An Improved Wideband MUSIC Method for Uniform Linear Array and Nested Array using Fibonacci Technique

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***Abstract* -** In this paper, we simulate the Wideband MUSIC (Multiple Signal Classification) method for uniform linear array and nested array. We improve the Wideband MUSIC method for uniform linear array and nested array by using Fibonacci method. Sharper peaks are obtained for improved Wideband MUSIC using ULA and nested array by applying the Fibonacci technique. The Direction of Arrival (DOA) estimation accuracy is increased by applying the Fibonacci technique.

***Keywords* -** *Wideband MUSIC; Direction of Arrival; Fibonacci method; Uniform linear array; Nested array.*

I. INTRODUCTION

Source localization utilizing a sensor array is used in numerous applications like sonar, radar, communications, oceanography, navigation and speech processing [1],[2]. For finding the direction of arrival (DOA) estimation of narrowband sources, a number of high resolution algorithms like maximum likelihood (ML) method [3], MUSIC [4], ESPIRIT[5] and subspace fitting[6] method has been developed. In case of a narrowband signal, the energy is located in a frequency band which is much smaller than the central frequency. In applications like high data rate communications, seismic signal processing, passive sonars and speech signal processing, wideband signals are frequently encountered. A Uniform linear array (ULA) with N sensors can resolve at most N-1 sources and then we can apply the Direction of Arrival (DOA) estimation methods like Wideband MUSIC (Multiple Signal Classification) to ULA. Nested arrays are obtained when we combine two or more ULAs with increased spacing. The nested array is able to detect more sources than the number of sensors. We decompose the wideband signal into different narrowband components and apply ULA and nested array [7] to each of those frequencies. A 2-level nested array is a linear array with sensor location given by union of sets ,

$S_1 = \{m\, d_1, m=1,\ldots,N_1\}$, $S_0 = \{ n(N+1)d_1, n=1,\ldots,N_2\}$.

A linear nested array with N sensors is assumed including two concatenated uniform linear arrays. Let us consider that the inner ULA has spacing d_1 and the number of sensors is N_1 and the outer ULA has a spacing d_0 and number of sensors is N_2 such that $d_0 = (N+1)d_1$. First, we consider the simulation of wideband MUSIC for Uniform linear Array. Then we consider the simulation of Wideband MUSIC for nested array. We improve the Wideband MUSIC method for Uniform Linear Array and nested array by using the Fibonacci method. Fibonacci method can be used to find the minimum of a function of one variable even if the function is not continuous. Then inverting the minima gives the maxima indicating peaks for DOA estimation for Wideband MUSIC spectrum. This method makes use of sequence of Fibonacci numbers $\{F_n\}$ such that,

$F_0=F_1=1$ and $F_n = F_{n-1} + F_{n-2}$, n=2,3,4.

Sharper peaks are obtained for improved Wideband MUSIC using ULA and nested array by applying the Fibonacci technique. The DOA estimation accuracy is increased by applying the Fibonacci technique. In this paper, section 1 contains Introduction, section 2 explains Signal Model, section 3 discusses Direction of arrival for Uniform linear array and nested array, section 4 explains Fibonacci method and section 5 discusses Simulations.

II. SIGNAL MODEL

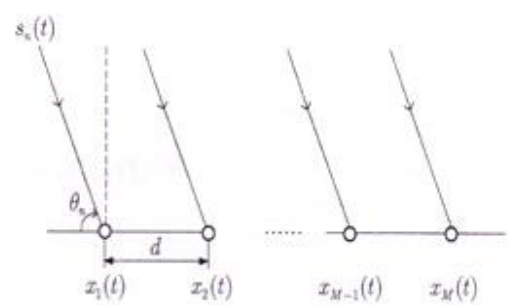

Fig.1: Uniform linear array of M=N sensors with interval d.

