

Performance Differences on Mathematics Vocabulary for English Learners and Non- English Learners

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Introductions

What is your name/job title?

What grade/subjects do you teach?

How many years have you been teaching math?

How do you teach mathematics vocabulary?

Which math vocabulary words are difficult for your students to understand?

Why is math vocabulary difficult for students?

Math Vocabulary Difficulties

Shared meaning in math and English

Different meanings in math and English

Multiple meanings in mathematics

Shared meaning with another content area

Homonyms with other terms

Different from terms in other languages

Math Vocabulary Difficulties

Longest

Plane

Quarter

Volume

Whole/hole

Cuarto (quarter)

Language of Mathematics

numerator

denominator

least common multiple

Fraction

mixed number

proper fraction

simplest form

multiple

decimal

improper fraction

degree

Mathematics Vocabulary

COMMON CORE STATE STANDARDS FOR

Mathematics



4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Mathematics Vocabulary

Kevin makes muffins.

- It takes 8 minutes to mix the batter.
- The muffins bake for 17 minutes.
- The muffins then cool for 5 minutes.

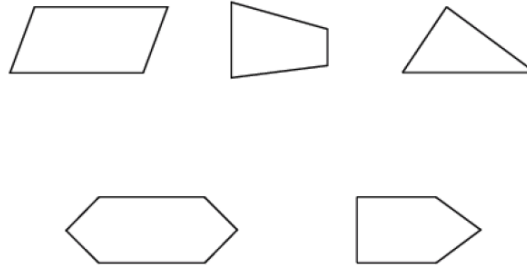
What is the total amount of time, in minutes, Kevin spends baking, and cooling the muffins?

Select the expression equivalent to $(4x + 3) + (-2x + 4)$.

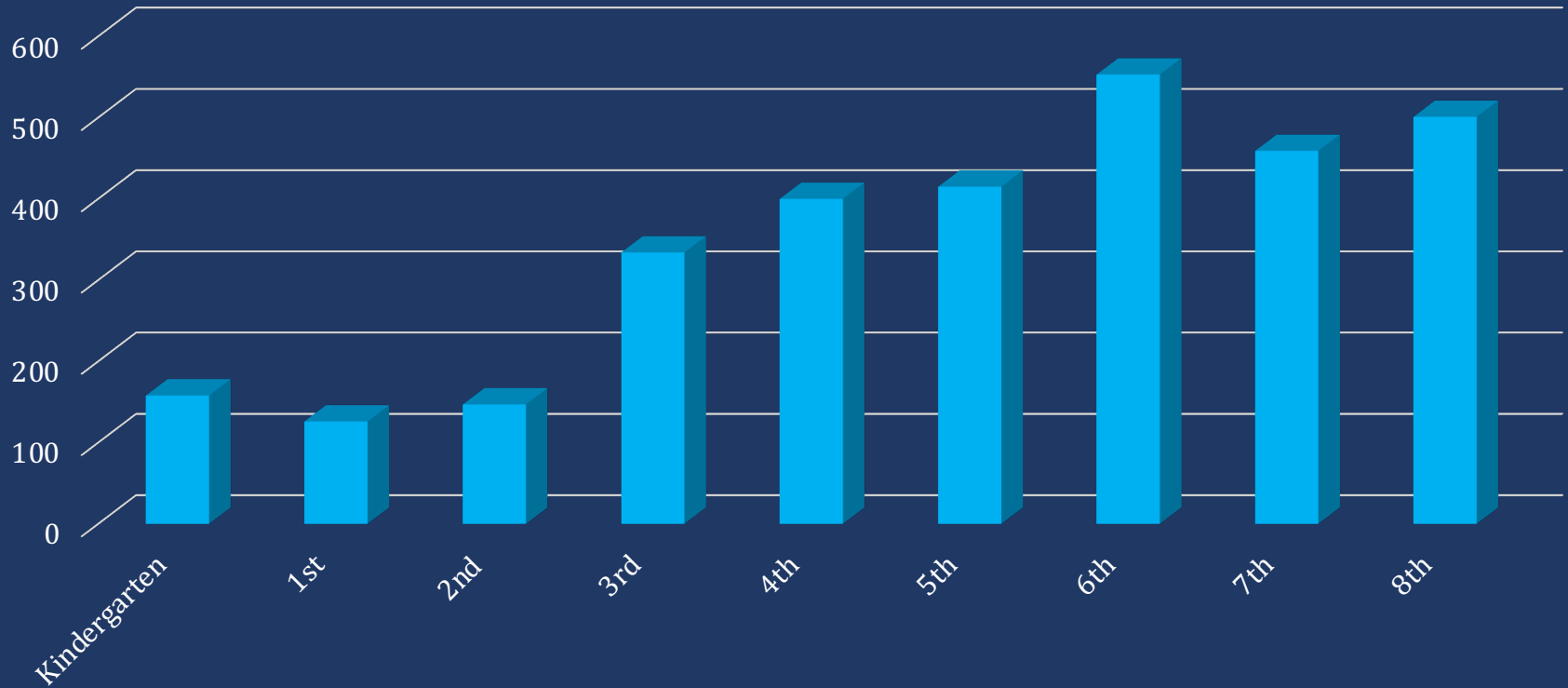
- (A) $-2x + 12$
- (B) $-8x + 12$
- (C) $6x + 7$
- (D) $2x + 7$

2003

Click all of the shapes that are quadrilaterals.



