

Best Practices in Math: Math Language and Fluency (Grades 6-8)



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Say hello.

Describe your role as an
educator and the mathematics
you support.





Schedule for This Year

December 2022

Best Practices in Math:
Math Language and Fluency

January 24, 2023

Best Practices in Math:
Modeling and Practice
Word-Problem Solving

TBD

Best Practices in Math:
Use of Multiple Representations



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving
instruction



Mathematical Language



Instructional Platform

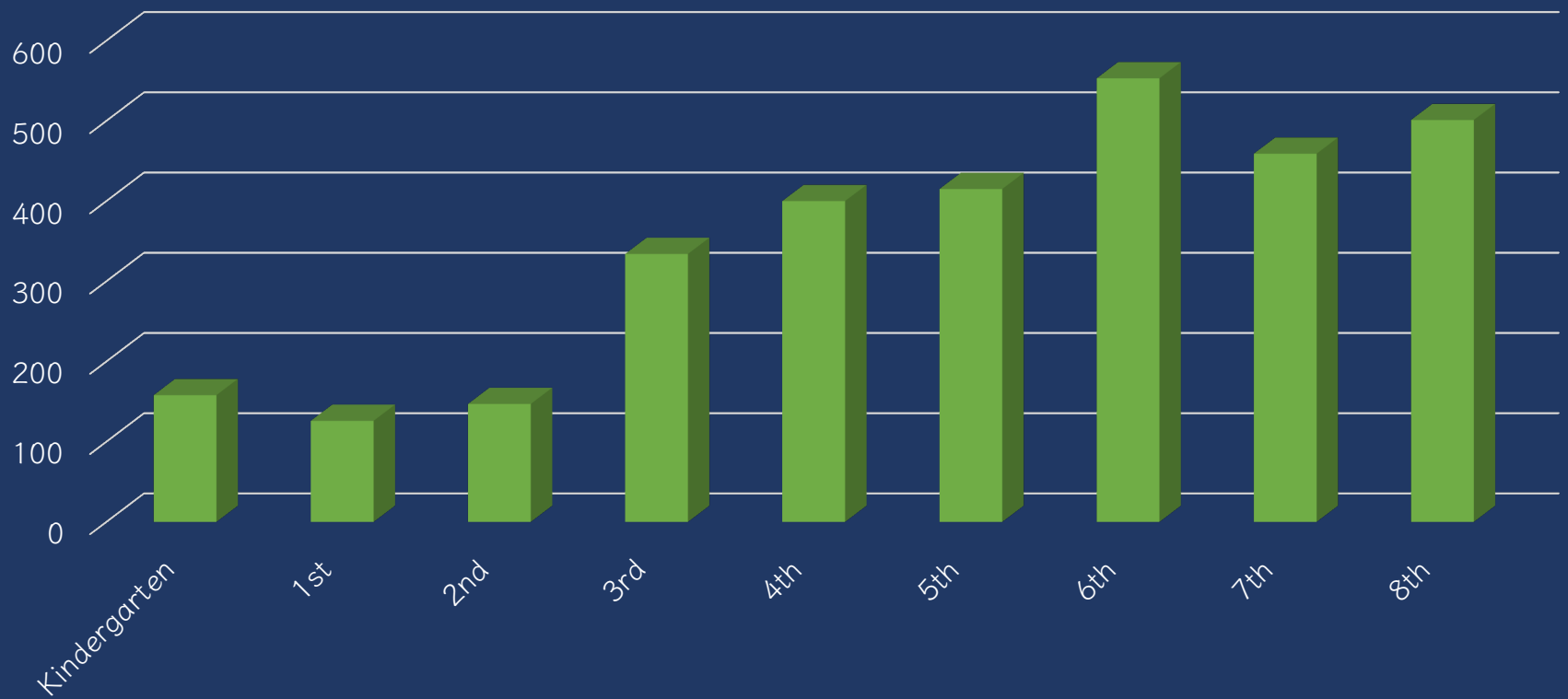
INSTRUCTIONAL DELIVERY

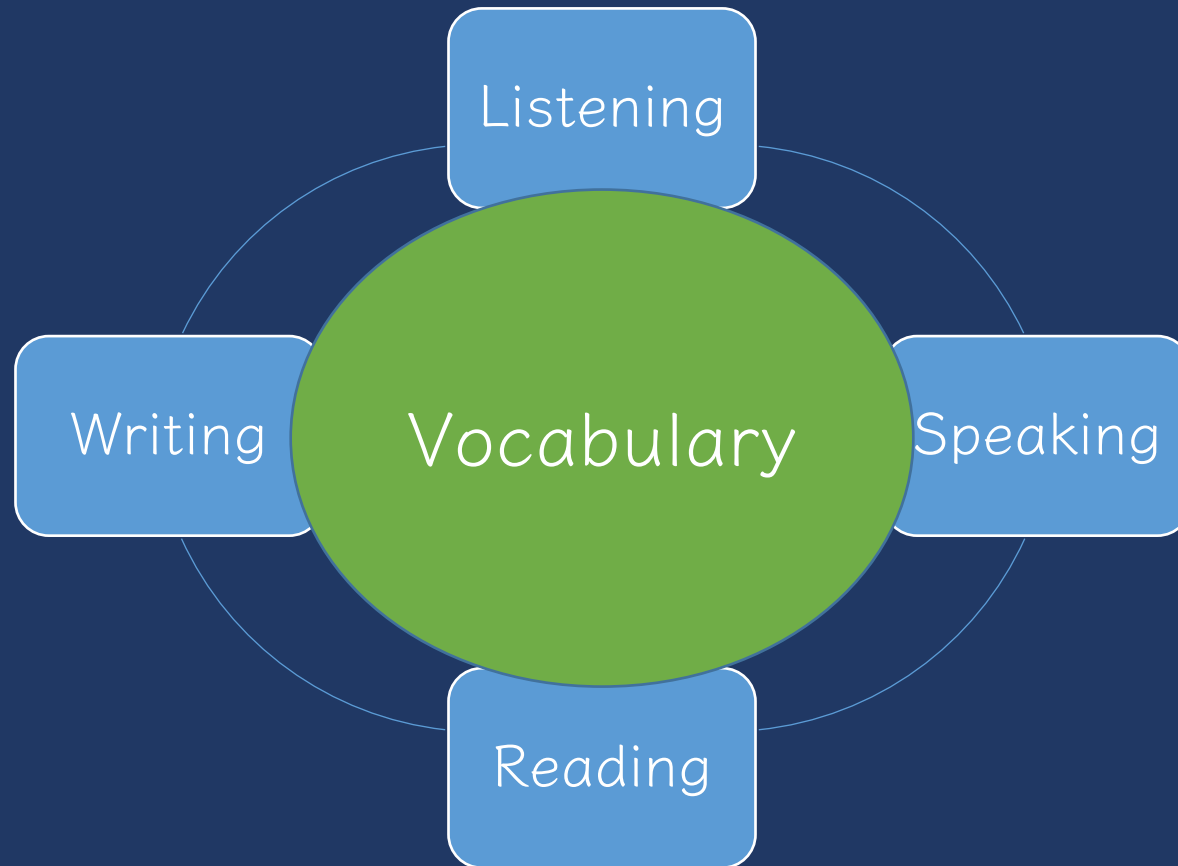
Explicit
instruction

Precise
language

INSTRUCTIONAL STRATEGIES









Best Practices in Math: Mathematics Language and Fluency Grades 6-8

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Difficulties with Mathematics Vocabulary

Use Formal Mathematics Language

Instead of that...	Say this...



