

#NCSIP2022

2022 NCSIP NETWORK CONFERENCE

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North Carolina Department of
PUBLIC INSTRUCTION



Critical Math Content Across Grades K-8



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Introduce yourself.
Describe your role as an educator.
Describe the mathematics you support.



Share fun things from today and tag
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Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

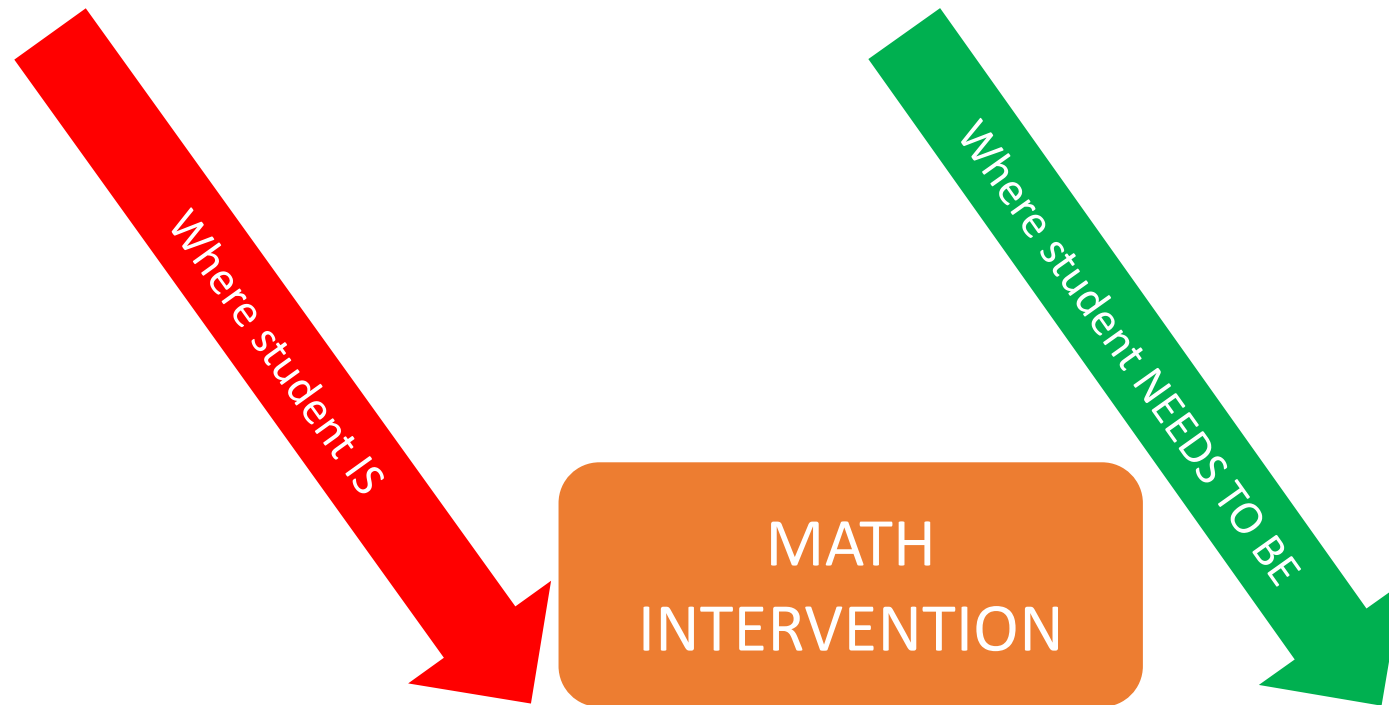
INSTRUCTIONAL STRATEGIES

Fluency
building

Problem solving
instruction



Determine critical content



Fluency



Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division...

Fluently multiply multi-digit whole numbers using the standard algorithm.

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships.

Fluently add and subtract within 5.

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm.



MATH INTERVENTION

Where student IS

Where student NEEDS TO BE

Fluently add and subtract within 5.

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships.

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division...

Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Fluently multiply multi-digit whole numbers using the standard algorithm.

Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm.





At your grade level, what is the critical content related to fluency and operations?

Fluency

Operations



Place Value



Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.