We assume a ULA with N sensors. Let us consider K wideband sources falling on this linear array from directions $\{\theta_k, k=1,\ldots,K\}$. Let us consider that the incident wideband signals have a common bandwidth B with the centre frequency f_c. Let $s_k(t)$ represent the kth baseband signal. Then the observed bandpass signal $x_k(t)$ at a reference point can be written as,

$$x_k(t)=s_k(t)e^{j2\pi f_c t} \quad (1)$$

If we observe the signal over interval $[t_1,t_2]$, then the baseband signal can be written as,

$$s_k(t)=\sum_{i=1}^{I} S_k(f_i)e^{j2\pi f_i t}\ ,\ t_1\le t\le t_2 \quad (2)$$

where,

$$S_k(f_i)=1/t_2-t_1\int_{t1}^{t2} s_k(t)\, e^{-j2\pi f_i t}dt \quad (3)$$

with $f_i = f_l + (i-1)B/(I-1)$, $i=1,\ldots,I$. f_l is the lowest frequency in the bandwidth B and I is the number of frequency components. We consider f_l and I so that the frequencies are symmetric about 0 Hz. For the kth signal and the nth sensor, the propagation delay is $\tau_{k,n}$. The modulated bandpass signal at the reference point can be written as ,

$$x_k(t,\tau_{k,n})=\sum_{i=1}^{I} S_k(f_i)e^{j2\pi(f_c+f_i)(t+\tau_{k,n})} \quad (4)$$

where $\tau_{k,n} = nd_I \sin(\theta_k)/c$, $k = 1,\ldots,K$ and $n=1,\ldots,N$ with c being the propagation speed. The demodulated signal can be expressed as,

$$x_k(t,\tau_{k,n})=x_k(t+\tau_{k,n})e^{-j2\pi f_c t} \quad (5)$$

Let $a(\theta_k, f_c +f_i)$ denote the N X 1 steering vector of the kth source and the ith frequency component:

$$a(\theta_k,f_i)=[e^{j2\pi(f_c+f_i)\tau_{k,1}},\ldots e^{j2\pi(f_c+f_i)\tau_{k,N}}]^T \quad (6)$$

The received data vector has the form,

$$x(t)= \sum_{k=1}^{K} x_k(t) = \sum_{i=1}^{I}[A(\theta,f_i)S(f_i) + E(f_i)]e^{j2\pi f_i t}$$
(7)

where $A(\theta, f_i) = [a(\theta_1, f_i),\ldots,a(\theta_k, f_i)]$

$S(f_i) = [S_1(f_i),\ldots,S_K(f_i)]^T$ is the K X 1 signal vector and $E(f_i) = [E_1(f_i),\ldots,E_K(f_i)]^T$ is the NX 1 noise Fourier coefficient vector. Let us consider,

$$y(i)=A(\theta,f_i)S(f_i)+E(f_i), i=1,\ldots,I \quad (8)$$

y(i) are by definition the N X 1 Fourier coefficient vectors of x(t). The source signals are independent of each other and the source autocorrelation matrix R_s is diagonal, $R_{si} = \mathrm{diag}(\sigma_{1,i}^2, \sigma_{2,i}^2, \sigma_{K,i}^2)$. $A(\theta,f_i)$ can be represented as A_i. The autocorrelation matrix of y(i) is,

$R_{yi} = A_iR_{si}A^H + \sigma_E^2 I$, where σ_E^2 is the noise power and I is the identity matrix.

III. DIRECTION OF ARRIVAL ESTIMATION FOR UNIFORM LINEAR ARRAY (ULA) AND NESTED ARRAY

For Wideband DOA estimation we consider a narrowband decomposition. For Uniform Linear Array (ULA), we use the narrowband signal subspace method MUSIC to estimate for each frequency component. $EVD(R_{yi}) = U_i\lambda_i U_i^T$, Where $\lambda_i = \mathrm{diag}(\lambda_i^1, \lambda_i^2, \lambda_i^3, \ldots, \lambda_i^N)$ are the eigen values.

$U_i = [u_i^1, u_i^2, u_i^3, \ldots, u_i^N]$ is the corresponding eigen vector matrix. We can get the noise subspace as,

$U_i^E = [u_i^{K+1}, u_i^{K+2}, \ldots, u_i^N]$ which consists of last N-K eigen vectors corresponding to smallest N-K eigen values. The estimated DOA can be found through an exhaustive search over all the direction space for the MUSIC spectrum.

$M_i(\theta)= 1/(a_i^\theta)^T U_i^E (U_i^E)^T a_i^\theta$, where ,$a_i^\theta = [1, a_i^\theta, \ldots, (a_i^\theta)^N]$.

