Fuzzy Theory Approach for Reliability Evaluation of Cube Based Networks

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Abstract- The uncertainty involved in any problem solving situation is a result of some information deficiency, which may be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory, or deficient in some other ways. This paper presents a new and simple method for evaluating the fuzzy reliability of cube based computer networks. An algorithm to evaluate the fuzzy reliability has been proposed. Then the proposed method is well illustrated through a simple example. Fuzzy reliabilities of some important cube based computer interconnection networks have been evaluated by the proposed method.

Keywords- Reliability, Fuzzy set, Interconnection network

A. NOTATIONS

X	a set containing a space of points in the probability domain
x	an element of X
p_i	fuzzy probability of an event i
$\overline{p}_i$	complement of fuzzy probability of an event i
$\mu_{p_i}(p)$	membership function of fuzzy probability p_i
N	number of nodes of the cube
R	fuzzy reliability of cube
G	reliability logic graph
V	vertex set
E	edge set
S	system success containing all paths between the source node(s) to destination node (t)
S_k	*kth minimal path set*
R_k	*the fuzzy reliability at kth step*
P_i	path at the i^{th} step
$W, \overline{W}$	indicator variables

B. ASSUMPTIONS

1. Initially, all components of the system are in good conditions.
2. The link failure and link success probability is assumed to be fuzzy numbers
3. Failures can not be determined with certainty.

I. INTRODUCTION

Uncertainty affects decision-making and appears in a number of different forms. The concept of information is fully connected with the concept of uncertainty. The most fundamental aspect of this connection is that the uncertainty involved in any problem solving situation is a result of some information deficiency, which may be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory, or deficient in some other way. So the effects of uncertainty in a system can be handled in a better way by using fuzzy logic. So the effects of uncertainty in a system can be handled in a better way by using the concept of fuzzy set theory [1].

To overcome this problem, the concept of fuzzy set approach is necessary to be used in the evaluation of the reliability of interconnection systems.

Fuzzy set theory was first introduced by Zadeh [1]. A fuzzy set can be defined as follows:

Let X be a space of points and an element of X be denoted by x, i.e. $X=\{x\}$. A fuzzy set A in X is characterized by a membership function $\mu_A(x)$, which is a real number in the interval [0,1] and represents the degree of membership of x in A.

In the conventional methods [2-4], it is required to find the minimized expression of system reliability using Boolean algebra. However, these expressions cannot be used in fuzzy set theory because of non-applicability of complementary laws. The expression used for fuzzy reliability of parallel systems has to be different from the expression of conventional probability analysis for obvious reason.

Keller and Kara-Zaitri [5] presented a method for assessment of reliability of a non-series parallel network using fuzzy logic. Soman and Misra analysed fuzzy fault tree using resolution identity [6], Tanaka et al [7] and Misra and Weber [8] showed how fuzzification can be carried out for the quantitative analysis of fault tree. Chowdhury and Mishra [9] evaluated the reliability of a non-series parallel network. Bastani et al [10] considered the reliability modeling continuous process-control system. Patra et al [11] presents a method for evaluating fuzzy reliability of a communication network with fuzzy element capacities and probabilities. Tripathy et al [12] evaluate the fuzzy reliability of multistage interconnection network. But none of methods discussed above considers the cube based interconnection networks and suggests a general method of evaluating fuzzy reliability of the said networks where there lies a large degree of uncertainty in system failure. So, there is always a need to search for a general and efficient method to evaluate the fuzzy reliability of such systems.

In this paper, a general and efficient method has been proposed to find an expression of fuzzy system reliability of cube based networks taking into consideration the special requirements of fuzzy sets. This method is supported by an efficient algorithm and well illustrated through an example network of 6 nodes and 9 links.