Understand that the two digits of a two-digit number represent amounts of tens and ones.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

Recognize that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right...

Compose and decompose numbers from 11 to 19 into ten ones and some further ones...

Use place value understanding to round whole numbers to the nearest 10 or 100.



Where student IS

MATH INTERVENTION

Where student NEEDS TO BE

Compose and decompose numbers from 11 to 19 into ten ones and some further ones...

Understand that the two digits of a two-digit number represent amounts of tens and ones.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

Use place value understanding to round whole numbers to the nearest 10 or 100.

Recognize that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right...

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.





At your grade level, what is the critical content related to place value?

Place Value



Problem Solving



Interpret and compute
quotients of fractions,
and solve word
problems involving
division of fractions by
fractions...

Solve multi-step
word problems
posed with whole
numbers and
having whole-
number answers
using the four
operations...

Solve word problems that
call for addition of three
whole numbers whose
sum is less than or equal
to 20...

Solve real-world and
mathematical problems
involving the four operations
with rational numbers.

Solve real-world
and mathematical
problems leading
to two linear
equations in two
variables.

Use multiplication and
division within 100 to
solve word problems...

Solve addition and
subtraction word
problems, and add and
subtract within 10...

Use addition and subtraction
within 100 to solve one- and
two-step word problems...

Solve word problems involving
addition and subtraction of
fractions referring to the same
whole, including cases of
unlike denominators...



Solve addition and subtraction word problems, and add and subtract within 10...

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20...

Use addition and subtraction within 100 to solve one- and two-step word problems...

Use multiplication and division within 100 to solve word problems...

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations...

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators

...

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions...

Solve real-world and mathematical problems involving the four operations with rational numbers.

Solve real-world and mathematical problems leading to two linear equations in two variables.





At your grade level, what is the critical content related to problem solving?

Problem Solving



Where student IS

Where student NEEDS TO BE

Use addition and subtraction within 100 to solve one- and two-step word problems...

Use multiplication and division within 100 to solve word problems...

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations...



Where student IS

Where student NEEDS TO BE

Explain addition and subtraction strategies, work, place value and properties of operations.

Understand that the two digits of a two-digit number represent amounts of tens and ones.

Use addition and subtraction within 100 to solve one- and two-step word problems...

Fluently add and subtract within 100 using strategies based on place value, properties of operations and/or relationships.

Apply properties of operations as strategies to multiply and divide.

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division...

Use multiplication and division within 100 to solve word problems...

Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Understand the place value of digits in multi-digit numbers and the relationship between addition, subtraction, multiplication and division.

Use whole number multiplication and division to solve problems involving multiplication and division.

Fluently multiply multi-digit whole numbers using the standard algorithm.

Use multi-digit multiplication and division to solve problems involving multiplication and division.



K-12

Vertical Progression

FOR THE NC STANDARD COURSE OF STUDY IN MATHEMATICS



Number and Operations in Base Ten	
Build foundation for place value.	
NC.K.NBT.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones by: <ul style="list-style-type: none"> Using objects or drawings. Recording each composition or decomposition by a drawing or expression. Understanding that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
Extend and recognize patterns in the counting sequence.	
NC.1.NBT.1	Count to 150, starting at any number less than 150.
NC.1.NBT.7	Read and write numerals, and represent a number of objects with a written numeral, to 100.
Understand place value.	
NC.1.NBT.2	Understand that the two digits of a two-digit number represent amounts of tens and ones. <ul style="list-style-type: none"> Unitize by making a ten from a collection of ten ones. Model the numbers from 11 to 19 as composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. Demonstrate that the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens, with 0 ones.
NC.1.NBT.3	Compare two two-digit numbers based on the value of the tens and ones digits, recording the results of comparisons with the symbols $>$, $=$, and $<$.
Use place value understanding and properties of operations.	
NC.1.NBT.4	Using concrete models or drawings, strategies based on place value, properties of operations, and explaining the reasoning used, add, within 100, in the following situations: <ul style="list-style-type: none"> A two-digit number and a one-digit number A two-digit number and a multiple of 10
NC.1.NBT.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
NC.1.NBT.6	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90, explaining the reasoning, using: <ul style="list-style-type: none"> Concrete models and drawings Number lines Strategies based on place value Properties of operations The relationship between addition and subtraction
Understand place value.	
NC.2.NBT.1	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. <ul style="list-style-type: none"> Unitize by making a hundred from a collection of ten tens. Demonstrate that the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds, with 0 tens and 0 ones. Compose and decompose numbers using various groupings of hundreds, tens, and ones.
NC.2.NBT.2	Count within 1,000; skip-count by 5s, 10s, and 100s.
NC.2.NBT.3	Read and write numbers, within 1,000, using base-ten numerals, number names, and expanded form.