1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
(but a more precise math meaning)

difference

even

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings
(but a more precise math meaning)
3. Some math terms are only used in math

trapezoid

numerator

parallelogram

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings
(but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning

round

square

second

base

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings

divide vs.
Continental
Divide

variable vs.
variably
cloudy

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

Rubenstein & Thompson (2002)



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2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings

factor vs.
multiple

hundreds vs.
hundredths

numerators
vs.
denominator

Rubenstein & Thompson (2002)



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4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings

mesa vs.
tabla

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
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(but a more precise math meaning)
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8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities

four vs. forty

Rubenstein & Thompson (2002)



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7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities
10. Some math concepts are verbalized in more than one way

skip count
vs. multiples

one-fourth
vs. one
quarter



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7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities
10. Some math concepts are verbalized in more than one way
11. Informal terms may be used for formal math terms

rhombus vs.
diamond

vertex vs.
corner





What are the ways you see your students experience difficulty with the vocabulary of math?






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Difficulties with Mathematics Vocabulary

Use Formal Mathematics Language

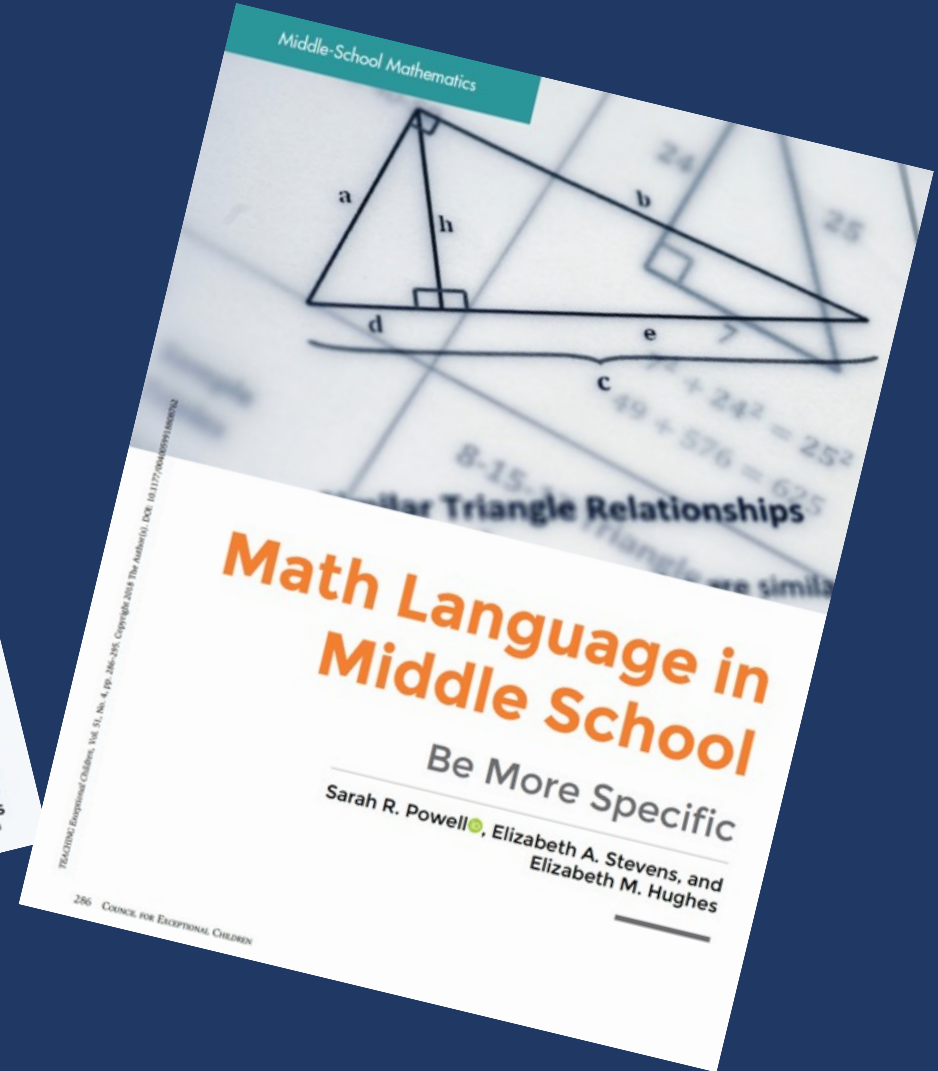
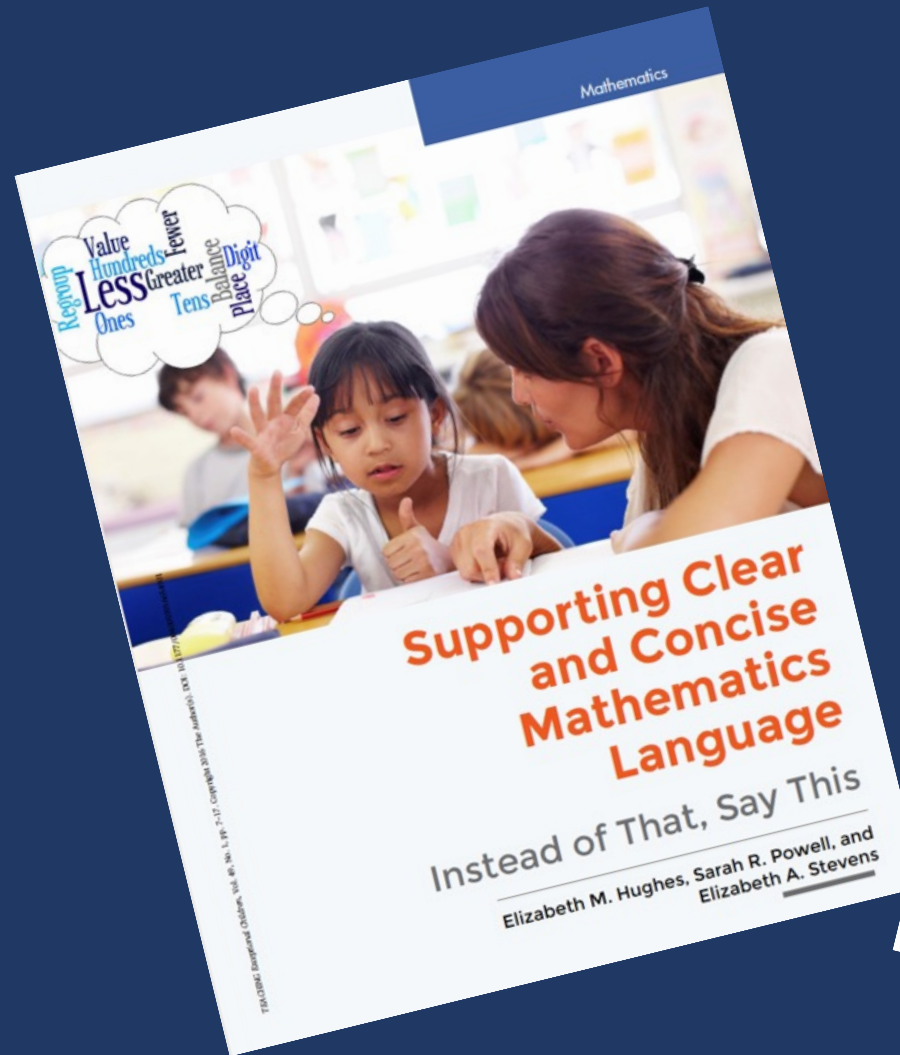
Instead of that...	Say this...
	




Use formal math language


Use terms precisely







What number is in
the tens place?




What digit is in the tens place?
What is the value of the digit in
the tens place?


135

Why this is important...

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.



The alligator eats the
bigger number



is less than
OR
is greater than

Why this is important...

