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Spring constant and damping constant tuning of nanomechanical resonators using a single-electron transistor

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By fabricating a single-electron transistor onto a mechanical system in a high magnetic field, it is shown that one can manipulate both the mechanical spring constant and damping constant by adjusting a potential of a nearby gate electrode. The spring constant effect is shown to be usable to control the resonant frequency of silicon-based nanomechanical resonators, while an additional damping constant effect is relevant for the resonators built upon carbon nanotube or similar molecular-sized materials. This could prove to be a very convenient scheme to actively control the response of nanomechanical systems for a variety of applications including radio-frequency signal processing, ultrasensitive force detection, and fundamental physics explorations. [DOI: 10.1063/1.1449533]

Nanomechanical resonators are routinely fabricated with frequencies above 500 MHz[1] and quality factors, Q, as large as 250 000[2] and typically $\sim$10 000.[1,3] These resonators are currently being applied for a variety of uses: as the front-end detector of weak forces,[4,5] for fundamental physics applications,[6,7] and for radio-frequency signal processing.[8] However, a major difficulty and limitation with this system is that the uncertainties and irreproducibility in nanofabrication prohibit one from producing devices of a desired resonant frequency or of a desired quality factor. It is shown here that by fabricating a single-electron transistor (SET) on a doubly clamped nanomechanical resonator in a magnetic field, one can engineer the spring and damping constant. By controlling these parameters, one can actively tune the resonant frequency or the quality factor by simply adjusting a gate voltage. This proposed technique, although more complex to fabricate, might be a more usable technique in practice as compared to other demonstrated techniques.[8–10]

Figure 1 shows the basic situation: a metallized doubly clamped, nanomechanical resonator (with mechanical spring constant k_M and damping constant β_M) with tunnel junctions at either end of the resonator, is immersed in a magnetic field perpendicular to the plane of the device. These junctions, with capacitance C_J and tunnel resistance, R_J, form a SET structure. Nearby, on the substrate, a gate electrode is located at an equilibrium distance x_0 and capacitance $C_G(x)$ to the SET island on the resonator. Once the temperature is lowered such that $k_B T \ll E_C = e^2/2C_\Sigma$, then single electron charging effects will strongly dominate the transport through the junctions[11] and the usual SET behavior is expected (e is the electron charge and $C_\Sigma = 2C_J + C_G$ is the total island capacitance.)

The current through the SET, I_{DS}, shows periodic modulation in the parameter $n(x) = C_G(x)V_G/e$, which is a measure of the number of excess electrons on the island as a result of the gate potential, V_G. Thus, by modifying either V_G or $C_G(x)$, one can strongly modify the conductance through the SET. The inset in Fig. 2 shows the calculated[12] dependence using the "orthodox" theory[13] of I_{DS} on V_G, when $V_{DS} = e/2C_\Sigma$, which is a typical bias potential for maximum SET responsivity.

When the resonator moves, the change in capacitance, $\delta C_G(x) \cong (\partial C_G/\partial x)\,\delta x$,[14] will lead to a change in I_{DS}: $\delta I_{DS} = (\partial I_{DS}/\partial x)\,\delta x$. Figure 2 shows the calculated[15] $\partial I_{DS}/\partial x$ for a particular situation. It is found that $\partial I_{DS}/\partial x$ is maximized for $V_{DS} \approx e/C_\Sigma$, and only weakly decreases as one decreases V_{DS}. This change in current will be transduced into a force on the resonator by the magnetic field, B, through the Lorentz force:

$$F = BI_{DS}l \Rightarrow \delta F = Bl\,\delta I_{DS} = Bl\,\frac{\partial I_{DS}}{\partial x}\,\delta x = k_L\,\delta x, \tag{1}$$

where l is the length of the resonator and $k_L = Bl(\partial I_{DS}/\partial x)$, the Lorentz spring constant.

The magnitude of this spring constant will be limited by the size of the potential one can apply between the nanomechanical resonator and the gate; for large enough gate voltage the nanomechanical resonator will "snap-in" toward the gate. It is assumed that this voltage $V_{\max}$ is limited by the displacement necessary to move the resonator to the gate by

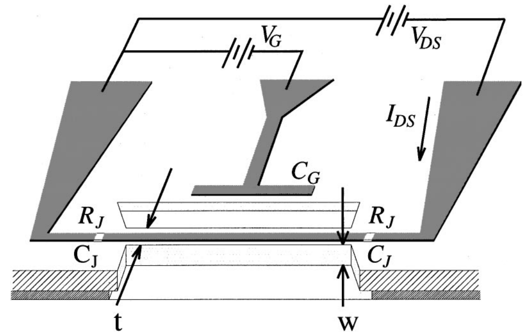

FIG. 1. Configuration: a single electron transistor formed by two tunnel junctions of capacitance C_J and resistance R_J fabricated onto a nanomechanical resonator, placed near a stationary gate, and immersed in a large magnetic field, B, oriented perpendicular to the plane of the device.