<https://www.dpi.nc.gov/media/465/open>

An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

K	1	2	3	4	5	6	7	8
Know number names and the count sequence	Represent and solve problems involving addition and subtraction	Represent and solve problems involving addition and subtraction	Represent & solve problems involving multiplication and division	Use the four operations with whole numbers to solve problems	Understand the place value system	Apply and extend previous understandings of multiplication and division to divide fractions by fractions	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers	Work with radical and integer exponents
Count to tell the number of objects	Understand and apply properties of operations and the relationship between addition and subtraction	Add and subtract within 20	Understand properties of multiplication and the relationship between multiplication and division	Generalize place value understanding for multi-digit whole numbers	Perform operations with multi-digit whole numbers and decimals to hundredths	Apply and extend previous understandings of numbers to the system of rational numbers	Analyze proportional relationships and use them to solve real-world and mathematical problems	Understand the connections between proportional relationships, lines, and linear equations**
Compare numbers	Add and subtract within 20	Use place value understanding and properties of operations to add and subtract	Multiply & divide within 100	Use place value understanding and properties of operations to perform multidigit arithmetic	Use equivalent fractions as a strategy to add and subtract fractions	Understand ratio concepts and use ratio reasoning to solve problems	Use properties of operations to generate equivalent expressions	Analyze and solve linear equations and pairs of simultaneous linear equations
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	Work with addition and subtraction equations	Measure and estimate lengths in standard units	Solve problems involving the four operations, and identify & explain patterns in arithmetic	Extend understanding of fraction equivalence and ordering	Apply and extend previous understandings of multiplication and division to multiply and divide fractions	Apply and extend previous understandings of arithmetic to algebraic expressions	Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Define, evaluate, and compare functions
Work with numbers 11–19 to gain foundations for place value	Extend the counting sequence	Relate addition and subtraction to length	Develop understanding of fractions as numbers	Build fractions from unit fractions by applying and extending previous understandings of operations	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	Reason about and solve one-variable equations and inequalities		Use functions to model relationships between quantities
	Understand place value		Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects	Understand decimal notation for fractions, and compare decimal fractions	Graph points in the coordinate plane to solve real-world and mathematical problems*	Represent and analyze quantitative relationships between dependent and independent variables		
	Use place value understanding and properties of operations to add and subtract		Geometric measurement: understand concepts of area and relate area to multiplication and to addition					
	Measure lengths indirectly and by iterating length units							

* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisk exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.

** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.



<https://achievethecore.org/category/774/mathematics-focus-by-grade-level>



Table A.3. Grades 6–8 Curriculum Focal Points and Connections Compared with the Expectations of the Content Standards in *Principles and Standards for School Mathematics*

Curriculum Focal Points and Connections	Expectations of the Content Standards
<p>Grade 6 Curriculum Focal Points</p> <p>Number and Operations: Developing an understanding of and fluency with multiplication and division of fractions and decimals</p> <p>Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain the procedures for multiplying and dividing decimals. Students use common procedures to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multistep problems and problems involving measurement.</p> <p>Number and Operations: Connecting ratio and rate to multiplication and division</p> <p>Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., “If 5 items cost \$3.75 and all items are the same price, then I can find the cost of 12 items by first dividing \$3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12”). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. Thus, they expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students solve a wide variety of problems involving ratios and rates.</p> <p>Algebra: Writing, interpreting, and using mathematical expressions and equations</p> <p>Students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and formulas to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they can rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information). Students know that the solutions of an equation are the values of the variables that</p>	<p>Number and Operations, Grades 6–8</p> <ul style="list-style-type: none"> ● ● Work flexibly with fractions, decimals, and percents to solve problems ● ● Compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line ● Develop meaning for percents greater than 100 and less than 1 ● ● Understand and use ratios and proportions to represent quantitative relationships ● ● Develop an understanding of large numbers [identified in Grades 4 and 5 Curriculum Focal Points] and recognize and appropriately use exponential, scientific, and calculator notation ● Use factors, multiples, prime factorization, and relatively prime numbers to solve problems ● Develop meaning for integers and represent and compare quantities with them ● ● Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers ● ● Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals ● ● Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems ● ● Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods