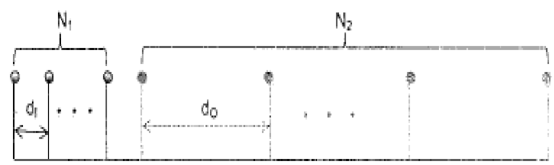

Fig.2: A two-level nested array with N_1 sensors in the inner ULA with inter sensor spacing d_1 and N_2 sensors in the outer ULA with inter sensor spacing d_0

For nested array, we consider the spatial smoothing matrix R_i^{ave} for ith frequency f_i , we do eigen value decomposition,

$$R_i^{ave} = \frac{1}{\frac{N^2}{4}+N/2}\sum_{l=1}^{\frac{N^2}{4}+\frac{N}{2}} R_i^l\ ,\ R_i^l = v_i^l v_i^{lT},\ v_i^l = A_i^l \emptyset^{l-1} p_i + \sigma_E^2 e_l,\ v_i = (A_i^* \otimes A_i)p_i + \sigma_E^2 1_e\ ,$$

Where,$p_i = [\sigma_{i,1}^2, \ldots, \sigma_{K,i}^2]^T$,

$1_e = e_2^T, \ldots, e_N^T]^T$. The symbol * denotes conjugation without transpose, and $\otimes$ denotes the Khatri-Rao product.

$EVD(R_i^{ave})=U_i\lambda_i U_i^T$, Where

$\lambda_i= \mathrm{diag}(\lambda_i^1, \lambda_i^2, \lambda_i^3, \ldots, \lambda_i^{\frac{N^2}{4}+N/2})$are the eigen values.

$U_i=[u_i^1, u_i^2, u_i^3, \ldots, u_i^{\frac{N^2}{4}+N/2}]$ is the corresponding eigen vector matrix. We can get the noise subspace as,

$U_i^E = \left[u_i^{K+1}, u_i^{K+2}, \ldots, u_i^{\frac{N^2}{4}+N/2}\right]$ which consists of last $N^2/4 + N/2$ - K eigen vectors corresponding to smallest $N^2/4$ + N/2 -K eigen values. The estimated DOA can be found through an exhaustive search over all the direction space for the MUSIC spectrum.

$M_i(\theta)=1/(a_i^\theta)^T U_i^E (U_i^E)^T a_i^\theta$,

where,$a_i^\theta = [1, a_i^\theta, \ldots, (a_i^\theta)^{\frac{N^2}{4}+\frac{N}{2}-1}]$,

$a_i^\theta = e^{-j2\pi(f_c+f_i)d_I \sin(\theta)/c}$.

Combining the resulting measurements for all the different frequencies, we construct the new combined MUSIC spectrum as:

$M_i(\theta)=1/\ 1/I\sum_{i=1}^{I}(a_i^{\theta})^T U_i^E\,(U_i^E)^T a_i^{\theta}$, then the estimated DOAs are corresponding to the K largest values of the spectrum M(θ).

IV. FIBONACCI METHOD

The Fibonacci method is used to find the minimum of a function of one variable even if the function is not continuous [8]. Then inverting the minima gives the maxima indicating peaks for DOA estimation for MUSIC spectrum. This method makes use of sequence of Fibonacci numbers $\{F_n\}$ such that,

$F_0=F_1=1$ and $F_n = F_{n-1} + F_{n-2}$, n=2,3,4.

Sharper peaks are obtained for improved Wideband MUSIC using ULA and nested array by applying the Fibonacci technique. The DOA estimation accuracy is increased by applying the Fibonacci technique.

Let the initial interval of uncertainty be L_o. Let it be defined by $a\leq x\leq b$ and let us suppose that the total number of experiments to be conducted as n. Let us take $L_2^* = (F_{n-2}/F_n)L_0$. Consider the first two experiments at point x_1 and x_2 which are positioned at distance of L_2^* from each end of L_o. It implies that ,

$x_1= a + L_2^* = a + (F_{n-2}/F_n)L_o$

$x_2=b-L_2^*=b-(F_{n-2}/F_n)L_o = a + (F_{n-1}/F_n)L_o$

If an experiment is located at a distance of $(F_{n-2}/F_n)L_o$ from one end , it will be at a distance $(F_{n-1}/F_n)L_o$ from the other end. Thus $L_2^* = (F_{n-1}/F_n)L_o$ will yield the same result as $L_2^* = (F_{n-2}/F_n)L_o$. We know that $L_2^* = (F_{n-2}/F_n)L_o \leq 1/2L_o$ for $n \geq 2$. A smaller level of uncertainity L_2 is given by ,

$$L_2 = L_o - L_2^* = L_0\left(1-\frac{F_{n-2}}{F_n}\right), \qquad L_2 = (\frac{F_{n-1}}{F_n})L_0$$

with one experiment left in it . This is at a distance of ,

$L_2^*= (F_{n-2}/F_n)L_o = (F_{n-2}/F_{n-1})L_2$ from one end,
and $L_2 - L_2^* = (F_{n-3}/F_n)L_o = (F_{n-3}/F_{n-1})L_2$ from other end.