Vocabulary Across Grades



Purpose

ELs

vs.

Non-ELs

No
difficulty

vs.

Equation
difficulty

vs.

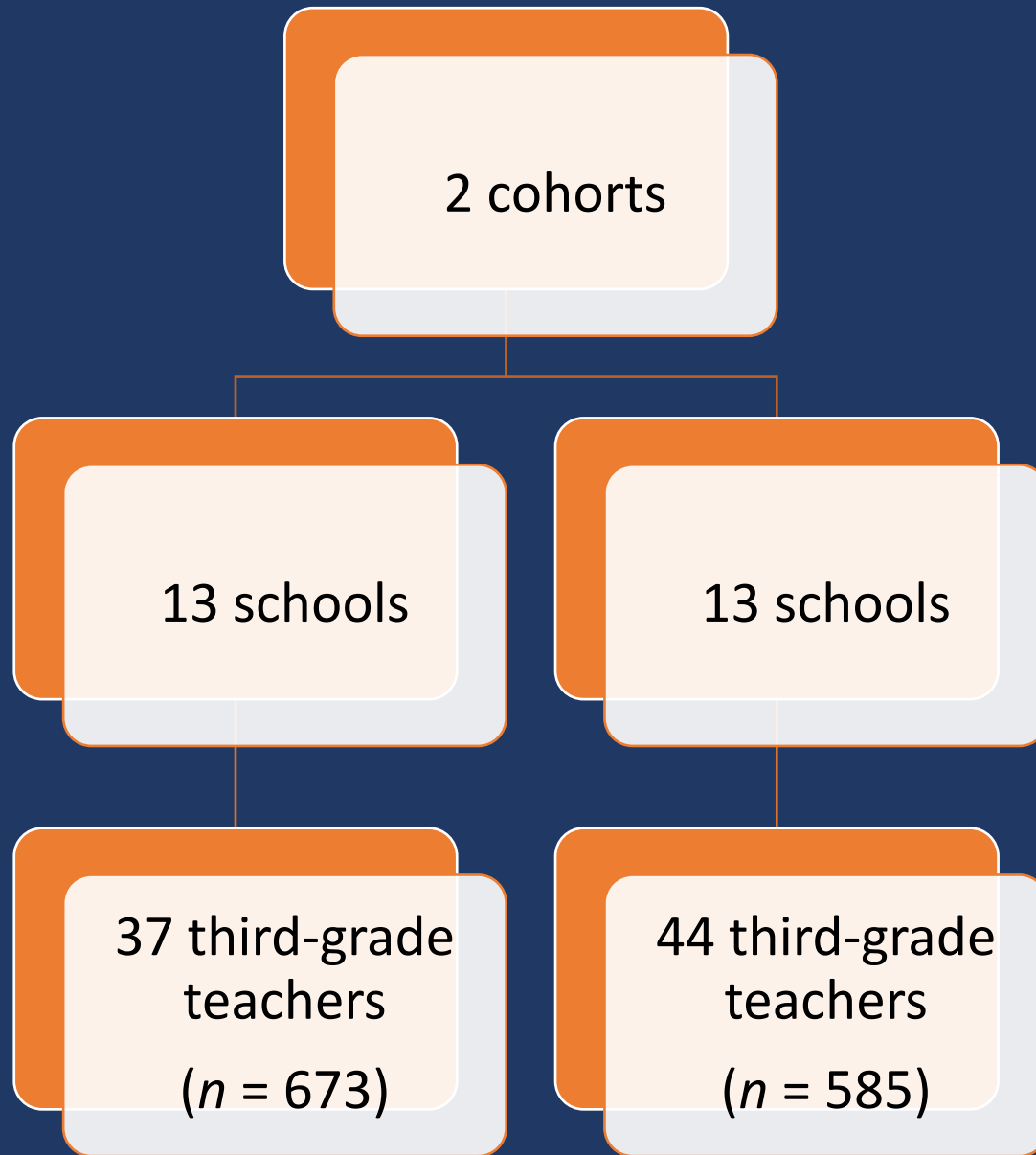
Word-
problem
difficulty

vs.

Word-
problem +
Equation
difficulty

Most difficult vocabulary terms

Sample



1. Identify English learners

English learners

Identified by classroom teacher as:

- English learner (EL)
- Non-English learner (non-EL)




2. Identify mathematics difficulty

Equation-Solving Difficulty

Open Equations (Powell, 2007)

Equations difficulty < 27th percentile; $\alpha = .93$

Open Equations



1. $___ + 3 = 7$

2. $2 = 7 - ___$

3. $___ = 4$

4. $6 = 2 + ___$

5. $___ - 4 = 3$

6. $3 + 5 = 4 + ___$

7. $___ = 7 - 4$

8. $___ - 6 = 2$

9. $9 = ___ + 4$


10. $8 - 6 = ___ - 3$

11. $___ - 3 = 8 - 2$


12. $5 = ___ + 3$

13. $5 = 9 - ___$

14. $3 + ___ = 8$



Open Equations



15. $5 + 4 = ___ + 2$

16. $9 - ___ = 6$

17. $7 + 2 = ___$

18. $___ + 4 = 5 + 2$

19. $7 = ___ - 2$

20. $7 - ___ = 5$

21. $5 + ___ = 9$

22. $3 + ___ = 2 + 7$

23. $6 = ___ - 2$

24. $9 - 6 = 7 - ___$

25. $___ + 6 = 9$


26. $7 = ___$

27. $___ = 2 + 6$

28. $8 - 3 = ___$

29. $6 - ___ = 7 - 3$


30. $7 = 4 + ___$



Word-Problem Difficulty

Single-Digit Word Problems (Jordan & Hanich, 2000)

