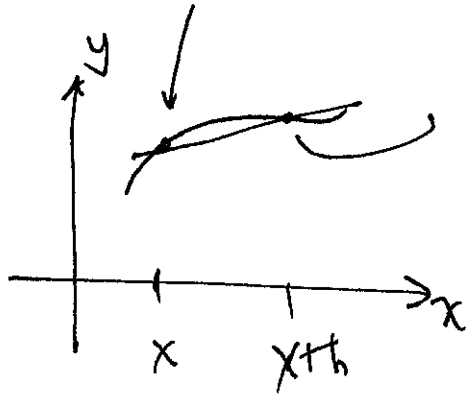
Math 2471 Calc 3

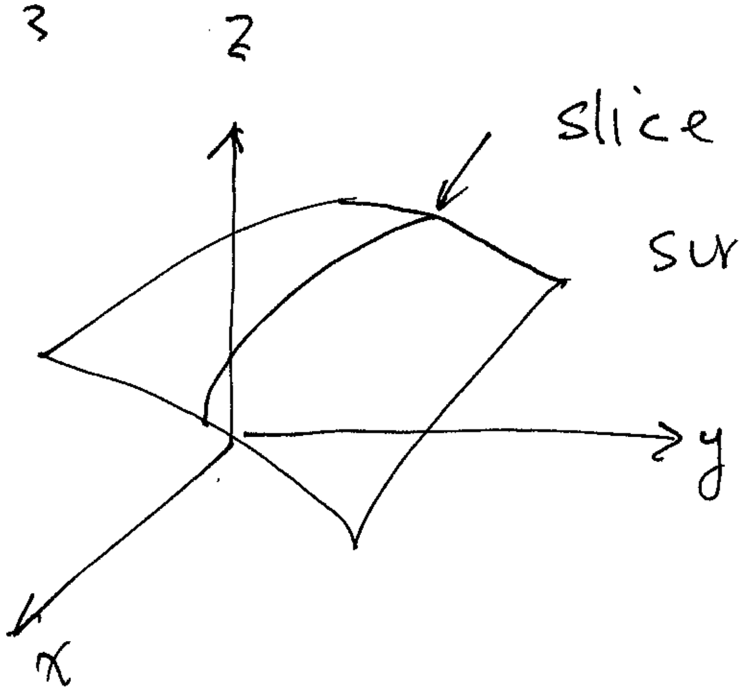
Derivatives

Calc 1



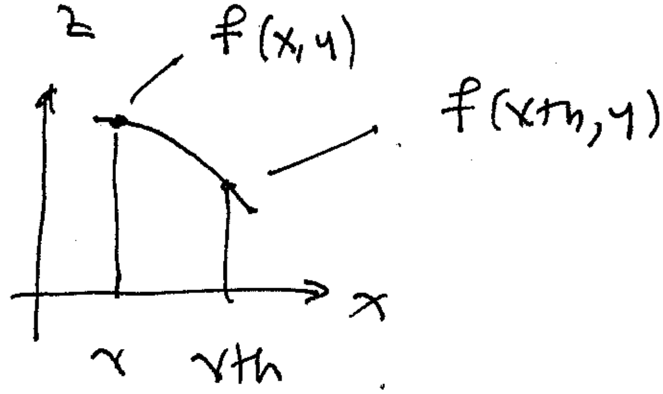
So $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = f'(x)$ the derv.

Now Calc 3



slice of the surface

fix y vary x



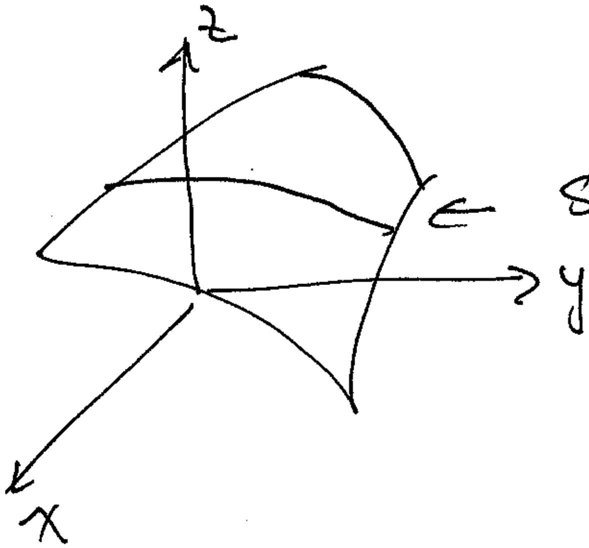
$$\lim_{h\to 0} \frac{f(x+h,y) - f(x,y)}{h} = \frac{\partial f}{\partial x} = f_x$$