We consider the third experiment in interval L_2 so that the current two experiments are located at a distance of $L_3^* = \left(\frac{F_{n-3}}{F_n}\right)L_0 = \left(\frac{F_{n-3}}{F_{n-1}}\right)L_2$ from each end of interval L_2.. The unimodal property allows us to minimize the interval of uncertainity to L_3.

$L_3 = L_2 - L_3^* = L_2\text{-}\left(\frac{F_{n-3}}{F_{n-1}}\right)L_2 = \left(\frac{F_{n-3}}{F_{n-1}}\right)L_2 = \left(\frac{F_{n-3}}{F_n}\right)L_0$.

The method of eliminating a certain interval and placing a new experiment in the remaining interval is being followed. This position of the j^{th} experiment and interval of uncertainity at the end of j^{th} experiment are given by,

$$L_j^* = \frac{F_{n-j}}{F_n-(j-2)}L_{j-1}$$

$$L_j = (F_n - \frac{j-1}{F_n})L_0$$

The ratio of interval of uncertainty remaining after conducting j experiments of the n predetermined experiments to the initial interval of uncertainity becomes,

$$\frac{L_j}{L_0} = \frac{F_{n-(j-1)}}{F_n}$$

For j=n,
$L_n/L_o= F_1/F_n = 1/F_n$

The symbol L_j represents the interval of uncertainity remaining after j experiments while the symbol L_j^* tells us the position of jth experiment. The ratio L_n/L_o will permit us to determine n, the required number of experiments to achieve desired accuracy in locating the optimum point. The last experiment has to be placed with care i.e.

$$\frac{L_n^*}{L_{n-1}} = \frac{F_0}{F_2} = \frac{1}{2} \quad \text{for all n.}$$

After performing n-1 experiments and eliminating the appropriate interval in each step , the remaining interval will contain one experiment exactly at its middle point. The final interval of uncertainity is given as ½ $L_{n-1.}$

V. SIMULATION RESULTS

Case 1:

We consider a Uniform Linear array (ULA) with 8 sensors and 6 Wideband sources impinging from directions θ= [-60°, -35°, -15°, 10°, 30°, 60°]. Consider that Wideband sources have the same center frequency f_c =100 Hz and the same bandwidth B=40 Hz. Zero mean Gaussian random process is being followed by the wideband sources with equal power. The demodulated data is sampled at a frequency of 300 Hz. The Wideband MUSIC spectrum for ULA is shown Fig 3. Now, we apply the Fibonacci search to fig3. We get Wideband MUSIC spectrum for ULA using Fibonacci search as shown in fig 4.. In this case, θ= [-70°, -50°, -30°, 10°, 30°, 50°]. We get sharper and more accurate peaks detecting the six wideband sources using Fibonacci search. The Direction of Arrival (DOA) estimation is improved.

Case 2:

We consider a two level nested array with N=6 sensors and K=7 wideband sources impinging from directions θ= [-60°, -35°, -15°, 5°, 30°, 45°,80°]. Consider that Wideband sources have the same center frequency f_c =100 Hz and the same bandwidth B=40 Hz. Zero mean Gaussian random process is being followed by the wideband sources with equal power. The demodulated data is sampled at a frequency of 300 Hz. In this nested array, both the inner and outer ULAs have 3 sensors. Fig 5 shows the spectrum for Wideband MUSIC for nested arrays. Fig 6 shows the spectrum for Wideband MUSIC for nested arrays using Fibonacci search. For fig 6, θ= [-70^0, -50^0, -30^0, 10^0, 30^0, 50^0,70^0]. From Fig 6, we get sharper and more accurate peaks detecting the 7 wideband sources using Fibonacci search. The Direction of Arrival (DOA) estimation accuracy is improved.

