Pre Calc 2

More Functions $y = f(x)$

so far linear $y = mx+b$

Quad $y = ax^2+bx+c$

Note suppose we want y intercept of the quad function so

$$y = ax^2+bx+c = 0$$ ← solve this for x

quad. formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So $2x^2-3x+1=0$

$$x = \frac{3 \pm \sqrt{3^2-8}}{4} = \frac{3 \pm 1}{2.2} = \frac{2}{2.2}, \frac{4}{2.2} = \frac{1}{2}, \frac{2}{2}$$

$$= \frac{1}{2}, 1$$

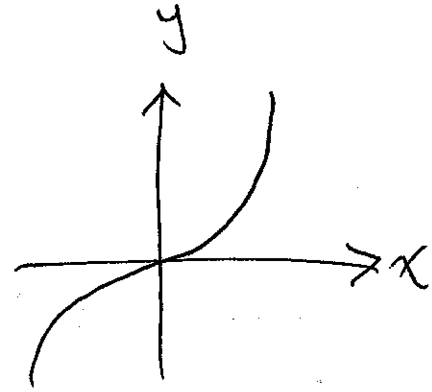
Factor

$$(2x-1)(x-1) = 0$$

same ans.

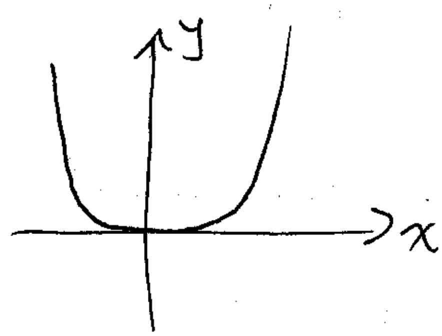
## General Power Law

$$y = ax^n$$




so $y = x^3$

& $y = x^4$



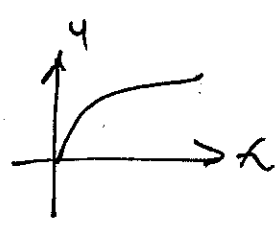

## Polynomial Function

$$y = a_n x^n + a_{n-1} x^{n-1} + \dots a_2 x^2 + a_1 x + a_0$$

$a_i$'s are #s

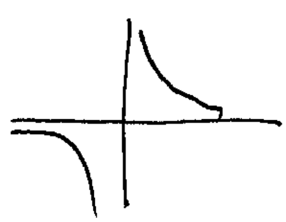
Also we could have fractional power

$y = x^{1/2} = \sqrt{x}$




or negative powers

$y = x^{-1} = \frac{1}{x}$

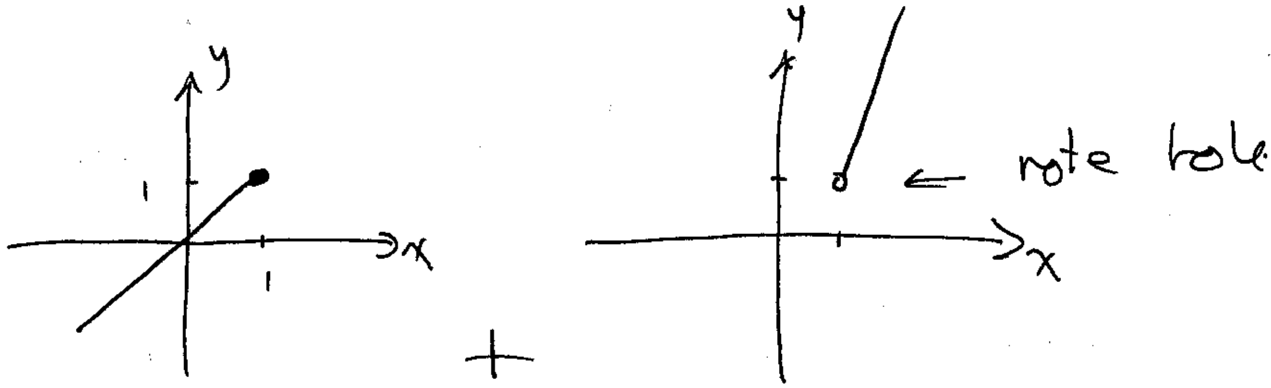


## Branch Function's

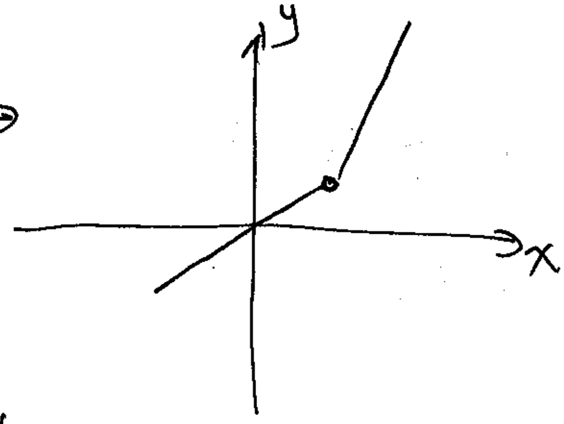
Function is defined for different interval.