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because $<$ and $>$ are two distinct symbols.





carry OR borrow



regroup OR
trade OR
exchange



$$\begin{array}{r} 167 \\ + 294 \\ \hline \end{array}$$

Why this is important...

- “Carry” or “borrow” is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.



top number and
bottom number



numerator and
denominator

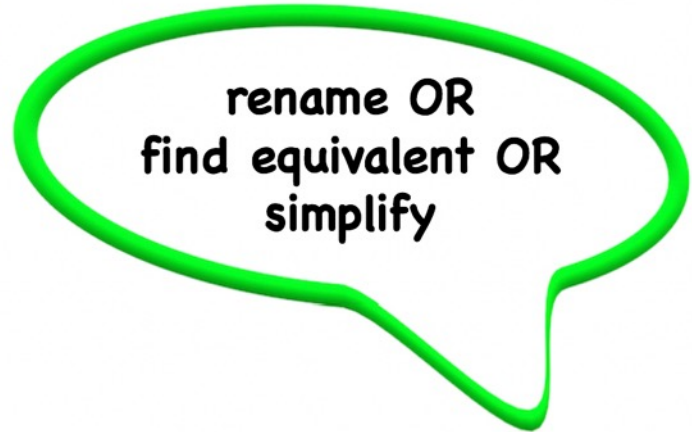

Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.





reduce the fraction




rename OR
find equivalent OR
simplify

Why this is important...

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.





Four point seven
Four point oh seven

Four and seven tenths
Four and seven hundredths



4.7
4.07

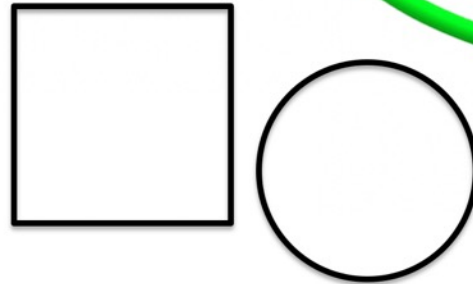
Why this is important...

- Accurately shares the magnitude of the decimal.
- Emphasizes place value.



box OR ball

square OR
circle

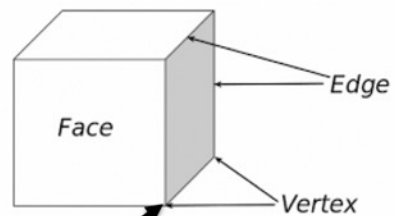


Why this is important...

- Use the formal language of shapes to confirm informal language.

point

vertex



Why this is important...

- This is the endpoint where two or more line segments or rays meet.



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Difficulties with Mathematics Vocabulary

--

Use Formal Mathematics Language

Instead of that...	Say this...



Identify examples of
“Instead of ____, say
____.”



Use formal math language

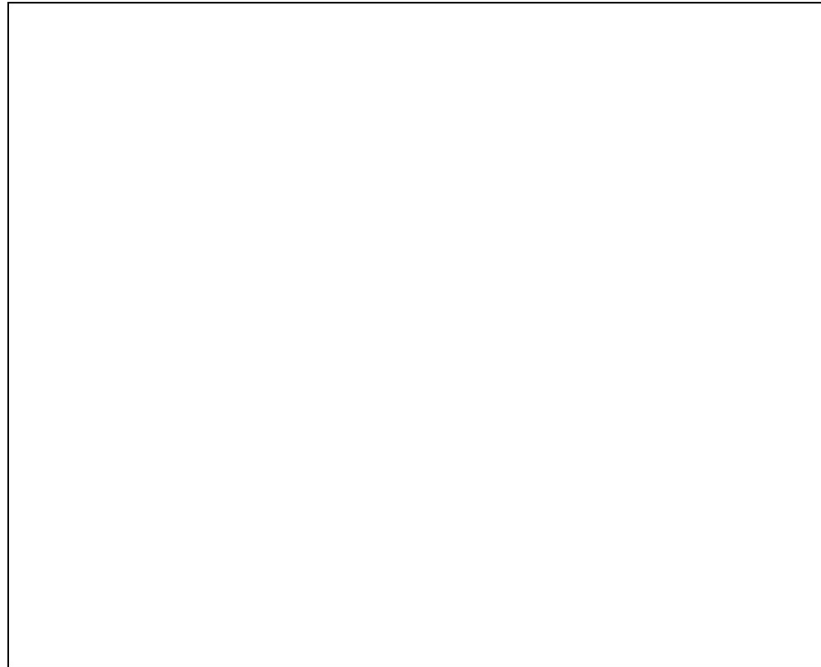
Use terms precisely



Use Mathematics Vocabulary With Precision



Strategies for Teaching Mathematics Language



Factor

$$1 \times 8 = 8$$

$$2 \times 4 = 8$$

factor *factor*

Multiple

$$8 \times 1 = 8$$

$$8 \times 2 = 16$$

multiples of 8

E



Improper fraction

$$\frac{8}{5}$$

Proportion

$$\frac{2}{5} = \frac{8}{20}$$

Mixed number

$$1\frac{3}{5}$$

Ratio

$$4:3$$

Proper fraction

$$\frac{2}{9}$$

Unit fraction

$$\frac{1}{6}$$

D



Coefficient
Constant
Term
Variable

$$\begin{array}{ccc} \text{term} & & \text{term} & & \text{term} \\ \text{---} & & \text{---} & & \text{---} \\ 2x^2 + x - 3 \\ \text{coefficient} & & \text{variable} & & \text{constant} \end{array}$$