II. PROPOSED METHOD FOR FUZZY RELIABILITY EVALUATION

First, the cube based interconnection network is converted into the equivalent reliability logic graph G {V, E}, where V is the vertex set and E is the link (edge) set. The edge (link) success and edge failure probability is assumed to be fuzzy numbers. Let S_k denote the k^{th} minimal path set and p_k be the fuzzy probability associated with S_k. Also let R_k be the fuzzy reliability at k^{th} step of the sum of fuzzy path probabilities. The reliability expression can be found out by a recursive formula:

$$R_k = R_{k-1} + \Pr\left\{ S_k \cap \left(\overline{\bigcup_{i}^{k-1} S_i} \right) \right\}$$
$$= R_{k-1} + \Pr\{S_k \cap \bar{S}_1 \cap \bar{S}_2 \cap \cdots \cap \bar{S}_{k-1}\} \quad (1)$$

Proposed algorithm

[1]. Convert the cube based network to a probabilistic graph.
[2]. Enumerate all the minimal path sets from the source to destination node.
[3]. Rearrange the total number of path sets (h) according to increasing order of cardinality.
[4]. Set k=1 and R_0=0
[5]. Repeat 2-3 for k=1 to h
[6]. The reliability expression can be given as

i. $R_k = (R_{k-1} + S_k)_{dis}$

b. 7. $p_k = R_k - R_{k-1}$

[7]. Compute the fuzzy reliability of the system as

$$R = 1 - \prod_{k=1}^{h} \bar{p}_k$$

Efficiency:

Finding the minimal paths using the above proposed algorithm requires O(N^3) operations. So, the running time of proposed algorithm is O(N^3) which is polynomial.

ILLUSTRATION

The proposed method is illustrated through an example network of 6 nodes and 9 edges.

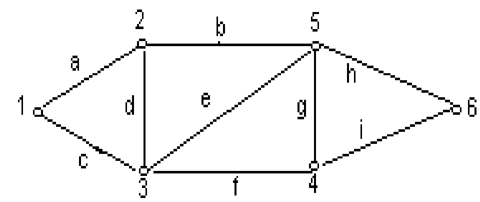

Fig.1: A network of 6 nodes and 9 links

Assuming the trapezoidal membership function as given in Equation 2, the fuzzy probabilities of links a, b, c, d, e, f, g, h, i of the network in Fig.1 are given as follows

p_a =(0.22, 0.8, 0.95, 0.99);
p_b =(0.11, 0.7, 0.88, 0.96);
p_c =(0.35, 0.85, 0.91, 0.98);
p_d =(0.1, 0.42, 0.65, 0.93);
p_e =(0.17, 0.6, 0.83, 0.96);
p_f =(0.3, 0.5, 0.7, 0.90);
p_g=(0.10,.76,0.85,0.97);
p_h=(0.1,0.64,0.7,0.9);
p_i=(0.1,0.55,0.60.9);

