CAP-5993 Homework 4
Deadline: 3:30PM on 4/13 (Thursday)

1. Exercise 8.1 from textbook. (10 pts)
2. Exercise 8.2 from textbook. (10 pts)
3. Exercise 8.3 from textbook. (10 pts)
4. Exercise 8.4 from textbook. (10 pts)
5. Exercise 8.5 from textbook. (10 pts)
6. Exercise 8.7 from textbook. (10 pts)
7. Exercise 8.8 from textbook. (10 pts)
8. Exercise 8.9 from textbook. (10 pts)
9. Exercise 8.10 from textbook. (10 pts)
10. Exercise 8.11 from textbook. (10 pts)
11. Exercise 8.14 from textbook. (10 pts)
12. Exercise 8.15 from textbook. (10 pts)
13. **Security Game (30 pts)** Consider the security game from lecture:

| | $L$(Attack Target 1) | $R$(Attack Target 2) |
|---|---|---|
| $U$(Defend Target 1) | $(4, -3)$ | $(-1, 1)$ |
| $D$(Defend Target 2) | $(-5, 5)$ | $(2, -1)$ |

   (a) Find all (mixed or pure) Nash equilibria. (5 pts)
   (b) Find all maxmin pure strategies for the Defender. (5 pts)
   (c) Find all maxmin mixed strategies for the Defender. (5 pts)
   (d) Find the optimal pure strategy for the Defender to commit to, assuming that the attacker will best respond. (5 pts)
   (e) Find the optimal mixed strategy for the Defender to commit to, assuming that the attacker will best respond. (10 pts)

14. **Big Match (50 pts)** In lecture we computed optimal strategies for the king and the minister in the "Big Match." As in lecture, let $s_2$ denote the initial game state, $s_1$ denote the state after the king returns to find the minister working hard, and $s_0$ the state after the king returns to find the minister enjoying life.

   (a) Consider a variation where, if the king returns and the minister is working hard, then the game transitions to $s_1$ with probability 0.9 and to $s_0$ with probability 0.1, while if the king returns to find the minister enjoying life, the game transitions to $s_0$ with probability 0.9 and to $s_1$ with probability 0.1. (20 pts)
      i. Write down all the game states for this game. (5 pts)
      ii. Find a Nash equilibrium for the full stochastic game. (15 pts)
   (b) Now consider a generalization where, if the king returns and the minister is working hard, then the game transitions to $s_1$ with probability $p$ and to $s_0$ with probability $1-p$, while if the king returns to find the minister enjoying life, the game transitions to $s_0$ with probability $q$ and to $s_1$ with probability $1-q$. Find a Nash equilibrium for this generalized game. (10 pts)
   (c) Now consider a variation where the king is more forgiving than in the initial game. If the king returns and the minister is working hard, then the game transitions to $s_1$ with probability 1, while if the king returns to find the minister enjoying life, the game transitions to $s_0$ with probability 0.9 and to $s_2$ with probability 0.1. Find a Nash equilibrium for this game. (10 pts)

(d) Now consider a generalization of the forgiving variation just described. If the king returns and the minister is working hard, then the game transitions to $s_1$ with probability $p$ and to $s_2$ with probability $1 - p$, while if the king returns to find the minister enjoying life, the game transitions to $s_0$ with probability $q$ and to $s_2$ with probability $1 - q$. Find a Nash equilibrium for this game. (10 pts)

15. **Largest Number (50 pts)** In lecture we saw that the continuous game "Choose the Largest Number" where both players could select any positive integer had no Nash equilibrium (even in mixed strategies). Recall that the player who chooses the smallest number pays \$1 to the player who chooses the largest number, and if the two players choose the same integer, no exchange of money occurs.

    (a) Now suppose that the players instead choose an integer from the set {1,...,k}. Prove that this game has a unique Nash equilibrium. (10 pts)

    (b) Consider the continuous game where both players must choose a real number from the interval [0,1]. Prove that this game also has a unique Nash equilibrium. (10 pts)

    (c) Now consider the game where players choose an integer from the set {1,...,k}, but the tiebreaking rule is different: if both players choose the same integer, then player 2 wins \$1. Find ALL Nash equilibria of this game. (10 pts)

    (d) Now consider the same game, but if both players choose the same integer then player 2 wins \$2. Does this game have a pure strategy Nash equilibrium? If so, find it, and if not then prove that one does not exist. (10 pts)

    (e) Now consider the following game. Both players select integers from the set {1,...,k}. If they select different integers then player 1 wins \$1, and if they select the same integer then player 2 wins \$1. Does this game have a Nash equilibrium in mixed strategies? If so, find one, and if not prove that one does not exist. (10 pts)