Ex $f(x,y) = x^2 y$

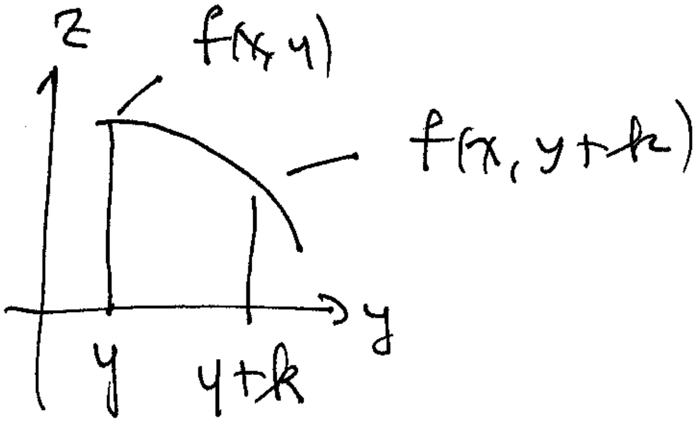
$$\lim_{h\to 0} \frac{(x+h)^2 y - x^2 y}{h}$$

$$\lim_{h\to 0} \frac{(\cancel{x^2} + 2xh + h^2 - \cancel{x^2})\,y}{h}$$

$$\lim_{h\to 0} \frac{2xyh + h^2 y}{h} = \lim_{h\to 0} 2xy + hy = 2xy$$



slide this way
fix x vary y



$$\lim_{k \to 0} \frac{f(x,y+k) - f(x,y)}{k} = \frac{\partial f}{\partial y} = f_y$$

Previous Ex $\displaystyle \lim_{k \to 0} \frac{x^2(y+k) - x^2 y}{k}$

$$= \lim_{k \to 0} \frac{x^2 \cancel{k}}{\cancel{k}} = x^2$$

note: The 2 derivatives are different

Quick Way

$$f(x,y) = x^2 y$$

$\frac{\partial f}{\partial x}$: hold y const $\quad \frac{\partial}{\partial x} x^2 c = 2xc$

or $f_x = 2xy$

$\frac{\partial f}{\partial y}$ hold x const $\quad \frac{\partial}{\partial x} c^2 y = c^2 = x^2$

or $f_y = x^2$

Ex $\quad f = \sin(xy) \quad$ chain Rule

$$\frac{\partial f}{\partial x} = \cos(xy) \frac{\partial}{\partial x} (xy)$$

$$= \cos(xy) \cdot y$$

$$\frac{\partial f}{\partial y} = \cos(xy)\,\frac{\partial}{\partial y}(xy)$$

$$= \cos(xy)\cdot x$$

Ex $\quad f(x,y) = \dfrac{x}{x+y}$

$$f_x = \frac{1\,(x+y) - x\,(1)}{(x+y)^2} = \frac{\cancel{x}+y-\cancel{x}}{(x+y)^2} = \frac{y}{(x+y)^2}$$

$$f_y = \frac{0\,(x+y) - 1\,(x)}{(x+y)^2} = \frac{-x}{(x+y)^2}$$

Higher Order Derivatives

$\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial y \partial x}, \frac{\partial^2 f}{\partial y^2}$

$f = x^3 y^2$

$f_x = 3x^2y^2$ — $f_{xx} = 6xy^2$, $f_{xy} = 6x^2y$

$f_y = 2x^3y$ — $f_{yx} = 6x^2y$, $f_{yy} = 2x^3$

$\left.\begin{array}{l} f_{xy} = 6x^2y \\ f_{yx} = 6x^2y \end{array}\right]$ same

they will always be if f_{xy} f_{yx} are cont's

HW pg 904

\# 11-27 odd

29, 31, 33, 43, 45, 49