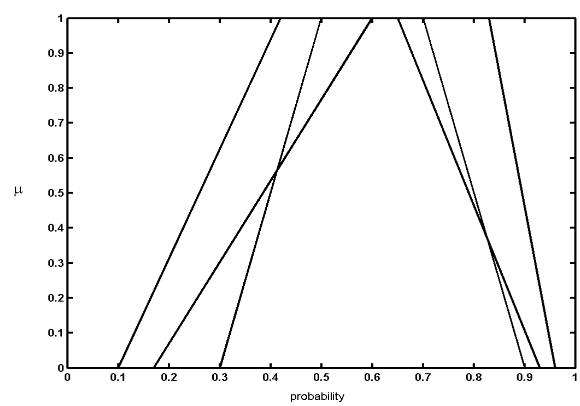

Fig.2: Fuzzy probabilities of links a,b,c

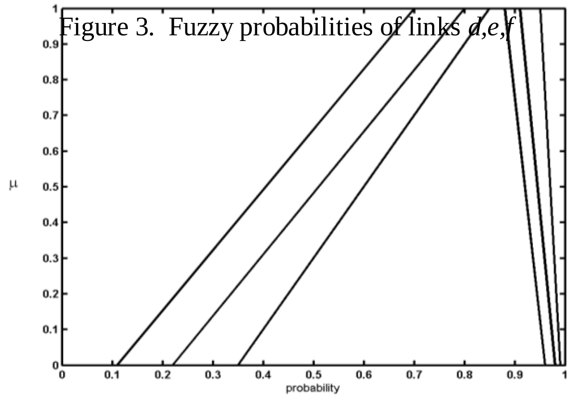

Figure 3. Fuzzy probabilities of links *d,e,f*

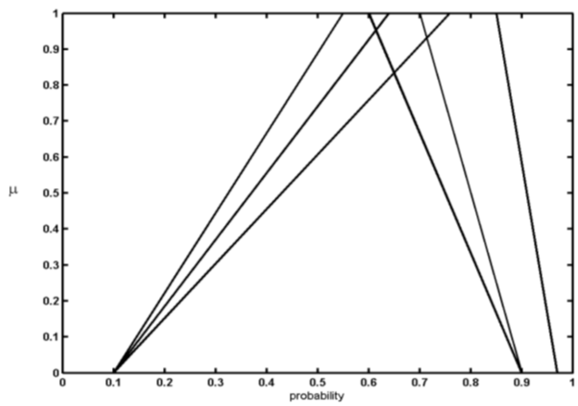

Fig.4. Fuzzy probabilities of links *g,h,i*

The path set is given by

P={abh, adeh,adfi,ceh,cfi,cegi, cfgh}

Using the proposed algorithm, the membership function of the fuzzy reliability of the network is given by

R= {0.4306, 0.5476, 0.9024, 0.9940}

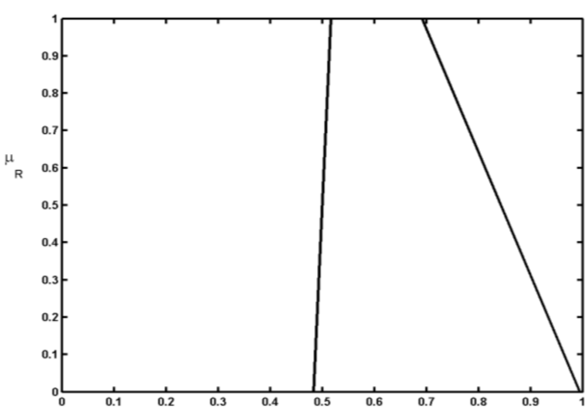

Fig.5: Fuzzy reliability of the network with 6 nodes and 9 edges

III. RESULTS AND DISCUSSIONS

The fuzzy reliability of five important cube based computer networks viz. hypercube (HC), Crossed cube (CQ), Fault-tolerant hypercube (FTH), Varietal hypercube (VH) and exchanged hypercube (EHC) have been evaluated by the proposed method. The membership functions of the said parallel computer interconnection networks are plotted against the probability (Figs. 6-10). The parameter functions of the said parallel computers are presented in Table 1. The inference that can be drawn from Table 1 is that the fuzzy reliability of hypercube lies between the limits 0.71-0.95 with a 100% possibility. Similarly the fuzzy reliability of FTH, CQ, VH and EHC lie between the limits 0.74-0.97, 0.76-0.96, 0.61-0.91, 0.51-0.84 respectively with a 100% possibility.