A



Equation $9x - 4 = 7x$

Expression $9x - 4$

Formula $a^2 + b^2 = c^2$

Function $f(x)$

Inequality $9x - 4 > 6x$

c



Quadrilaterals

Kite



Rhombus



Parallelogram



Square



Rectangle

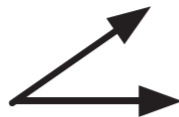


Trapezoid

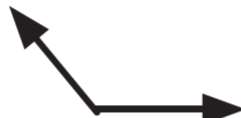


A

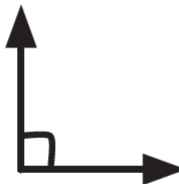
Acute angle



Obtuse angle



Right angle



Straight angle



B



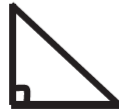
Acute triangle



Obtuse triangle



Right triangle



Equilateral triangle



Isosceles triangle



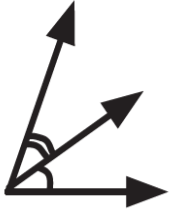
Scalene triangle



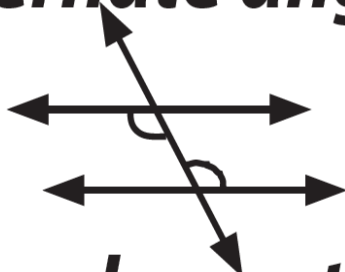
C



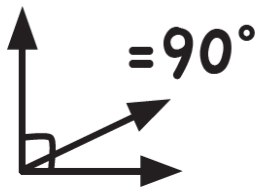
Adjacent angles



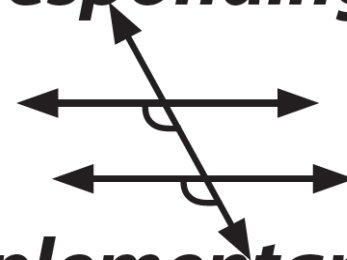
Alternate angles



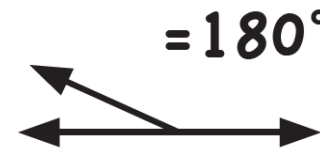
Complementary angles



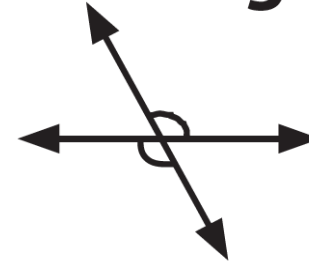
Corresponding angles



Supplementary angles



Vertical angles



D

Congruent figures

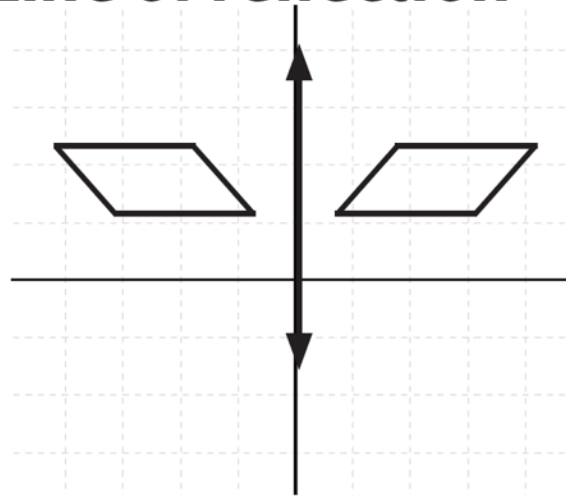


Similar figures

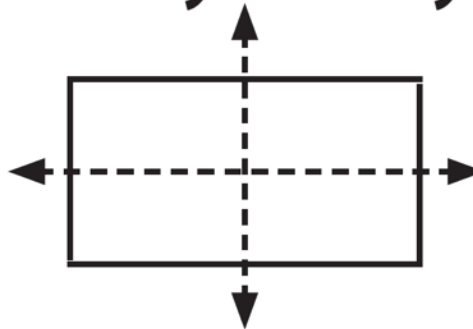


E

Line of reflection

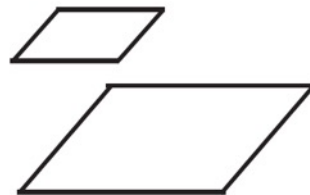


Line of symmetry



f

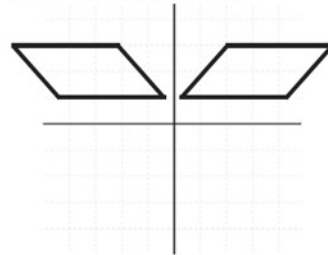
Dilation



Scale factor

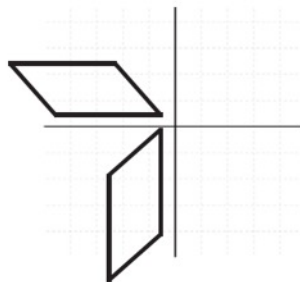
← scale factor
is 1:2

Reflection

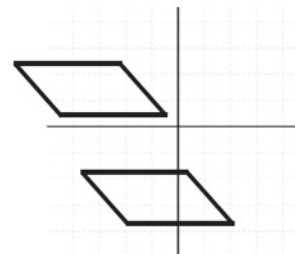


Transformation

Rotation

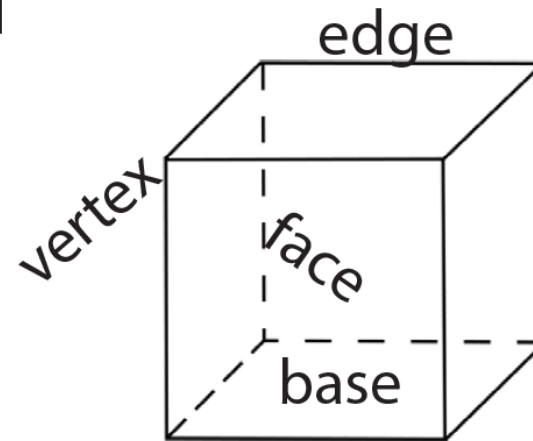
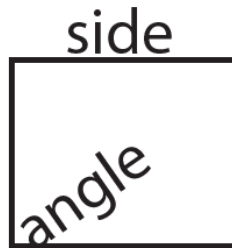


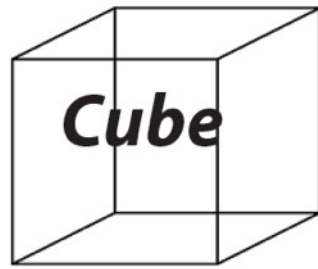
Translation



G

Angle
Base
Edge
Face
Side
Vertex





Cube



Prism



Pyramid



Cylinder



Sphere

I

Coordinate plane

Ordered pair Quadrant 2

Quadrants

x

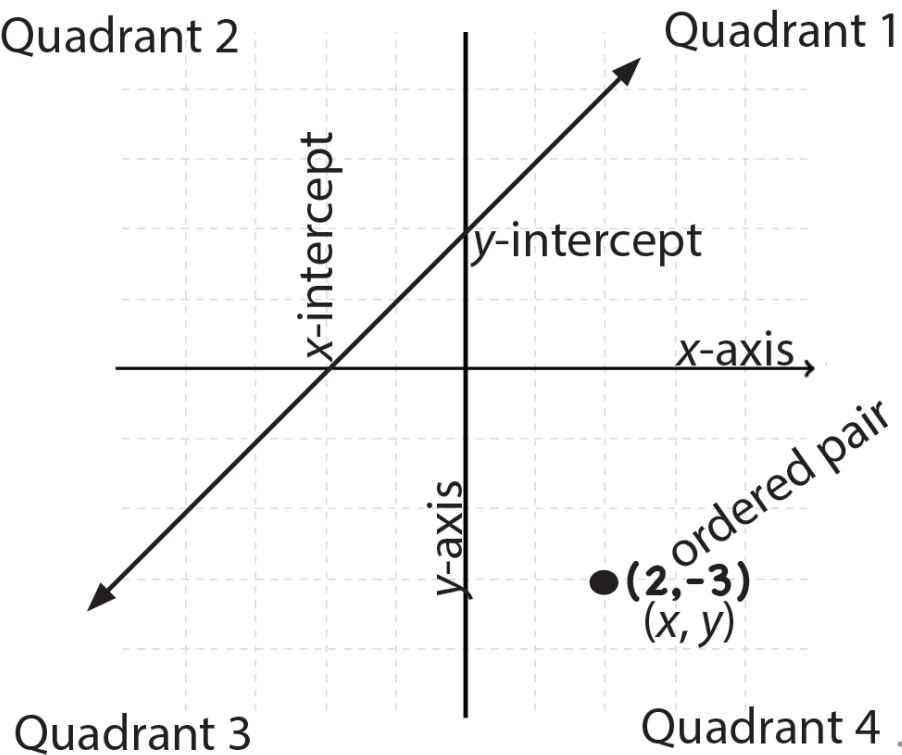
x-axis

x-intercept

y

y-axis

y-intercept



Use Mathematics Vocabulary With Precision

Strategies for Teaching Mathematics Language



Discuss terms you want
your students to use
with precision.