Word-problem difficulty <28th percentile; $\alpha = .89$

FIRST: _____ LAST: _____ 

PT TWP OE

____ 1. Alex has 8 pennies. Kris has 6 pennies. How many pennies does Alex need to give away to have as many as Kris?

____ 2. Sue had 5 pennies. Then Mike gave her 2 more pennies. How many pennies does Sue have now?


____ 3. Chelsea has 6 pennies. Max has 4 pennies. How many pennies does Max have less than Chelsea?

____ 4. Nina had 9 pennies. Then she gave 3 pennies to Anthony. How many pennies does Nina have now?

____ 5. Janet has 3 pennies. Andy has 5 more pennies than Janet. How many pennies does Andy have?

____ 6. Carol had 4 pennies. Then Nick gave her some more pennies. Now Carol has 6 pennies. How many pennies did Nick give her?

____ 7. Claire has 4 pennies. Ben has 9 pennies. How many more pennies does Claire need to have as many as Ben?



____ 8. Jen had 7 pennies. Then she gave some pennies to Joe. Now Jen has 2 pennies. How many pennies did she give to Joe?

____ 9. Emily has 3 pennies. John has 6 pennies. How many pennies do they have altogether?

____ 10. Maria and Kevin have 8 pennies together. Maria has 3 pennies. How many pennies does Kevin have?

____ 11. Ashley has 7 pennies. Jason has 4 pennies less than Ashley. How many pennies does Jason have?

____ 12. Dennis has 7 pennies. Molly has 5 pennies. How many pennies does Dennis have more than Molly?

____ 13. Karen had some pennies. Then Matt gave her 4 more pennies. Now Karen has 6 pennies. How many pennies did she have to start with?

____ 14. Lisa had some pennies. Then she gave 3 pennies to Bill. Now Lisa has 5 pennies. How many pennies did Lisa have to start with?

Demographics by Difficulty Status and English Learner Status

Variable	No difficulty (n = 788)				Equation difficulty (n = 125)				Word problem difficulty (n = 135)				Word problem + Equation difficulty (n = 211)			
	EL (n = 242)		Non-EL (n = 546)		EL (n = 36)		Non-EL (n = 89)		EL (n = 101)		Non-EL (n = 34)		EL (n = 136)		Non-EL (n = 75)	
	n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%
Female	117	48.3	262	48.0	25	69.4	45	50.6	39	38.6	23	67.7	76	55.9	44	58.7
Race																
African American	5	2.1	62	11.3	1	2.8	18	20.2	1	1.0	4	11.8	4	2.9	26	34.7
Asian American	13	5.4	27	4.9	1	2.8	3	3.4	2	2.0	1	2.9	6	4.4	1	1.3
Caucasian	7	2.9	307	56.2	1	2.8	43	48.3	2	2.0	6	17.6	1	0.7	9	12.0
Hispanic	207	85.5	86	15.8	29	80.6	14	15.7	95	94.1	18	52.9	114	83.8	34	45.3
Multi-racial	5	2.1	54	9.9	4	11.1	9	10.1	1	1.0	4	11.7	5	3.7	3	4.0
Other	5	2.1	10	1.8	0	0.0	2	2.2	0	0.0	1	2.9	6	4.4	2	1.3
School-identified disability	3	1.2	21	3.8	1	2.8	7	7.9	5	5.0	5	14.7	17	12.5	22	29.3
English learner	242	100.0	0	0.0	36	100.0	0	0.0	101	100.0	0	0.0	136	100.0	0	0.0
Retained	8	3.3	11	2.0	2	5.6	1	1.1	10	9.9	4	11.7	16	6.3	6	8.0

Note. EL = English Learner.

Demographics by Difficulty Status and English Learner Status

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	n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%
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Texas English Language Proficiency Assessment System (TELPAS) Scores for English Learners

Variable	No difficulty (<i>n</i> = 242)		Equation difficulty (<i>n</i> = 36)		Word problem difficulty (<i>n</i> = 101)		Word problem + Equation difficulty (<i>n</i> = 136)	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Listening	3.16	0.76	3.03	0.80	2.50	0.97	2.30	0.93
Speaking	2.86	0.86	2.68	0.95	2.19	0.95	2.05	0.83
Reading	2.65	0.94	2.29	0.87	1.67	0.70	1.55	0.67
Writing	2.45	0.89	2.18	0.90	1.69	0.75	1.68	0.77
Composite	2.68	0.76	2.47	0.75	1.87	0.67	1.73	0.62

4 = advanced high

3 = advanced

2 = intermediate

1 = beginning

Mathematics Vocabulary

Third-Grade Mathematics Vocabulary (Powell & Tran, 2016)

$\alpha = .92$

NUMBER AND
OPERATIONS

NUMBER AND
OPERATIONS -
FRACTIONS

GEOMETRY

MEASUREMENT

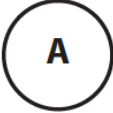


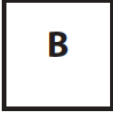


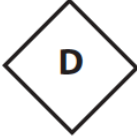

Mathematics Vocabulary - Grade 3

Answer the questions. Try the easy problems first, then go back and try the harder problems.