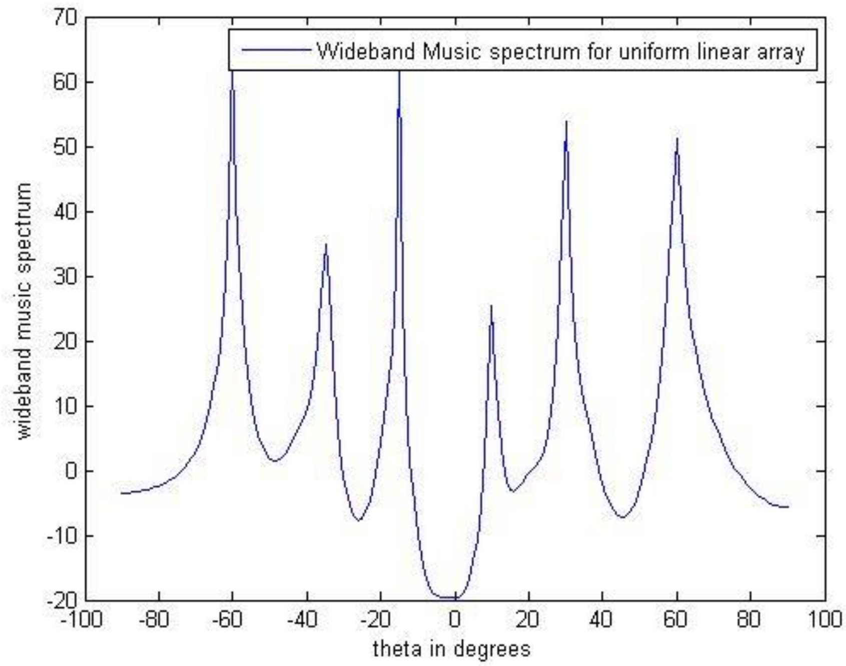

Fig.3: Wideband MUSIC spectrum for uniform linear array

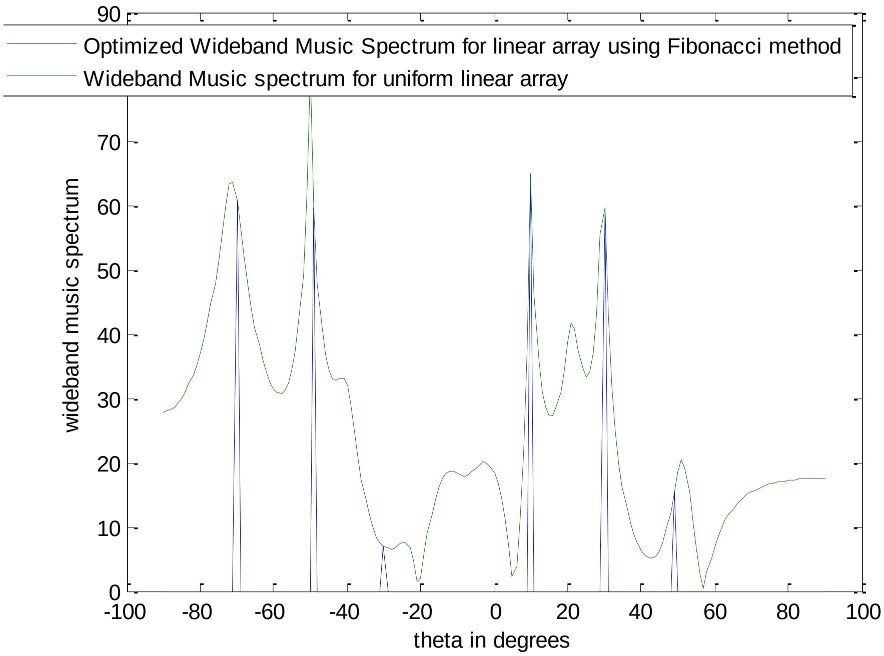

Fig.4: Wideband MUSIC spectrum for uniform linear array by using Fibonacci method

