Chapter 7
Rational Functions

Section 7-4
Adding and Subtracting Rational Expressions

Adding or Subtracting Rational Expressions

As with numerical fractions, the procedure used to add (or subtract) two rational expressions depends upon whether the expressions have like or unlike denominators. To add (or subtract) rational expressions with like denominators, simply add (or subtract) their numerators. Then place the result over the common denominator.

Core Concept

Adding or Subtracting with Like Denominators

Let a, b, and c be expressions with $c \neq 0$.

Addition

$$\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}$$

Subtraction

$$\frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}$$

EXAMPLE 1 Adding or Subtracting with Like Denominators

a. $\frac{7}{4x} + \frac{3}{4x} =$

b. $\frac{2x}{x + 6} - \frac{5}{x + 6} =$

To add (or subtract) two rational expressions with *unlike* denominators, find a common denominator. Rewrite each rational expression using the common denominator. Then add (or subtract).

Core Concept

Adding or Subtracting with Unlike Denominators

Let a, b, c, and d be expressions with $c \neq 0$ and $d \neq 0$.

Addition

$$\frac{a}{c} + \frac{b}{d} = \frac{ad}{cd} + \frac{bc}{cd} = \frac{ad + bc}{cd}$$

Subtraction

$$\frac{a}{c} - \frac{b}{d} = \frac{ad}{cd} - \frac{bc}{cd} = \frac{ad - bc}{cd}$$

You can always find a common denominator of two rational expressions by multiplying the denominators, as shown above. However, when you use the least common denominator (LCD), which is the least common multiple (LCM) of the denominators, simplifying your answer may take fewer steps.

To find the LCM of two (or more) expressions, factor the expressions completely. The LCM is the product of the highest power of each factor that appears in any of the expressions.

EXAMPLE 2 Finding a Least Common Multiple (LCM)

Find the least common multiple of $4x^2 - 16$ and $6x^2 - 24x + 24$.

SOLUTION

Step 1 Factor each polynomial. Write numerical factors as products of primes.

$$4x^2 - 16 = 4(x^2 - 4) = (2^2)(x + 2)(x - 2)$$

$$6x^2 - 24x + 24 = 6(x^2 - 4x + 4) = (2)(3)(x - 2)^2$$

Step 2 The LCM is the product of the highest power of each factor that appears in either polynomial.

$$\text{LCM} = (2^2)(3)(x + 2)(x - 2)^2 = 12(x + 2)(x - 2)^2$$

EXAMPLE 3 Adding with Unlike Denominators

Find the sum $\frac{7}{9x^2} + \frac{x}{3x^2 + 3x}$.

EXAMPLE 4 Subtracting with Unlike Denominators

Find the difference $\frac{x+2}{2x-2} - \frac{-2x-1}{x^2-4x+3}$.

COMMON ERROR

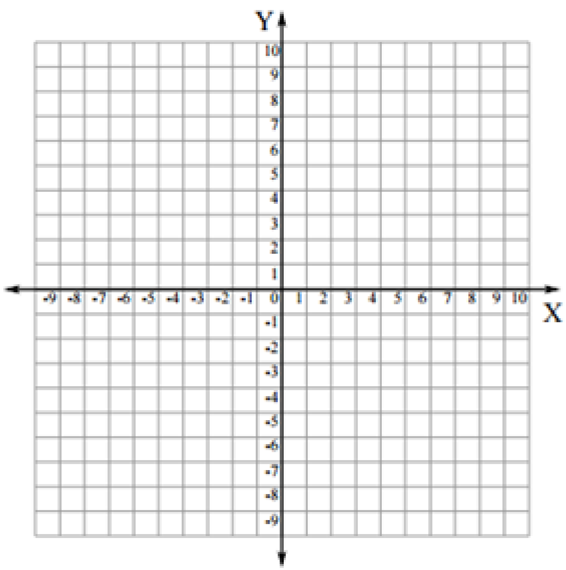
When subtracting rational expressions, remember to distribute the negative sign to all the terms in the quantity that is being subtracted.

Rewriting Rational Functions

Rewriting a rational expression may reveal properties of the related function and its graph. In Example 4 of Section 7.2, you used long division to rewrite a rational expression. In the next example, you will use inspection.

EXAMPLE 5 Rewriting and Graphing a Rational Function

Rewrite the function $g(x) = \dfrac{3x + 5}{x + 1}$ in the form $g(x) = \dfrac{a}{x - h} + k$. Graph the function. Describe the graph of g as a transformation of the graph of $f(x) = \dfrac{a}{x}$.



X	Y

Complex Fractions

A **complex fraction** is a fraction that contains a fraction in its numerator or denominator. A complex fraction can be simplified using either of the methods below.

Core Concept

Simplifying Complex Fractions

Method 1 If necessary, simplify the numerator and denominator by writing each as a single fraction. Then divide by multiplying the numerator by the reciprocal of the denominator.

Method 2 Multiply the numerator and the denominator by the LCD of *every* fraction in the numerator and denominator. Then simplify.

EXAMPLE 6 Simplifying a Complex Fraction

Simplify $\dfrac{\frac{5}{x+4}}{\frac{1}{x+4}+\frac{2}{x}}$.

SOLUTION

$$\frac{\frac{5}{x+4}}{\frac{1}{x+4}+\frac{2}{x}} = \frac{\frac{5}{x+4}}{\frac{3x+8}{x(x+4)}}$$ Add fractions in denominator.

$$= \frac{5}{x+4} \cdot \frac{x(x+4)}{3x+8}$$ Multiply by reciprocal.

$$= \frac{5x\cancel{(x+4)}}{\cancel{(x+4)}(3x+8)}$$ Divide out common factors.

$$= \frac{5x}{3x+8},\ x \neq -4,\ x \neq 0$$ Simplify.

Simplify the complex fraction.

10. $\dfrac{\frac{x}{6}-\frac{x}{3}}{\frac{x}{5}-\frac{7}{10}}$

11. $\dfrac{\frac{2}{x}-4}{\frac{2}{x}+3}$

12. $\dfrac{\frac{3}{x+5}}{\frac{2}{x-3}+\frac{1}{x+5}}$

Section 7-4 Homework #3-43 Every other odd and 45,47