Fall 2016



1. Match the letter of each shape with the name.

circle	<input type="text"/>			
triangle	<input type="text"/>			
rectangle	<input type="text"/>			
parallelogram	<input type="text"/>			
rhombus	<input type="text"/>			
square	<input type="text"/>			
trapezoid	<input type="text"/>			

2. Write an **odd** number.

Write an **even** number.

3. Write a **fraction** for the picture.



4. In the box, draw a **line**.

In the box, draw a **line segment**.



circle
triangle
rectangle
parallelogram
rhombus
square
trapezoid

odd
even

fraction

line
line segment

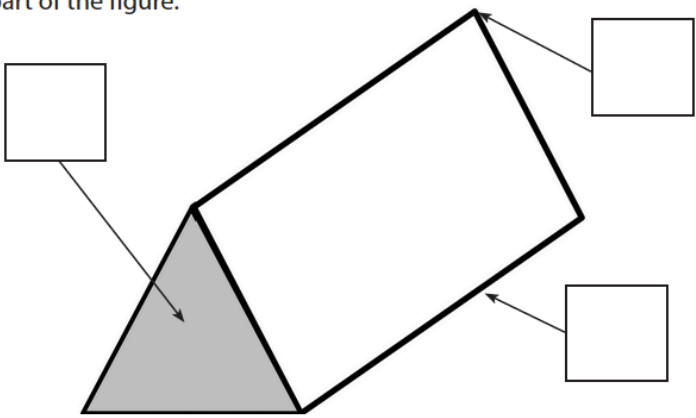
5. Write 537 in **expanded form**.

6. Write a **unit fraction**.

7. Draw an **array** for 4 times 2.

8. Match the letter with each part of the figure.

- A edge
- B face
- C side
- D vertex



9. Draw a **polygon**.



expanded form

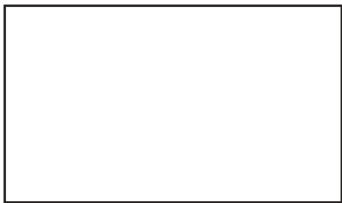
unit fraction

array

edge
face
vertex

polygon

11. Draw a **right angle**.



12. Write an **equation**.

13. Write *three-hundred, twenty-five* in **standard form**.

14. Mark the **perimeter** of the shape.



Mark the **area** of the shape.



15. Draw a **quadrilateral**.



16. Circle the set of **equivalent fractions**.

A. $\frac{3}{4} = \frac{3}{8}$

B. $\frac{3}{4} = \frac{8}{12}$

C. $\frac{3}{4} = \frac{6}{8}$

right angle

equation

standard form

perimeter
area

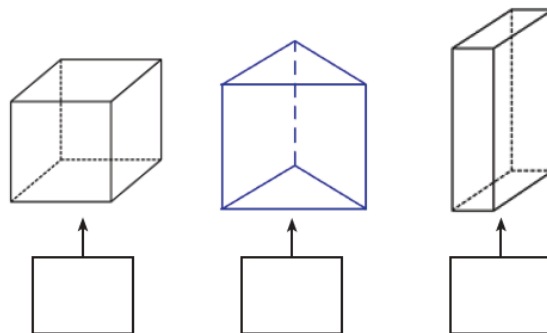
quadrilateral

equivalent
fractions



















17. Write the letter of each shape.


- A cube**
- B rectangular pyramid**
- C rectangular prism**
- D triangular prism**

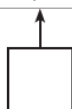
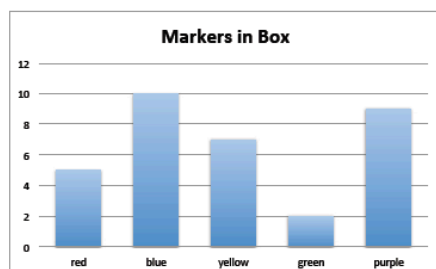



















18. Write the letter that matches each graph.

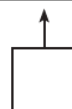
- A bar graph**
- B dot plot**
- C pictograph**
- D tally chart**

Red roses	     
Yellow roses	  
White roses	
Pink roses	     

Each  stands for 5 roses.



