Math 3331 - ODE3

Previously $ay'' + by' + cy = 0$ homogeneous

$y = e^{mx}$ gives $am^2 + bm + c = 0$

Sol^n (i) $m = r_1, r_2$ $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$

(ii) $m = r, r$ $y = c_1 e^{rx} + c_2 x e^{rx}$

iii) $m = \alpha \pm i\beta$ $y = c_1 e^{\alpha x} \cos\beta x + c_2 e^{\alpha x} \sin\beta x$

Now we solve

$ay'' + by' + cy = g(x)$

Sol^n of this — 1 sol^n of entire ODE

y_c + y_p

↑

and we add together

Techniques to y_p - particular sol^n

(i) Method of undetermined Coeff.

(ii) reduction of order

(iii) variation of parameter

(i) <u>method of U.C.</u>

Solve $\quad y'' - 3y' + 2y = 12e^{4x}$

1st find y_c

Solve $\quad y'' - 3y' + 2y = 0$

$m^2 - 3m + 2 = 0 \qquad (m-1)(m-2) = 0$

$m = 1, 2$

$y_c = c_1 e^x + c_2 e^{2x}$

for y_p try $y_p = Ae^{4x}$

so $\quad y_p' = 4Ae^{4x}, \quad y_p'' = 16Ae^{4x}$

$16Ae^{4x} - 12Ae^{4x} + 2Ae^{4x} = \qquad \Rightarrow \quad 6Ae^{4x} = 12e^{4x}$

$A = 2$

so $y_p = 2e^{4x}$

∴ G.S. $y = y_c + y_p = c_1 e^{x} + c_2 e^{2x} + 2e^{4x}$

So when RHS is exponentail try

y_p = exponentail

E! $y'' + 2y' + y = 4x^2 + 15x + 9$

$\underline{y_c}$ $y'' + 2y' + y = 0$

$m^2 + 2m + 1 = 0$

$(m+1)^2 = 0$ $m = -1, -1$ $y_c = c_1 e^{-x} + c_2 x e^{-x}$

for $y_p = Ax^2 + Bx + C$

$y_p' = 2Ax + B$, $y_p'' = 2A$

sub

$2A + 2(2Ax + B) + Ax^2 + Bx + C = 4x^2 + 15x + 9$

$Ax^2 + (4A + B)x + 2A + 2B + C = 4x^2 + 15x + 9$

$A = 4$ $4A + B = 15$ $2A + 2B + C = 9$

$B = 15 - 4(4) = -1$ $C = 9 - 2(4) - 2(-1) = 9 - 8 + 2$

$= 3$

so $y_p = 4x^2 - x + 3$

G.S. $y = y_c + y_p = c_1 e^{-x} + c_2 x e^{-x} + 4x^2 - x + 3$

Ex 3 $y'' - 4y' + 5y = 16 \sin x$

y_c $y'' - 4y' + 5y = 0$

$m^2 - 4m + 5 = 0$ $m = \frac{4 \pm \sqrt{16 - 4(5)}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i$

$y_c = c_1 e^{2x} \sin x + c_2 e^{2x} \cos x$

for y_p if we try $y_p = A \sin x$

$y_p' = A \cos x$ $y_p'' = -A \sin x$

Sub $-A \sin x - 4A \cos x + 5A \sin x \stackrel{?}{=} 16 \sin x$

problem

try $y_p = A \sin x + B \cos x$

$y_p' = A \cos x - B \sin x$

$y_p'' = -A \sin x - B \cos x$

Sub

$$-A\sin x - B\cos x - 4(A\cos x - B\sin x) + 5(A\sin x + B\cos x)$$

$$= 16\sin x$$

$$(-A + 4B + 5A)\sin x + (-B - 4A + 5B)\cos x = 16\sin x$$

$$\left.\begin{array}{l} +4A + 4B = 16 \\ -4A + 4B = 0 \end{array}\right\} \quad 8B = 16 \quad B = 2$$

$$\longrightarrow A = B = 2$$

So $y_p = 2\sin x + 2\cos x$

G.S. $y = C_1 e^{2x}\sin x + C_2 e^{2x}\cos x + 2\sin x + 2\cos x$

Solve $y'' - 3y' + 2y = e^x$

C.E. $m^2 - 3m + 2 = 0$ $m = 1, 2$ as before

$y_c = c_1 e^x + c_2 e^{2x}$

try $y_p = Ae^x$ (but part of y_c)

$y_p' = Ae^x$ $y_p'' = Ae^x$

sub $y'' - 3y' + 2y = e^x$

$Ae^x - 3Ae^x + Ae^x \stackrel{?}{=} e^x$

$0 = e^x$

so we need to do more when RHS has parts of y_c in it!