<https://www.nctm.org/curriculumfocalpoints/>





What is the critical content for the following four areas?

early numeracy



whole numbers



rational numbers



algebra

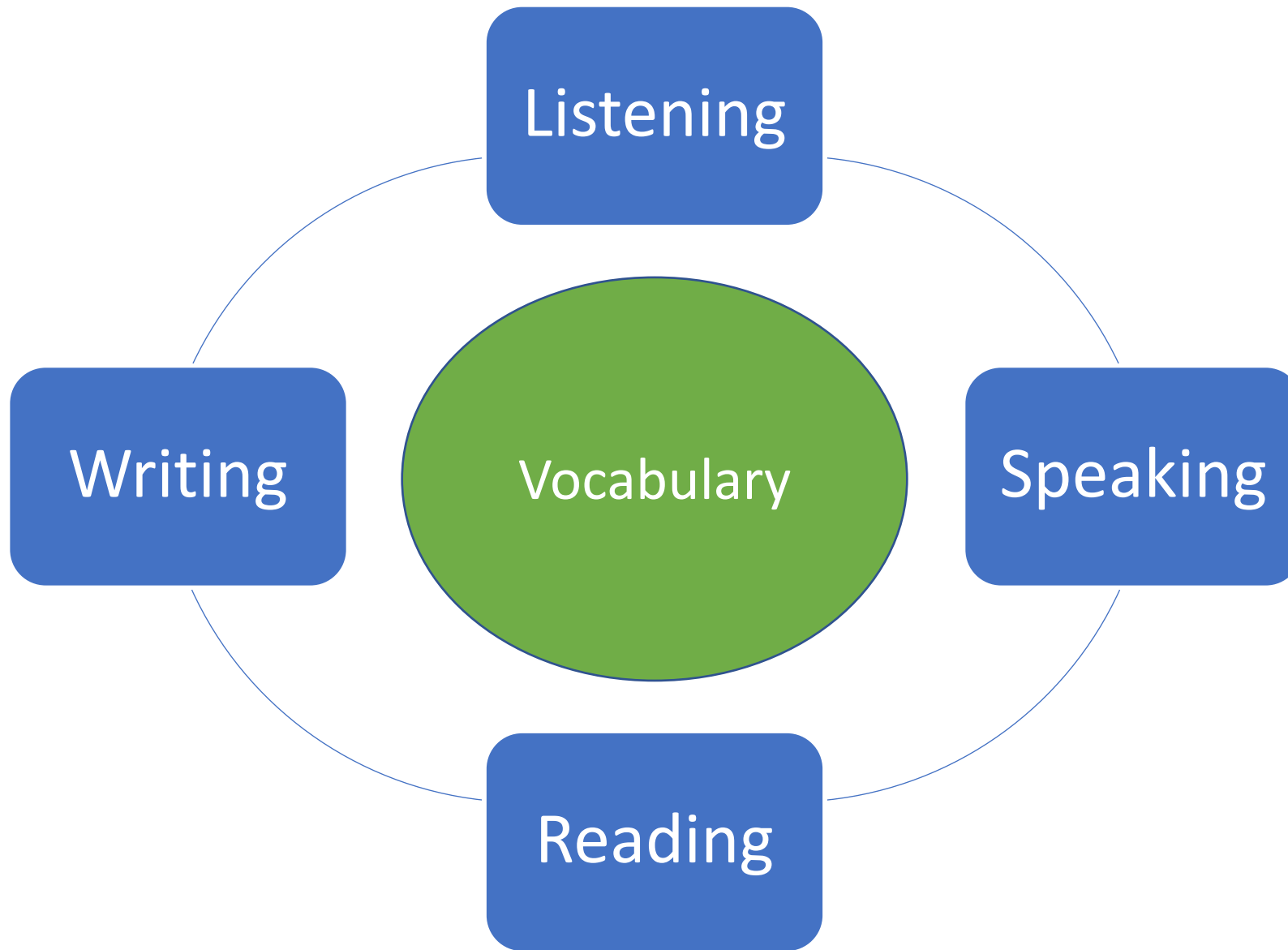




Meet with a group representing a range of grade levels.

Share your critical content and place in order from easier to more difficult mathematics.