Fruit	Total Number
Apple	    
Banana	   
Orange	   
Mango	   



19. Draw an **angle**.



cube
rectangular pyramid
rectangular prism
triangular prism

bar graph
dot plot
pictograph
tally chart

angle

20. Write the letter for each part of a number sentence.

- A addend
- B difference
- C dividend
- D divisor
- E factor
- F minuend
- G product
- H quotient
- J sum

$$\begin{array}{c} \uparrow \qquad \qquad \uparrow \\ \square \qquad \qquad \square \\ 5 + 6 = 11 \end{array}$$

$$\begin{array}{c} \uparrow \qquad \uparrow \qquad \uparrow \\ \square \qquad \square \qquad \square \\ 24 \div 4 = 6 \end{array}$$

$$\begin{array}{c} \uparrow \qquad \qquad \uparrow \\ \square \qquad \qquad \square \\ 3 \times 9 = 27 \end{array}$$

$$\begin{array}{c} \qquad \qquad \uparrow \\ \qquad \qquad \square \\ 14 - 5 = 9 \end{array}$$

21. What is the name of this?

$$\begin{array}{r} 20 \text{ R}5 \\ 6 \overline{)125} \\ \square \end{array}$$

22. Write the **numerator**.

$$\frac{3}{8} \quad \square$$

Write the **denominator**.

$$\frac{6}{9} \quad \square$$

23. Draw a shape with three **sides**.



addend
difference
dividend
divisor
factor
minuend
product
quotient
sum

remainder

numerator
denominator

sides

For non-English learners

Means and Standard Deviations by Difficulty Status and English Learner Status

Variable	No difficulty (<i>n</i> = 787)				Equation difficulty (<i>n</i> = 125)				Word problem difficulty (<i>n</i> = 135)				Word problem + Equation difficulty (<i>n</i> = 211)			
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	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Word problems	11.43	1.96	12.71	1.57	10.44	1.99	11.27	2.02	5.05	1.72	5.18	2.05	4.12	1.99	4.41	1.97
Open equations	13.29	5.59	15.49	7.31	3.97	1.13	3.35	1.35	9.85	4.04	9.38	4.13	2.82	1.55	2.57	1.54
Math vocabulary	15.02	6.41	20.49	7.90	11.06	5.79	14.52	6.78	9.31	4.89	10.74	3.85	7.22	4.51	8.19	4.80

Note. EL = English learner.

For English learners

Means and Standard Deviations by Difficulty Status and English Learner Status

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	EL		Non-EL		EL		Non-EL		EL		Non-EL		EL		Non-EL	
	(<i>n</i> = 242)		(<i>n</i> = 545)		(<i>n</i> = 36)		(<i>n</i> = 89)		(<i>n</i> = 101)		(<i>n</i> = 34)		(<i>n</i> = 136)		(<i>n</i> = 75)	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Word problems	11.43	1.96	12.71	1.57	10.44	1.99	11.27	2.02	5.05	1.72	5.18	2.05	4.12	1.99	4.41	1.97
Open equations	13.29	5.59	15.49	7.31	3.97	1.13	3.35	1.35	9.85	4.04	9.38	4.13	2.82	1.55	2.57	1.54
Math vocabulary	15.02	6.41	20.49	7.90	11.06	5.79	14.52	6.78	9.31	4.89	10.74	3.85	7.22	4.51	8.19	4.80

Note. EL = English learner.

Means and Standard Deviations by Difficulty Status and English Learner Status

Variable	No difficulty (<i>n</i> = 787)				Equation difficulty (<i>n</i> = 125)				Word problem difficulty (<i>n</i> = 135)				Word problem + Equation difficulty (<i>n</i> = 211)			
	EL		Non-EL		EL		Non-EL		EL		Non-EL		EL		Non-EL	
	(<i>n</i> = 242)		(<i>n</i> = 545)		(<i>n</i> = 36)		(<i>n</i> = 89)		(<i>n</i> = 101)		(<i>n</i> = 34)		(<i>n</i> = 136)		(<i>n</i> = 75)	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Word problems	11.43	1.96	12.71	1.57	10.44	1.99	11.27	2.02	5.05	1.72	5.18	2.05	4.12	1.99	4.41	1.97
Open equations	13.29	5.59	15.49	7.31	3.97	1.13	3.35	1.35	9.85	4.04	9.38	4.13	2.82	1.55	2.57	1.54
Math vocabulary	15.02	6.41	20.49	7.90	11.06	5.79	14.52	6.78	9.31	4.89	10.74	3.85	7.22	4.51	8.19	4.80

Note. EL = English learner.

$$F = 89.85, p < .001$$

Means and Standard Deviations by Difficulty Status and English Learner Status

Variable	No difficulty (<i>n</i> = 787)				Equation difficulty (<i>n</i> = 125)				Word problem difficulty (<i>n</i> = 135)				Word problem + Equation difficulty (<i>n</i> = 211)			
	EL		Non-EL		EL		Non-EL		EL		Non-EL		EL		Non-EL	
	(<i>n</i> = 242)		(<i>n</i> = 545)		(<i>n</i> = 36)		(<i>n</i> = 89)		(<i>n</i> = 101)		(<i>n</i> = 34)		(<i>n</i> = 136)		(<i>n</i> = 75)	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Word problems	11.43	1.96	12.71	1.57	10.44	1.99	11.27	2.02	5.05	1.72	5.18	2.05	4.12	1.99	4.41	1.97
Open equations	13.29	5.59	15.49	7.31	3.97	1.13	3.35	1.35	9.85	4.04	9.38	4.13	2.82	1.55	2.57	1.54
Math vocabulary	15.02	6.41	20.49	7.90	11.06	5.79	14.52	6.78	9.31	4.89	10.74	3.85	7.22	4.51	8.19	4.80

Note. EL = English learner.