Use formal math language

Use terms precisely



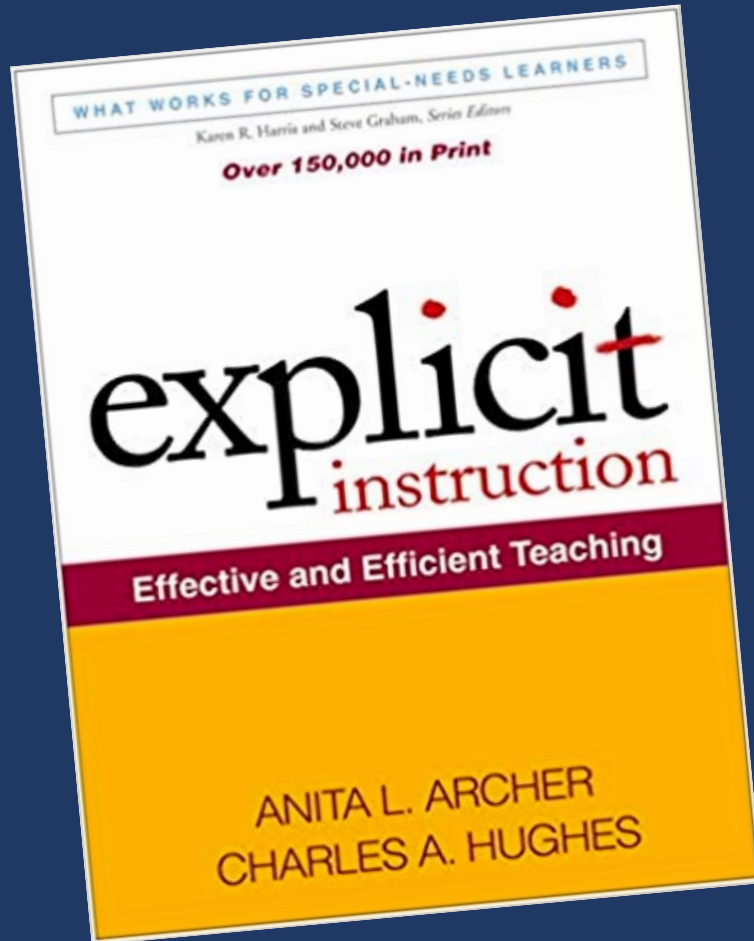
Use Mathematics Vocabulary With Precision



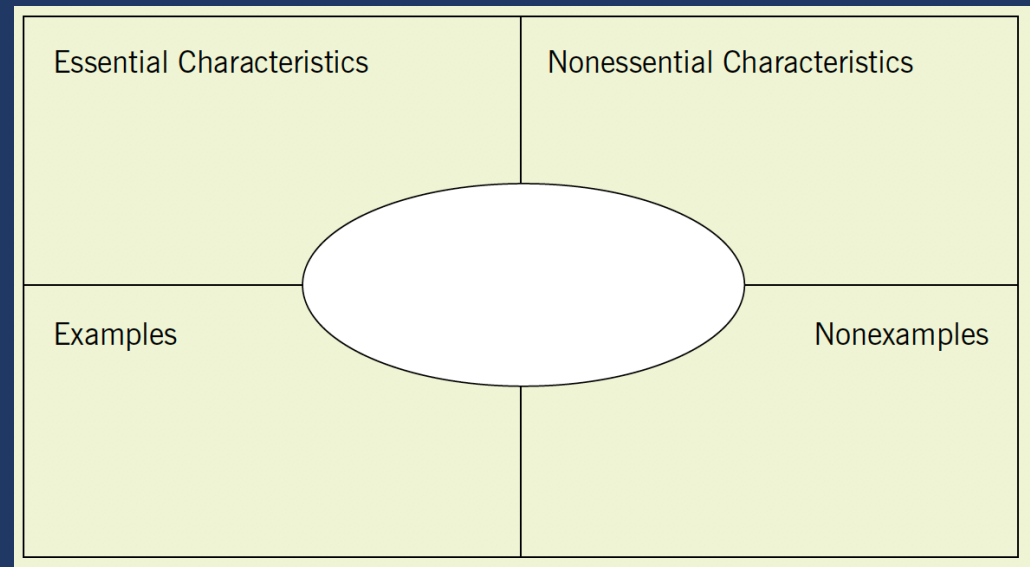
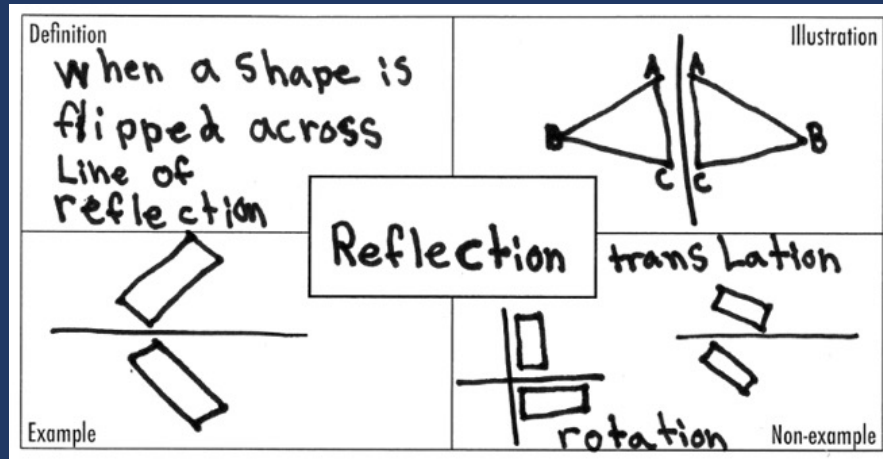
Strategies for Teaching Mathematics Language



1. Use explicit instruction



2. Use graphic organizers



Dunston & Tyminski (2013)



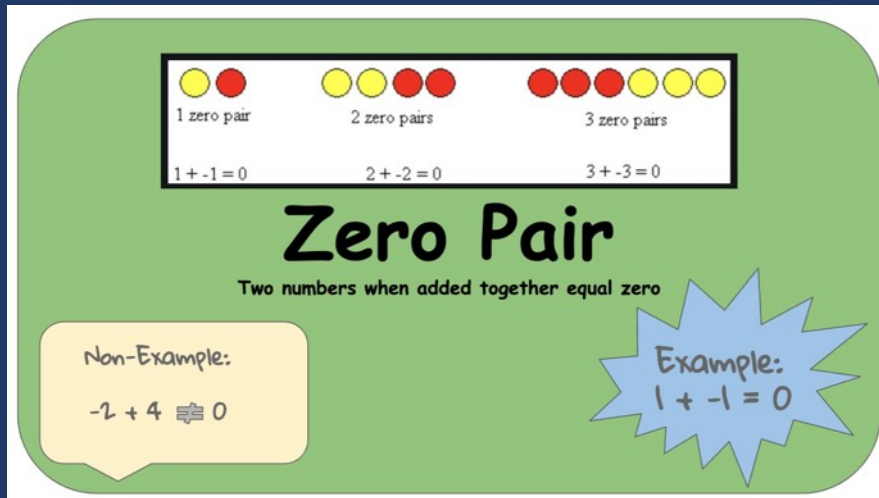
2. Use graphic organizers

Word	Lightbulb Word
Definition	Picture

Dunston & Tyminski (2013)



3. Have students create vocabulary cards



A green-bordered card titled "Zero Pair" with the subtitle "Two numbers when added together equal zero". At the top, it shows three examples of zero pairs using colored circles: 1 pair (1 yellow, 1 red), 2 pairs (2 yellow, 2 red), and 3 pairs (3 yellow, 3 red). Below each pair is the equation $1 + -1 = 0$, $2 + -2 = 0$, and $3 + -3 = 0$ respectively. In the bottom left, a yellow box labeled "Non-Example:" shows $-2 + 4 \neq 0$. In the bottom right, a blue starburst labeled "Example:" shows $1 + -1 = 0$.