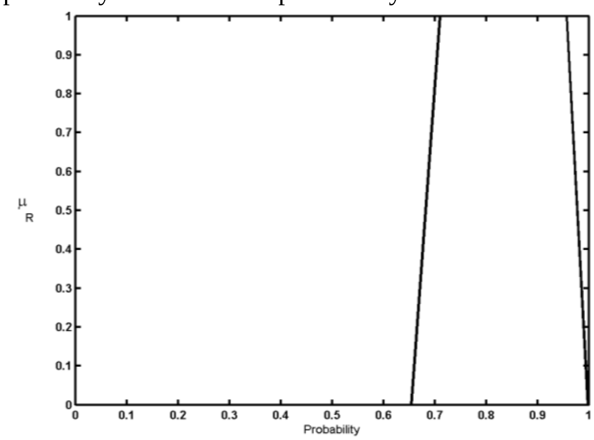

Fig.6: Fuzzy reliability of

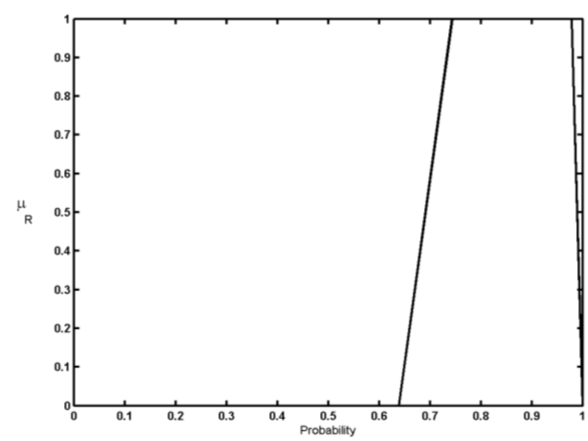

Fig.7: Fuzzy reliability of Fault-Tolerant Hypercube

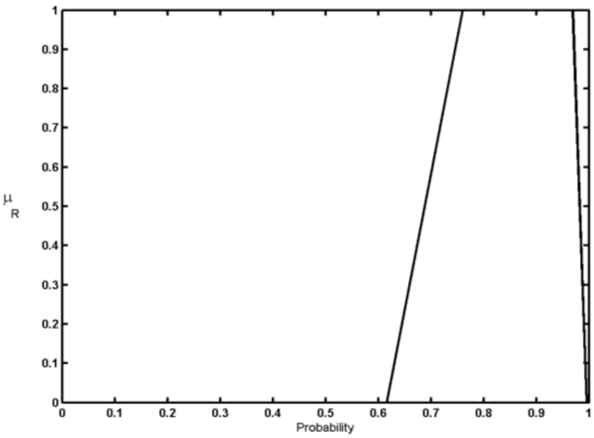

Fig.8: Fuzzy reliability of Crossed Cube

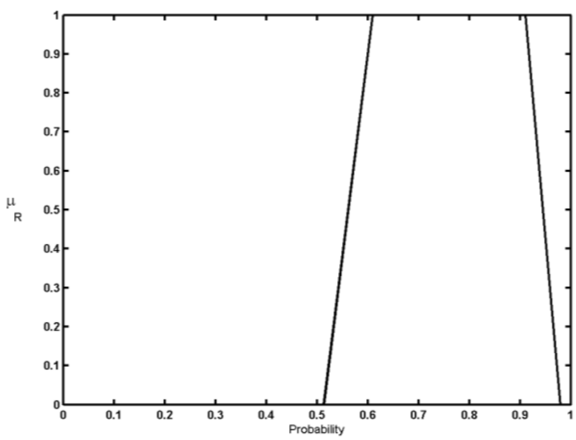

Fig.9: Fuzzy reliability of Varietal Hypercube

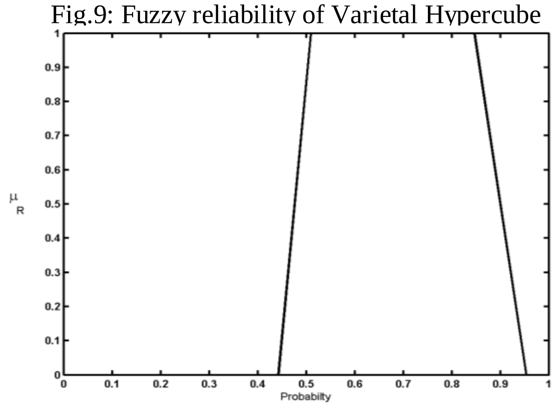

Fig.10. Fuzzy reliability of Exchanged Hypercube

Table1. Parameter functions of the Cube based interconnection networks

Cubes	α_1	α_2	β_2	β_1
HC	0.6544	0.7105	0.9566	0.9982
CQ	0.6166	0.7604	0.9689	0.9954
FTH	0.6388	0.7440	0.9781	0.9998
VH	0.5138	0.6102	0.9105	0.9794
EHC	0.4426	0.5101	0.8466	0.9533

IV. CONCLUSION

In this paper, a general and efficient method has been proposed to analyze the fuzzy reliabilities of cube based parallel computer interconnection networks. Basically, the proposed method uses the path enumeration technique in evaluating fuzzy reliability. The link failure and link success probabilities are assumed to be fuzzy numbers. The method is followed by mathematical basis, algorithm and illustration.

Using the proposed techniques, the fuzzy reliabilities of five number of loosely coupled parallel computer interconnection networks are evaluated. The proposed method can also be extended further to evaluate all categories of cube based parallel computer networks.

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