$$F = 7.23, p = .008$$

Means and Standard Deviations by Difficulty Status and English Learner Status

Variable	No difficulty (<i>n</i> = 787)				Equation difficulty (<i>n</i> = 125)				Word problem difficulty (<i>n</i> = 135)				Word problem + Equation difficulty (<i>n</i> = 211)			
	EL		Non-EL		EL		Non-EL		EL		Non-EL		EL		Non-EL	
	(<i>n</i> = 242)		(<i>n</i> = 545)		(<i>n</i> = 36)		(<i>n</i> = 89)		(<i>n</i> = 101)		(<i>n</i> = 34)		(<i>n</i> = 136)		(<i>n</i> = 75)	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Word problems	11.43	1.96	12.71	1.57	10.44	1.99	11.27	2.02	5.05	1.72	5.18	2.05	4.12	1.99	4.41	1.97
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Math vocabulary	15.02	6.41	20.49	7.90	11.06	5.79	14.52	6.78	9.31	4.89	10.74	3.85	7.22	4.51	8.19	4.80

Note. EL = English learner.

$$F = 2.39, p = .124$$

Means and Standard Deviations by Difficulty Status and English Learner Status

Variable	No difficulty (<i>n</i> = 787)				Equation difficulty (<i>n</i> = 125)				Word problem difficulty (<i>n</i> = 135)				Word problem + Equation difficulty (<i>n</i> = 211)			
	EL		Non-EL		EL		Non-EL		EL		Non-EL		EL		Non-EL	
	(<i>n</i> = 242)		(<i>n</i> = 545)		(<i>n</i> = 36)		(<i>n</i> = 89)		(<i>n</i> = 101)		(<i>n</i> = 34)		(<i>n</i> = 136)		(<i>n</i> = 75)	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Word problems	11.43	1.96	12.71	1.57	10.44	1.99	11.27	2.02	5.05	1.72	5.18	2.05	4.12	1.99	4.41	1.97
Open equations	13.29	5.59	15.49	7.31	3.97	1.13	3.35	1.35	9.85	4.04	9.38	4.13	2.82	1.55	2.57	1.54
Math vocabulary	15.02	6.41	20.49	7.90	11.06	5.79	14.52	6.78	9.31	4.89	10.74	3.85	7.22	4.51	8.19	4.80

Note. EL = English learner.

$$F = 2.21, p = .147$$

NUMBER AND
OPERATIONS

NUMBER AND
OPERATIONS -
FRACTIONS

GEOMETRY

MEASUREMENT

Easier Terms for ELs and non-ELs

even
expanded form
odd

circle
cube
edge
face
rectangle
square
triangle
triangular prism
vertex

perimeter

Difficult Terms for ELs and Non-ELs with MD

array
difference
dividend
divisor
factor
product
quotient
remainder
sum

denominator
numerator
unit fraction

line
line segment

angle
right angle

Difficult Terms for ELs and Non-ELs with EQD + WPD

addend
array
difference
dividend
divisor
equation
factor
product
quotient
remainder
sum

denominator
equivalent
fraction
fraction
numerator
unit fraction

line
line segment

angle
area
bar graph
pictograph
right angle
tally chart

Implications for Practice

Use formal math language

Use explicit instruction

Formal Math Language

Use terms specifically

Use terms precisely

- Technical terms

trapezoid

rhombus

numerator

addend

subtract

- Subtechnical terms

base

degrees

cube

plane

arc

- Symbolic terms

plus

zero

twelve

dollars

and

- General terms

above

measure

answer

longest

outside

Instead of...

“And the last one is 10.”

“What number is in the tens place?”

“Six hundred and forty-eight”

“Bigger number and smaller number”

Say...

“8, 9, 10. We’ll stop counting there but we could count more.”

“What digit is in the tens place?”

“Six hundred forty-eight”

“Number that is greater and the number that is less”

Instead of...

“Numbers in the fraction”

“Top number and bottom number”

“Reduce”

“One point two nine”

Say...

“This fraction is one number.”

“Numerator and denominator”

“Find an equivalent fraction”

“One and twenty-nine hundredths”

Instead of...

“Corner”

“Flips, slides, and turns”

“Box or ball”

“Long hand and short hand”

Say...

“Angle”

“Reflections, translations, and rotations”

“Cube or sphere”

“Minute hand and hour hand”

[illegible][illegible]

Explicit Instruction

Modeling

Clear
Explanation

Planned
Examples

Practice

Guided
Practice

Independent
Practice

Supporting Practices

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Modeling

Clear
Explanation

Planned
Examples

“To solve 26 **plus** 79, I first decide about the **operation**. Do I **add, subtract, multiply, or divide**?”

“The **plus sign** tells me to **add**. So, I’ll **add** 26 **plus** 79. I’ll use the **partial sums strategy**. First, I **add** 20 **plus** 70. What’s 20 **plus** 70?”

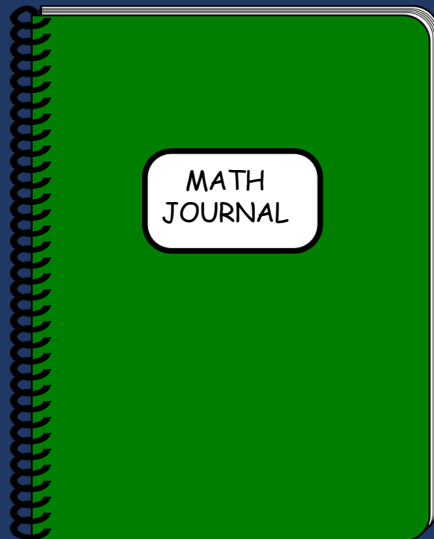
“20 **plus** 70 is 90. I write 90 right here under the **equal line**.”