1 zero pair 2 zero pairs 3 zero pairs

$1 + -1 = 0$ $2 + -2 = 0$ $3 + -3 = 0$

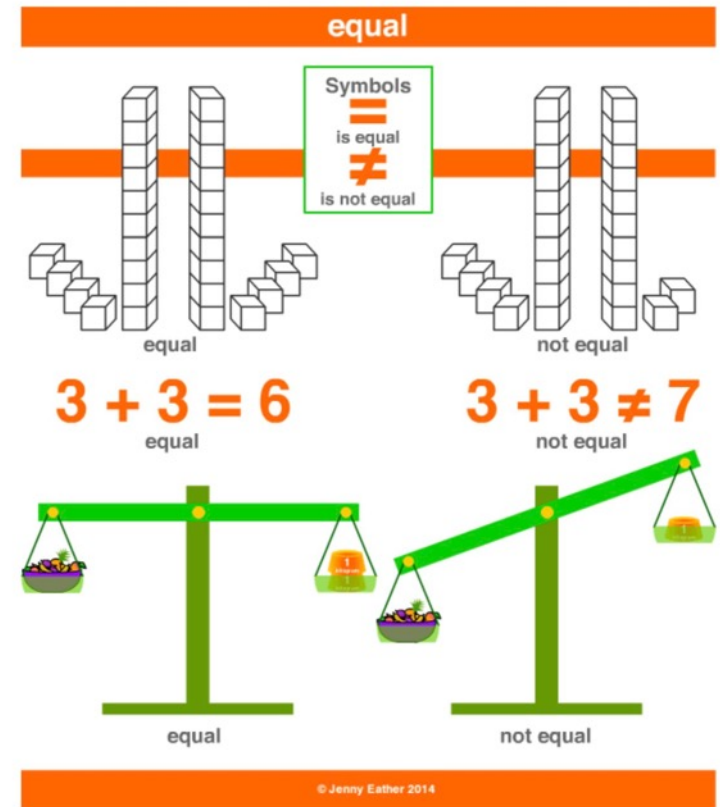
Zero Pair

Two numbers when added together equal zero

Non-Example:
 $-2 + 4 \neq 0$

Example:
 $1 + -1 = 0$

6. **Equal**: having the same amount or value.



A white card with orange borders illustrating the concept of equality. At the top, an orange bar says "equal". Below it, two sets of blocks are shown. The first set has two equal stacks of 3 blocks each, labeled "equal" with the equation $3 + 3 = 6$. The second set has two unequal stacks (one of 3, one of 4), labeled "not equal" with the equation $3 + 3 \neq 7$. A green box in the center lists symbols: "=" is equal, and " \neq " is not equal. At the bottom, two balance scales are shown. The first is balanced, labeled "equal". The second is tilted, labeled "not equal". An orange bar at the very bottom says "© Jenny Esther 2014".

equal

Symbols
= is equal
 \neq is not equal

equal not equal

$3 + 3 = 6$ $3 + 3 \neq 7$

equal not equal

© Jenny Esther 2014

4. Have students create glossaries


Integer Definitions

Zero Pairs
A positive and negative cancel one another;

Positive
A number that is greater than zero.

Absolute Value
The distance of a number from zero on a number line; shown as $||$

Negative
A number that is less than zero. Identified by a minus sign.



Numerator: how many parts of the whole

- Ex. $\frac{4}{10}$

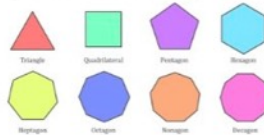
Odd number: a number not divided evenly by 2

- Ex. 1, 3, 5, 7, 9....