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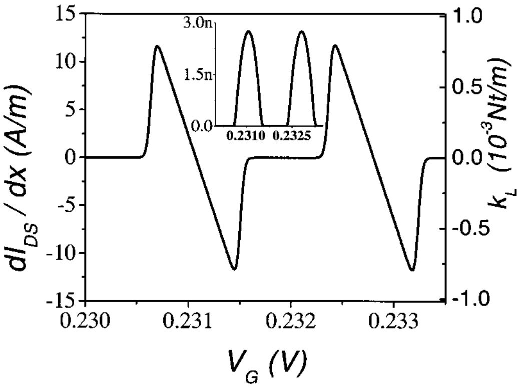

FIG. 2. The calculated $\partial I_{DS}/\partial x$ and k_L vs V_G for resonator Si-10, assuming $C_J = 100$ aF, $C_G = 60$ aF, $R_J = 25$ kΩ, $V_{DS} = e/2C_\Sigma$, $x_0 = 100$ nm, $V_G = 0.23$–0.2335 V, and $T = 50$ mK. Shown in the inset is the calculated current through the SET, I_{DS}, as a function of gate voltage, V_G.

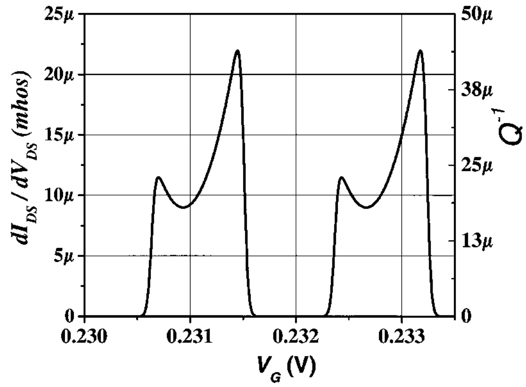

FIG. 3. The calculated values of $\partial I_{DS}/\partial V_{DS}$ and Q^{-1} vs V_G. The parameters from Si-10 in Table I are used to calculate Q^{-1}.

$x_0/3$.[6] Table I shows the maximum values of the bias voltage and the maximum Lorentz spring constant for various realizable nanomechanical resonators.

This effect can be used to rapidly adjust the resonant frequency by making a small change in the gate potential. The fractional tunability is given by

$$\frac{\Delta\omega}{\omega_0} = \sqrt{1+\frac{k_L}{k_M}} - \sqrt{1-\frac{k_L}{k_M}}. \tag{2}$$

For all but the highest frequency silicon resonator in the table, Si-715, one can easily achieve frequency shifts that are greater than the natural linewidth of the resonance, which for typical nanomechanical resonators, $\Delta\omega_0/\omega_0 = 1/Q \approx 10^{-4}$–$10^{-5}$.[16]

In addition to the restoring force caused by the $I_{DS}(x)$, there will be an additional damping force caused by the EMF developed across the SET island as a result of the metallic SET island moving through the magnetic field: $\delta V_{DS} = -d\Phi/dt = Blv$, where v is the resonator velocity. This EMF will lead to a change in $\delta I_{DS} = \partial I_{DS}/\partial V_{DS} Blv$. Through the magnetic field, this will produce a force on the mechanical resonator which is always opposed to the velocity and leads to dissipation: $\delta F = (\partial I_{DS}/\partial V_{DS})(Bl)^2 v = \beta_L v$, where β_L is an additional damping constant and leads to a limitation of the quality factor, Q:

$$\frac{1}{Q} = \frac{1}{Q_M} + \frac{1}{Q_L} = \frac{1}{\omega_0 m}\left[\beta_M + \frac{\partial I_{DS}}{\partial V_{DS}}(Bl)^2\right]. \tag{3}$$

Figure 3 shows both the calculated values of $\partial I_{DS}/\partial V_{DS}$ and the effect on Q_L as a function of gate voltage for resonator Si-10. One can see that the effect on Q_L can range from no effect, to a maximum damping depending on the exact bias conditions of SET. Table I also shows the maximum effect of Q_L for various nanomechanical resonators.[17]

From Table I and Eq. (3), it is clear that this damping constant effect is particularly important for the very low mass nanomechanical systems, such as carbon nanotube-base nanomechanical resonators. In addition, this "Q-spoiling" effect should be rather simple to implement since it does not depend on the application of a large static electric potential between the mechanical resonator and the nearby gate which is essential for a large spring constant effect.