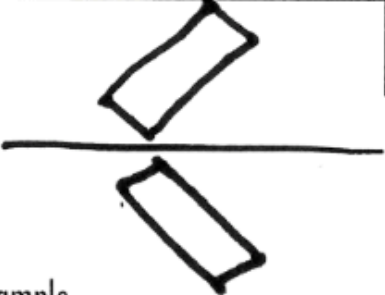


“Then I **add** 6 **plus** 9. What’s 6 **plus** 9?”

“6 **plus** 9 is 15. So, I write 15 here.”

“Finally, we **add** the **partial sums**: 90 and 15. 90 **plus** 15 is 105. So, 26 **plus** 79 **equals** 105.”

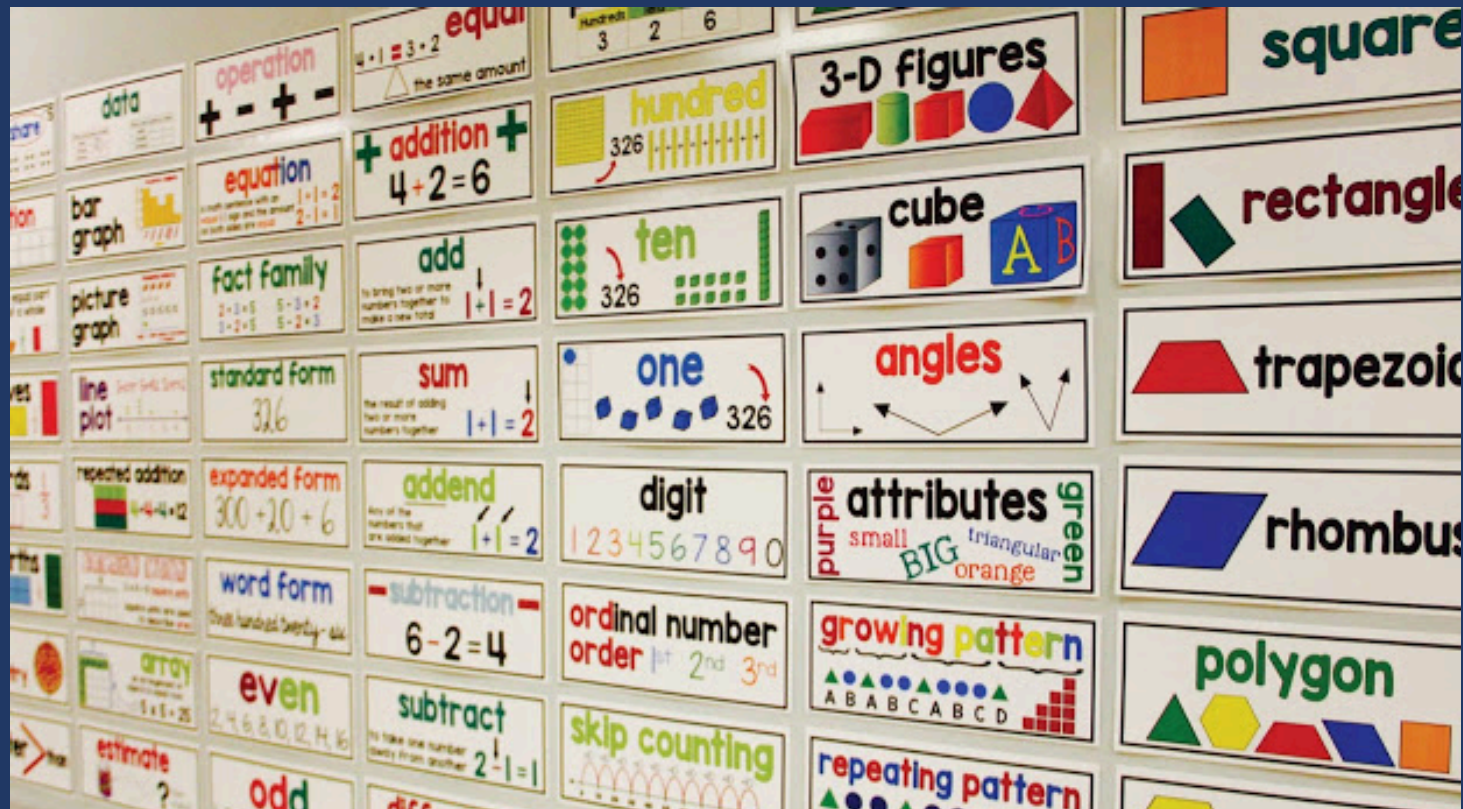
Math Journals



<p>Definition</p> <p>When a shape is flipped across Line of reflection</p>	<p>Illustration</p> 
<p>Example</p> 	<p>trans lation</p>  <p>rotation</p>  <p>Non-example</p>

