Math 2371 Calc III
Sample Test 3

1.(i) Is the following vector field conservative?

$$\vec{F} =< yz + 3, xz + 4y, xy + 3z^2 > .$$

If so, find the potential ϕ. Use this to evaluate

$$\int_c (yz + 3)dx + (xz + 4y)dy + (xy + 3z^2)dz$$

where c is any path from $(0, 0, 0)$ to $(1, 2, 3)$.

1. (ii) Is the following vector field conservative?

$$\vec{F} =< 2xy, x^2 + z^2, 2yz > .$$

If so, find the potential ϕ. Use this to evaluate

$$\int_c 2xydx + (x^2 + z^2)dy + 2yzdz$$

where c is any path from $(0, 0, 0)$ to $(1, 2, 3)$.

2. Evaluate the following line integral $\int_c xy\, ds$ where c is counterclockwise direction around a circle of radius 1 from $(1, 0)$ to $(0, 1)$.

3. Green's Theorem is

$$\int_C P\, dx + Q\, dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA.$$

Verify Green's Theorem where $\vec{F} =< y^2, x^2 + 2xy >$ and R is the region bound by the curves $y = x^2$, $y = 1$ and $x = 0$ in $Q1$.

4. Evaluate $\iint_S z\, dS$ where S is the surface of the paraboloid $z = 1 - x^2 - y^2, z \geq 0$.

5. Find the flux $\iint_S \vec{F} \cdot \hat{n} dS$ of the vector field $\vec{F} =< 2x, 2y, 2z + 2 >$ through the surface of the plane $x + y + z = 1$ in the first quadrant.