Percent: a specific number in comparison to 100

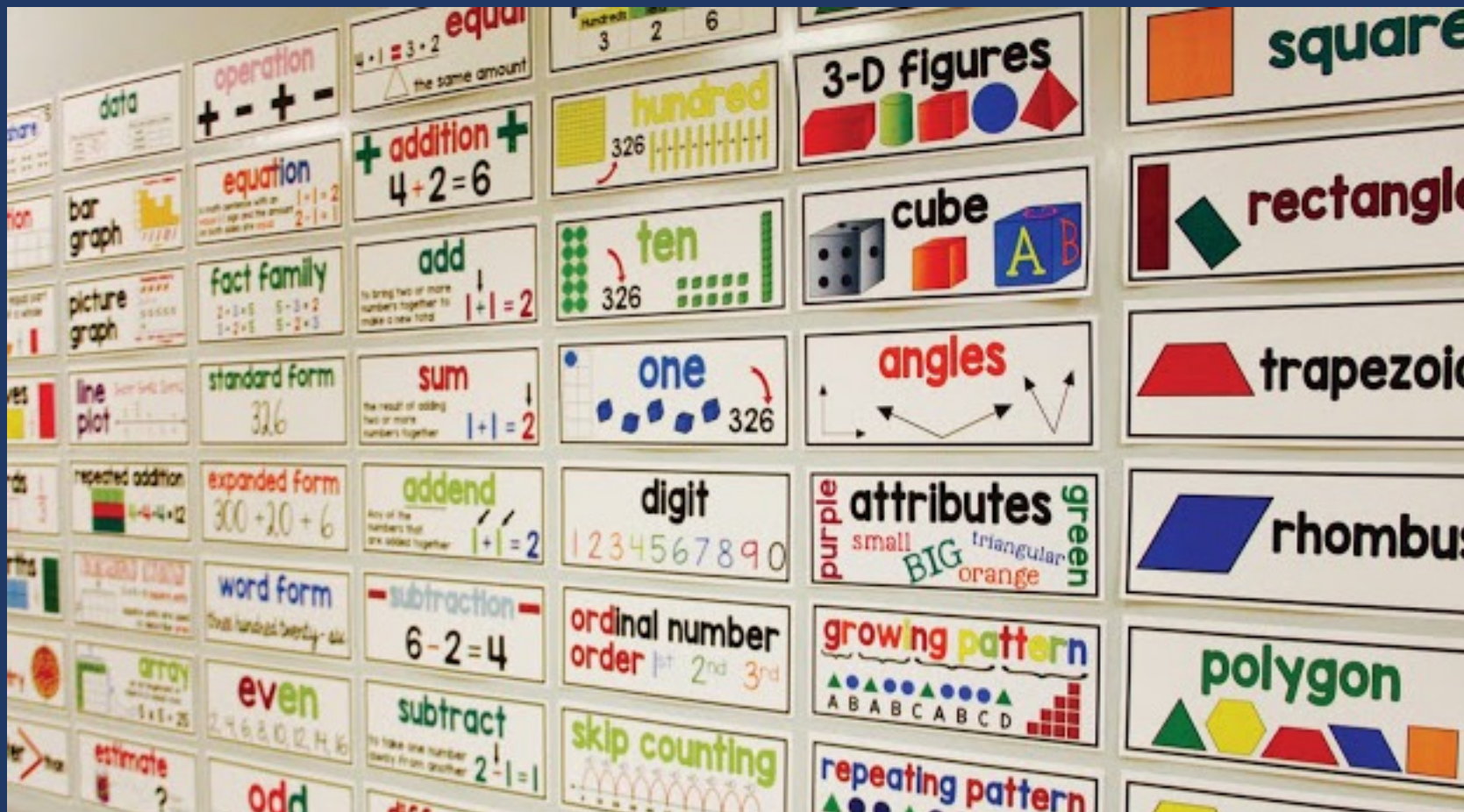
- 74%

Polygon: any enclosed shape that is made up of 3 or more straight lines

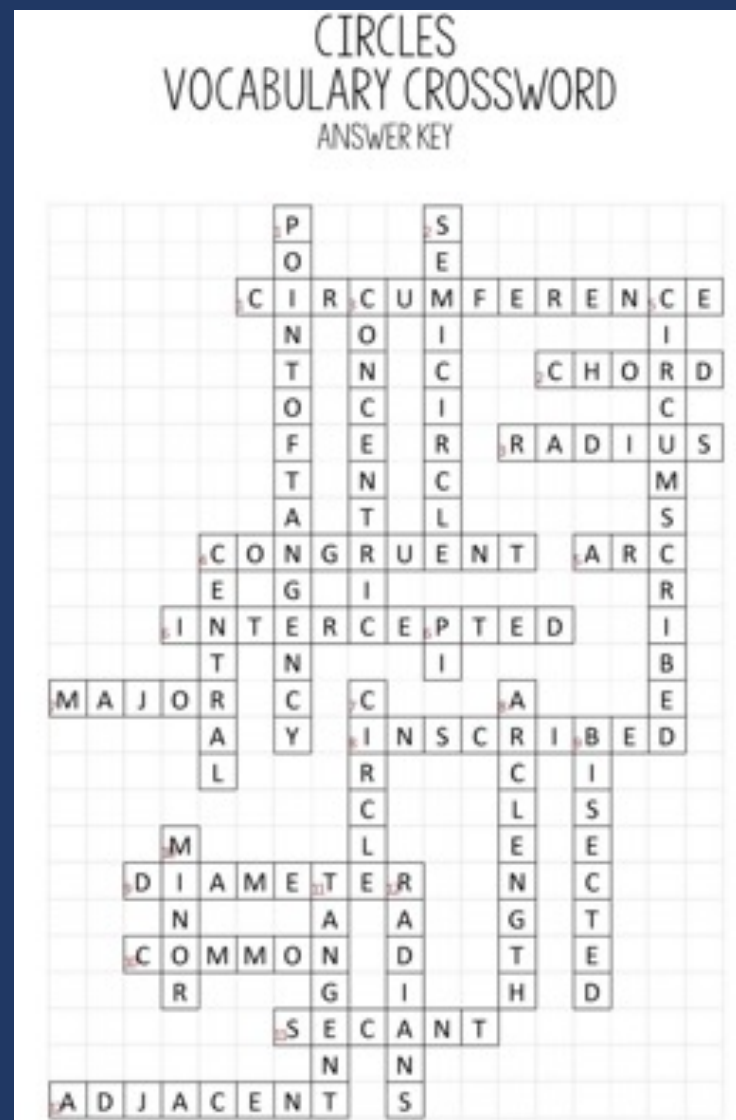
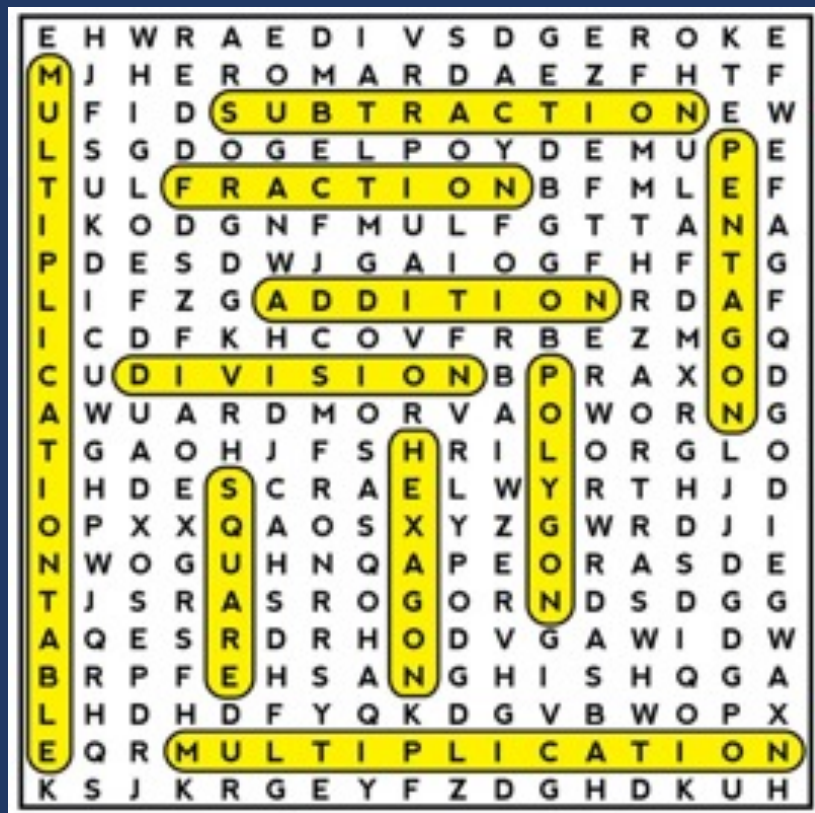


- Ex.

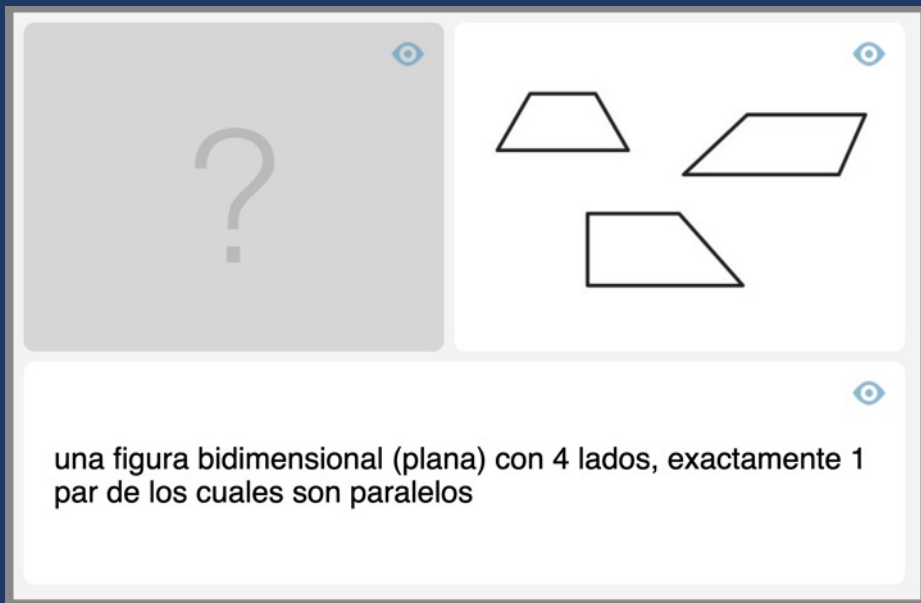
5. Create a word wall



6. Do word games



7. Use technology



una figura bidimensional (plana) con 4 lados, exactamente 1 par de los cuales son paralelos

Math Learning Center



Houghton Mifflin Math eGames

Math Lingo

How to Play New Game

60 minutes = 1 ____

change	minute hand	hour hand
hour	equal amounts	second
quarter-hour	half-hour	minute

Math Lingo



Use Mathematics Vocabulary With Precision

Strategies for Teaching Mathematics Language



Discuss your strategy
for focusing on
mathematical language
in your teaching.



Fluency



Fluency



What is your mathematical language goal for January?

What is your fluency goal for January?

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

INSTRUCTIONAL STRATEGIES

Fluency building



Building Fluency

Fluency is
doing
mathematics
easily and
accurately.

Fluency in
mathematics
makes
mathematics
easier.

Fluency
provides less
stress on
working
memory.

Fluency
helps
students
build
confidence
with
mathematics.

With fluency, it is important to emphasize both
conceptual learning and procedural learning.