Reflection

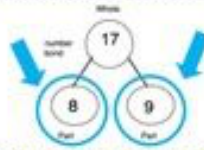
Word Walls



Vocabulary Cards

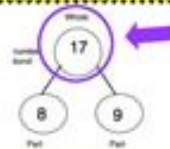
One of two or more smaller units that make up a whole.

part



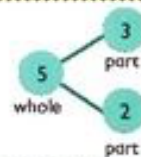
A complete unit

whole

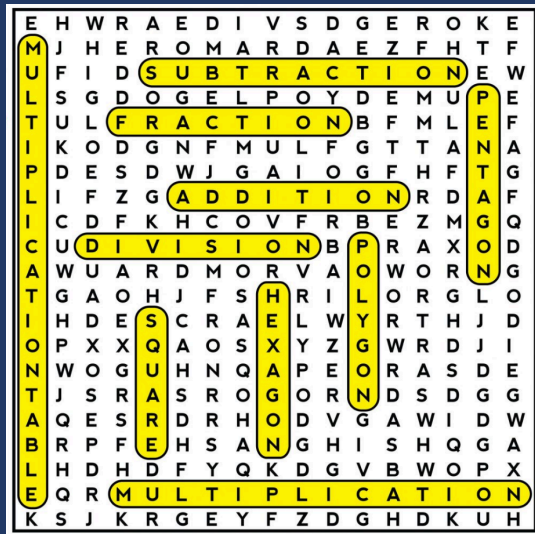


A number relationship between parts and the whole.

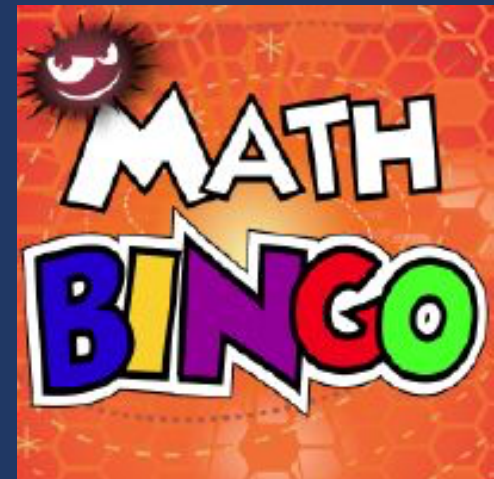
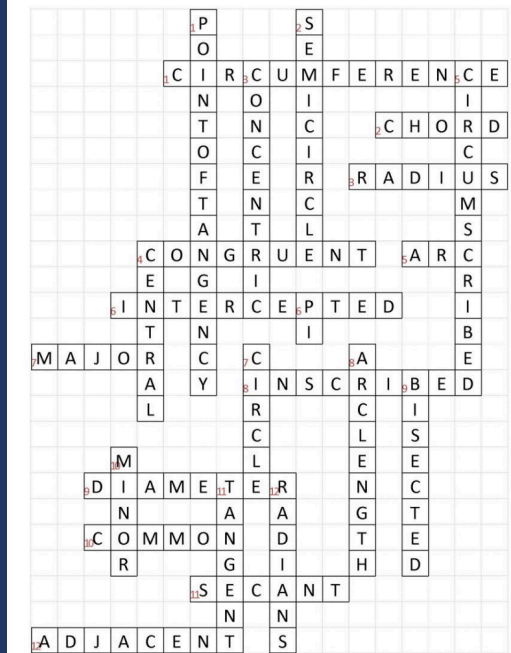
number bond



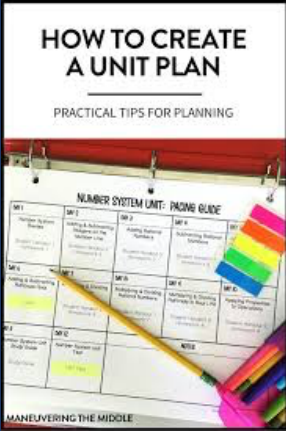
Math Games




CIRCLES VOCABULARY CROSSWORD ANSWER KEY



Other Ideas




Circumference and Area of Circle



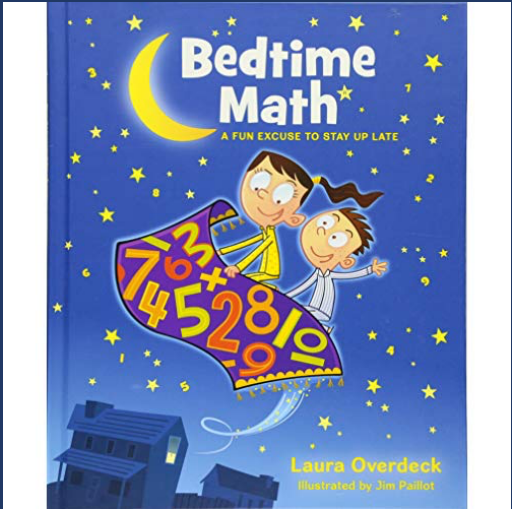
Cherry pie's delicious

$$C = \pi d$$



Apple pies are too

$$A = \pi r^2$$



Problem type	Definition	Examples			Equation	Graphic organizer
		Total unknown	Part unknown			
Total						

Thanks



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Exit Slip

1. What was most valuable part of today's session?
2. What questions do you still have about the topics or the discussion?
3. Is there anything else you want me to know?