Ex

$$f(x) = \begin{cases} x & x \le 1 \\ 2x-1 & x > 1 \end{cases}$$

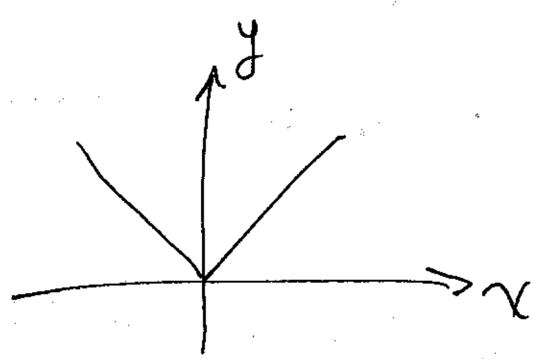



this is the branch function f(x) →



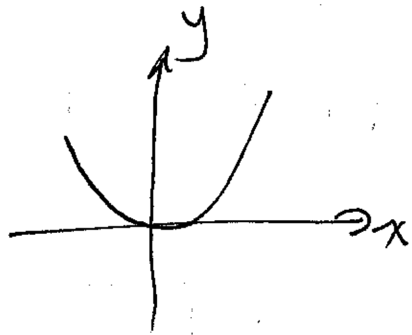

## Absolute function

$$f(x) = |x| = \begin{cases} x & x \ge 0 \\ -x & x < 0 \end{cases}$$



# Transformation of Functions

Given $y = f(x)$

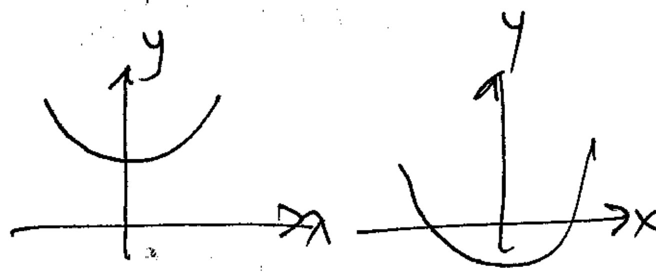


## Vertical Shift

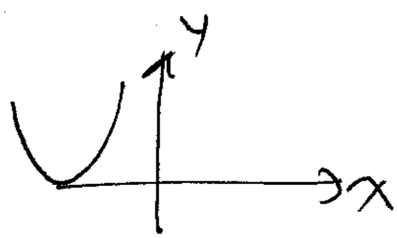
$y = f(x) + c$

$c > 0$ shift up

$c < 0$ shift down



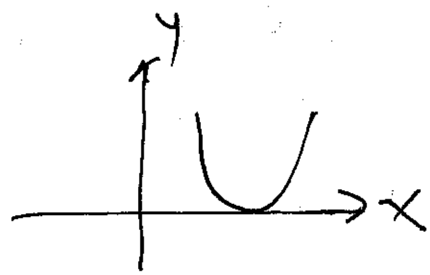
## Horizontal Shift



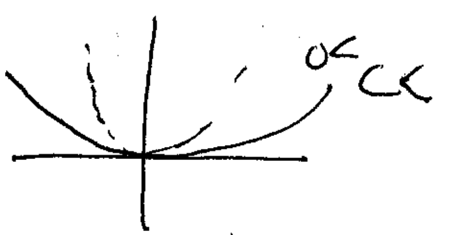
$y = f(x+c)$

$c > 0$ shift left

$c < 0$ shift right

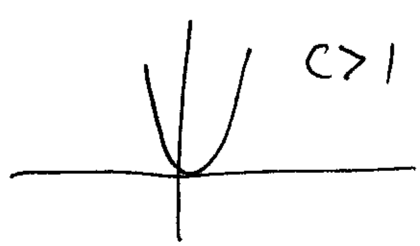


## Stretch (or shrink)




$y = cf(x)$ vertical

$y = f(cx)$ horizontal




more on this later

## Rational Functions

$$f(x) = \frac{p(x)}{q(x)}, \quad q(x) \neq 0 \qquad p, q \text{ poly's}$$

Ex $\quad f(x) = \dfrac{2x-1}{x+1}$

## Domain & Range

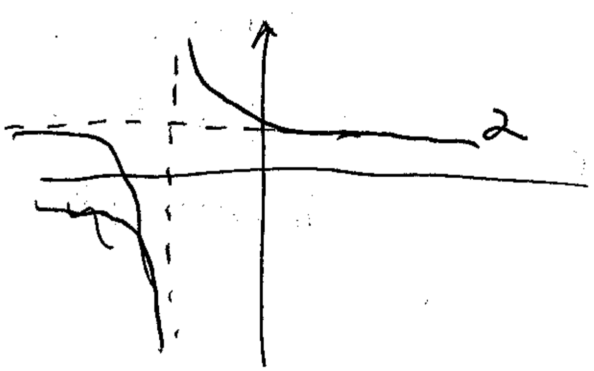
Domain - set of values $x$ where the function is defined so in previous ex.

$D = \{x \mid x \neq -1\}$ all $x$ except $x = -1$

Range - All possible $y$ values

A graph of the example above is

$R = \{y \mid y \neq 2\}$



to show this suppose

$$\frac{2x-1}{x+2} = k \quad \text{so} \quad 2x-1 = k(x+2)$$