Because of the incoherent nature of the tunneling process and the large change in island potential for each tunneling electron, e/C_Σ, the nanomechanical resonator will experience a white-noise force noise driving by these SET island potential fluctuations.[18] This fundamental noise is the origin of the backaction noise in a SET amplifier[19] and will drive the resonator.[7] For Si-10, this force will drive the resonator to an amplitude of ~ 12 pm$_{\rm rms}$, which is equivalent to the thermomechanical temperature of $T^* = k_M x_{\rm rms}^2/k_B \approx 4K$.

The ability to tune the resonant frequency and the quality factor using an integrated SET-nanomechanical resonator is not limited to the milli-kelvin temperature regime. Nanotube-based nanomechanical resonators could be particularly interesting, as a SET (operating at temperature $T \approx 10$ K) is naturally formed when one places metallic leads onto nanotube bundles.[20] Nanotube resonators have very low spring constants due to their molecular size and in this system, it should be possible to engineer the situation where k_L dominates over k_M. In addition, using silicon-on-insulator and a well-controlled oxidation process, researchers at NTT

TABLE I. Various nanomechanical resonators composed of silicon (Si), single wall nanotubes (SWNTs), nanotube bundles (B-SWNTs), with length l, width w, thickness t, mass m, resonant frequency ω_0, mechanical spring constant k_M, maximum bias voltage $V_{\max}$, Lorentz spring constant k_L, range of frequency tuning $\Delta\omega_0/\omega_0$, and least upper bound on quality factor Q_L. Parameters picked according to what is achievable in practice using electron beam lithography. For the nanotube resonators, it is assumed that the density is ~ 1100 kg/m^3, Young's modulus 1000 GPa, $C_G = 10$ aF, $C_J = 10$ aF.

Material	$l(\mu\text{m})\times w(\text{nm})\times t(\text{nm})$	m (kg)	$\omega_0/2\pi$ (MHz)	k_m (Nt/m)	$V_{\max}$ (V)	k_L (Nt/m)	$\Delta\omega/\omega_0$	Q_L
Si-1	$30\times200\times104$	1.4×10^{-15}	1	0.090	0.58	0.027	0.30	4.4×10^{3}
Si-10	$7\times50\times56$	4.5×10^{-17}	10	0.28	3.6	0.013	0.05	5.2×10^{4}
Si-100	$1.3\times20\times20$	1.2×10^{-18}	100	0.79	17	0.0014	0.002	3.0×10^{5}
Si-715	$0.5\times10\times20$	2.3×10^{-19}	715	6.9	10	8.2×10^{-5}	8.2×10^{-6}	2.4×10^{6}
B-SWNT	$3.0\times5\times5$	8.5×10^{-20}	17	1.4×10^{-3}	0.81	7×10^{-4}	0.52	737
SWNT	$3.0\times1.2\times1.2$	4.7×10^{-21}	4.0	4.6×10^{-6}	0.044	2.8×10^{-5}	$\cdots$	5.2

have been able to fabricate doped silicon bridges that show room temperature Coulomb blockade.[21] Together with a rare-earth magnet with surface fields ~ 1 T, one may be able to implement this technique at room temperature.

A limitation of these effects is the fact that the spring constant and damping constant modifications are not independent; they are both functions of V_{DS} and V_G. However, given their different functional dependence on parameters such as gate voltage, magnetic field, and bias voltage, one can emphasize one effect over the other.

Since the spring constant can be significantly modified by small change in gate potential, one can easily implement parametric amplification.[22,23] In addition, the modulation of the spring constant as a function of displacement may prove to be very useful for fundamental physics applications. The periodic spring constant forms a very interesting potential surface which may be useful for exploring the macroscopic quantum tunneling of a mechanical device at ultralow temperatures.[24] The effects described here demonstrate the rich phenomenology that is possible between nanomechanical and single electron devices.[25]

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