Addition	Subtraction
Multiplication	Division

Counting

Comparing
numbers

Counting
coins

Telling
time

Identifying
equivalent
fractions

Knowing
multiples

Identifying
shapes

Knowing
formulas



100 addition facts

Single-digit addends sum to a single- or double-digit number

$$\begin{array}{r} 5 \\ + 4 \\ \hline 9 \end{array}$$

(addend)
(addend)
(sum)



100 subtraction facts

Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

$$\begin{array}{r} 16 \\ - 8 \\ \hline 8 \end{array}$$

(minuend)
(subtrahend)
(difference)



100 multiplication facts

Multiplication of single-digit factors results in a single- or double-digit product

2	(<u>factor</u>)
$\times 3$	(factor)
<hr/>	
6	(<u>product</u>)



90 division facts

Divisor and quotient are single-digit numbers
and dividend is single- or double-digit number

$$8 \div 4 = 2$$

(dividend) (divisor) (quotient)



Addition	Subtraction
Multiplication	Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

$$\begin{array}{r} 5 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ - 4 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 56 \\ \div 8 \\ \hline \end{array}$$



Cover, Copy, Compare

$$\begin{array}{r} 9 \\ \times 6 \\ \hline 54 \end{array}$$

$$\begin{array}{r} 7 \\ \times 8 \\ \hline 56 \end{array}$$

$$\begin{array}{r} 9 \\ \times 9 \\ \hline 81 \end{array}$$

$$\begin{array}{r} 6 \\ \times 7 \\ \hline 42 \end{array}$$

$$\begin{array}{r} 8 \\ \times 8 \\ \hline 64 \end{array}$$

$$\begin{array}{r} 8 \\ \times 6 \\ \hline 48 \end{array}$$

$$\begin{array}{r} 6 \\ \times 5 \\ \hline 30 \end{array}$$

$$\times \begin{array}{r} 6+4= \end{array}$$

$$7+3=$$

$$2+7=$$

$$5+6=$$

$$4+7=$$

$$7+8=$$

$$6+7=$$

$$7+9=$$

$$7+6=$$

$$8+7=$$

$$7+0=$$

$$9+6=$$

$$6+0=$$

$$6+8=$$

File Folder

$$6+3=$$

$$1+7=$$

Taped Problems

$$\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$$

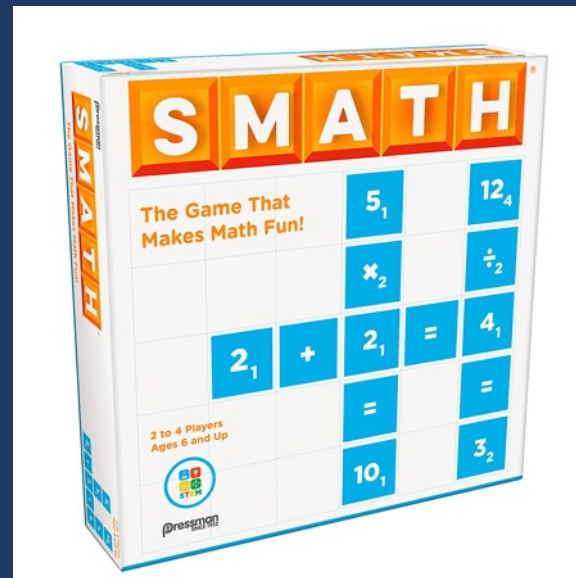
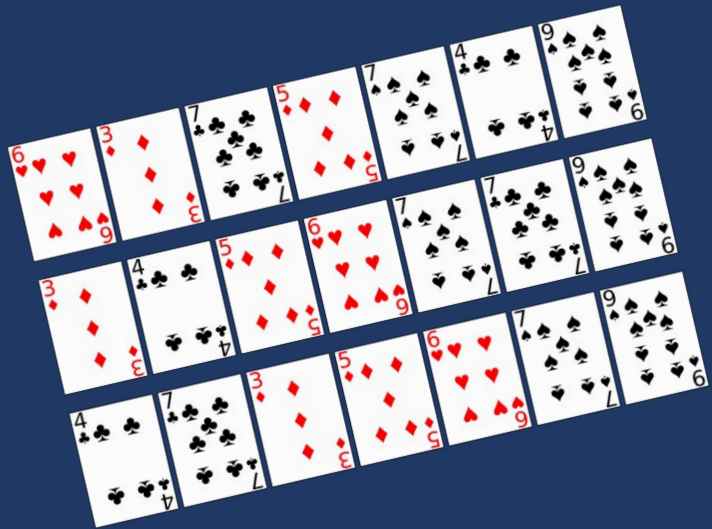
$$\begin{array}{r} 4 \\ \times 8 \\ \hline \end{array}$$

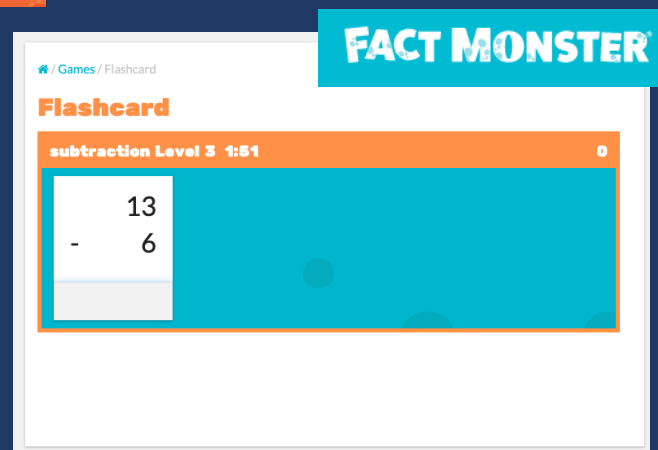
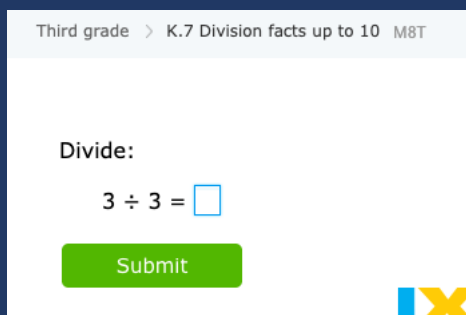
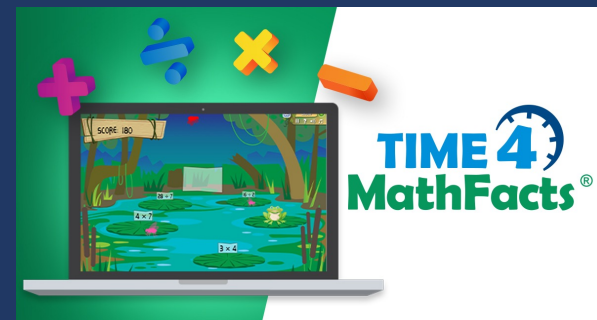
$$\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$$





[illegible]





Addition

Subtraction

Multiplication

Division

Build fluency with whole-number computation

$$\begin{array}{r} 15 \\ + 28 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 1009 \\ - 724 \\ \hline \end{array}$$

$$\begin{array}{r} 7250 \\ \div 15 \\ \hline \end{array}$$



Addition

Subtraction

Multiplication

Division

Build fluency with rational-number computation

$$\begin{array}{r} 1.4 \\ + 3.9 \\ \hline \end{array}$$

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{9}{4} - \frac{3}{8}$$

$$\begin{array}{r} 7.892 \\ \div 0.14 \\ \hline \end{array}$$



Addition

Subtraction

Multiplication

Division

Build fluency with integer computation

$$\begin{array}{r} 1.4 \\ + -3.9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times -12 \\ \hline \end{array}$$

$$-14 - (-7) =$$

$$-135 \div 2 =$$



Addition	Subtraction
Multiplication	Division



What fluency practice do your students need?



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

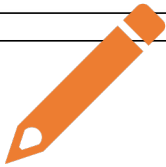
INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving
instruction



Fluency



What is your mathematical language goal for January?

What is your fluency goal for January?



What is your mathematical language goal for January?



What is your fluency goal for January?





Schedule for This Year

December 2022

Best Practices in Math:
Math Language and Fluency

January 24, 2023

Best Practices in Math:
Modeling and Practice
Word-Problem Solving

TBD

Best Practices in Math:
Use of Multiple Representations



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