$$2x-1 = kx + 2k$$

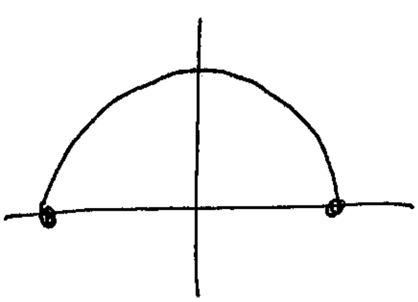
$$(2-k)x = 1+2k \Rightarrow \quad x = \frac{2k+1}{2-k}$$

and $x$ is defined except when $k=2$

ex. $y = \sqrt{1-x^2}$

if $x^2+y^2=1$ (a circle of radius 1 center $(0,0)$)

so solving for $y$ gives our ex.



$$D = \{x \mid -1 \le x \le 1\}$$

$$R = \{y \mid 0 \le y \le 1\}$$

btw- $x^2+y^2=1$ is an example of a ~~function~~ relation between $x$ & $y$ that is given implicitly.

# Evaluating Functions

if $f(x) = 2x^2+3x$

find $f(1)$, $f(a+1)$, $\frac{f(a+h)-f(a)}{h}$

For this example we sub & simplify

(i) $f(1) = 2(1)^2+3(1) = 5$

ii $f(a+1) = 2(a+1)^2 + 3(a+1)$

$= 2(a^2+2a+1)+3a+3$

$= 2a^2+7a+5$

(iii) $f(a+h) = 2(a+h)^2+3(a+h)$

$f(a) = 2a^2+3a$

$$\frac{f(a+h)-f(a)}{h} = \frac{2(a+h)^2+3(a+h)-(2a^2+3a)}{h}$$

$$= \frac{\cancel{2a^2}+4ah+2h^2+\cancel{3a}+3h-\cancel{2a^2}-\cancel{3a}}{h}$$

$$= \frac{\cancel{h}(4a+2h+3)}{\cancel{h}}$$

## Composition

Given $f(x)$ & $g(x)$ then

$(f \circ g)(x) = f(g(x))$ is the composite of $f$ & $g$

Ex $f(x) = 2x+1$ $\quad g(x) = \dfrac{x-1}{3}$

$$f(g(x)) = 2 \cdot g(x) + 1 = 2\left(\frac{x-1}{3}\right)+1 = \frac{2x}{3} + \frac{1}{3}$$

$$g(f(x)) = \frac{f(x)-1}{3} = \frac{2x+\cancel{1}-\cancel{1}}{3} = \frac{2x}{3}$$

In general

$$f(g(x)) \neq g(f(x))$$