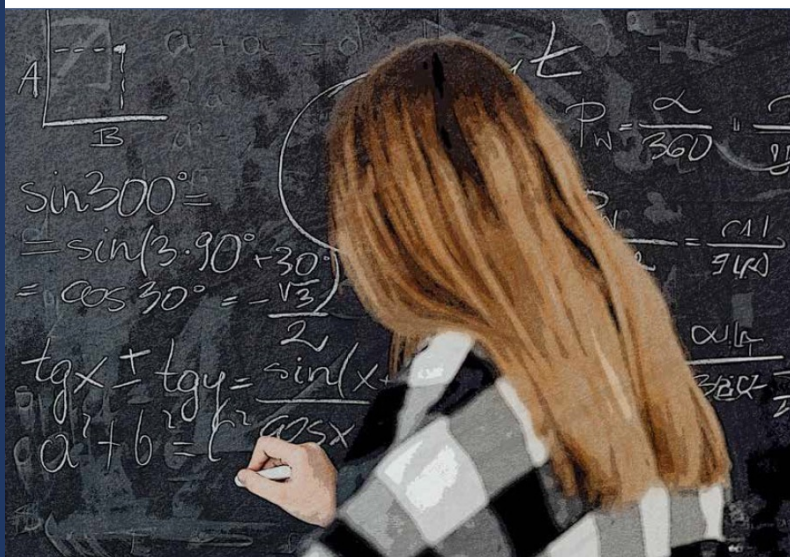
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MATHS PRACTICES YOU CAN COUNT ON

A Guide to Five Research-Validated
Practices in Mathematics

Sarah R. Powell, Sarah G. King, and Sarah A. Benz



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IES Institute of Education Sciences

What Works Clearinghouse™

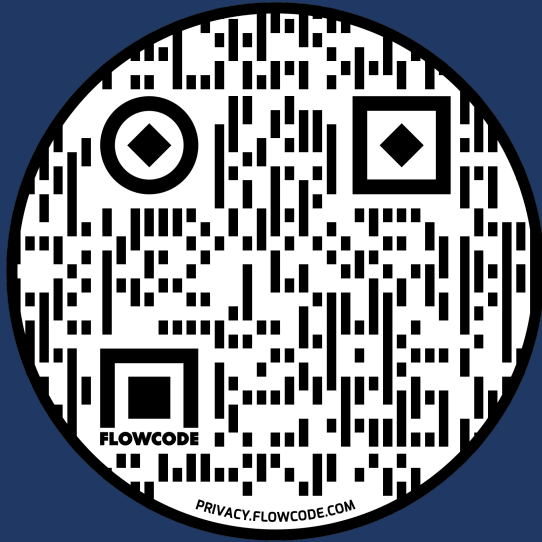
Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades

Educator's Practice Guide

WWC 2021006
U.S. DEPARTMENT OF EDUCATION

A publication of the National Center for Education Evaluation and Regional Assistance (NCEE) at IES





bit.ly/srpowell

Virtual Manipulatives

Help students see and learn math using different tools!

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- Number & Operations
- Place Value
- Fractions & Decimals
- Integers & Algebra
- Geometry
- Time & Money
- Data & Probability
- Extras

<h2>Fractions & Decimals</h2>	<p>fraction strips</p>	<p>fraction strips</p>	<p>fraction strips</p>	<p>Cuisenaire rods</p>	
	<p>fraction circles</p>	<p>geoboard</p>	<p>geoboard</p>	<p>geoboard</p>	<p>pattern blocks</p>
	<p>two-color counters</p>	<p>decimal strips</p>	<p>place value disks</p>	<p>percentage strips</p>	



Resources



Instructional Platform



Explicit Instruction



Language



Multiple Representations



Fluency



Problem Solving



Additional Resources



Coaching



<https://www.mathspiral.com>



<p>STAIR Tailored: Culturally Responsive Teaching Part 1: What is it?</p> <p>Culturally Responsive Teaching</p> <p>View full playlist</p>	<p>STAIR Tailored: Multiplying Linear Expressions – Part I: Using Algebra Blocks</p> <p>Teaching Quadratic Expressions</p> <p>View full playlist</p>	<p>STAIR Tailored: Introduction to the Coordinate Plane Using CPA and Measurement</p> <p>Coordinate Plane/Grids</p> <p>View full playlist</p>	<p>STAIR Tailored Practice to Research... and Back to Practice: Concrete Learning</p> <p>Practice to Research & Back to Practice</p> <p>View full playlist</p>
<p>STAIR Tailored: Exploring Properties of Quadrilaterals Using the CPA Framework and AngLegs</p> <p>Introduction to Geometry</p> <p>View full playlist</p>	<p>STAIR Tailored: Defining Data-Based Individualization (DBI)</p> <p>Data-Based Individualization</p> <p>View full playlist</p>	<p>Project STAIR: One-Step Equations with an Addition Operator using Cups and Counters</p> <p>Introduction to Equations</p> <p>View full playlist</p>	<p>Project STAIR: Adding Integers Using a Positive and Negative Mat</p> <p>Integers</p> <p>View full playlist</p>
<p>Project STAIR: Representing Fractions with the Area Model</p> <p>Fraction Fundamentals</p> <p>View full playlist</p>	<p>STAIR Tailored: Do Not Use Key Words</p> <p>Word Problem Instruction</p> <p>View full playlist</p>	<p>Project STAIR: Whole-Number Computation: Addition with Partial Sums</p> <p>Whole Number Computation</p> <p>View full playlist</p>	<p>Project STAIR: Explicit Instruction</p> <p>Best Practices for Math Teachers</p> <p>View full playlist</p>

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Elementary Math Leads Survey

March 11, 2024



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