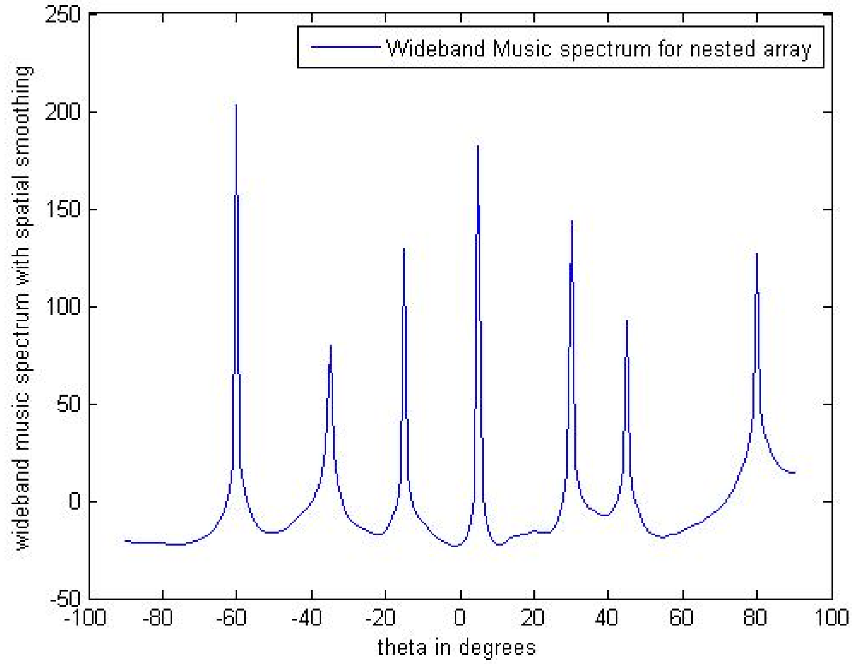

Fig.5: Wideband MUSIC spectrum for nested array

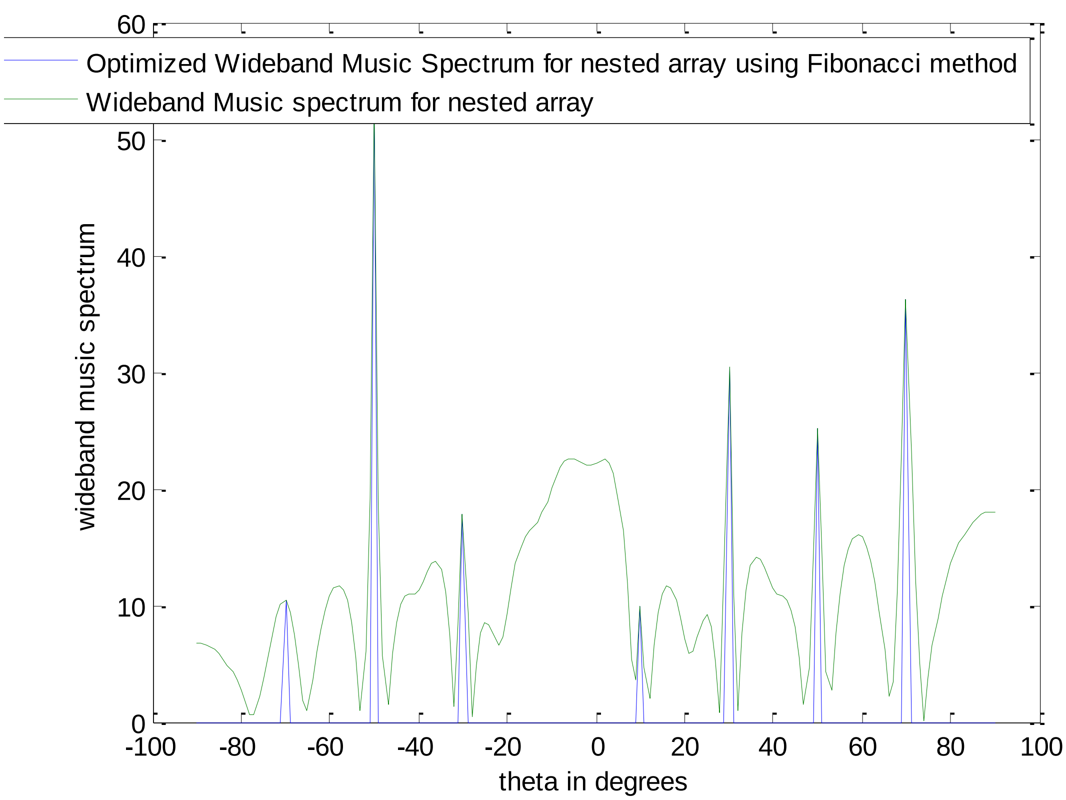

Fig.6: Wideband MUSIC spectrum for nested array using Fibonacci method

VI. CONCLUSION

We simulate the WMUSIC method for ULA. Then we improve the WMUSIC for ULA by using the Fibonacci method. Similarly, we simulate the WMUSIC for nested array. Then , we improve the WMUSIC for nested array by using Fibonacci method. Sharper and more precise peaks are obtained for WMUSIC by using Fibonacci method. The DOA estimation accuracy is improved by using Fibonacci method.

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