Use formal math language

Use terms precisely

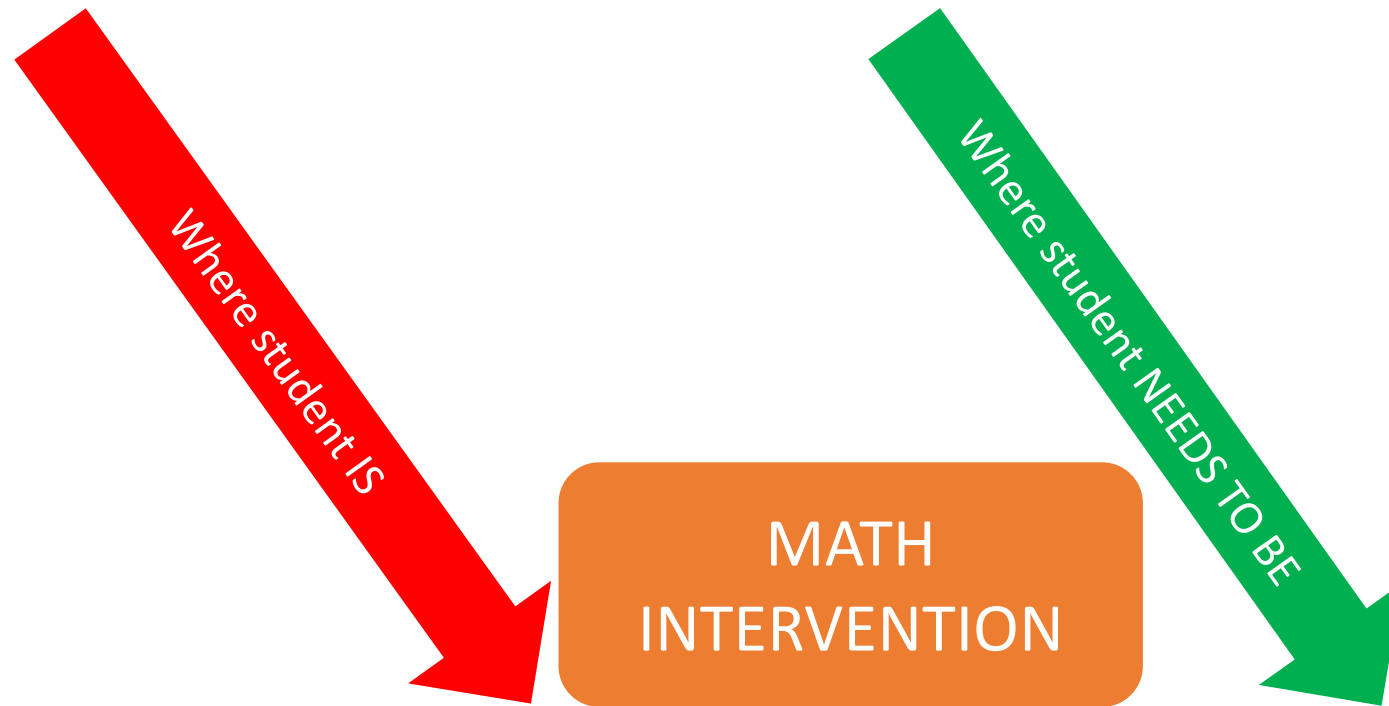




In your group, provide the mathematics language that is essential to understand the critical content.



Determine critical content



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