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Research Article

Graphs of Continuous Functions in Topological Spaces

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Abstract

In the present paper, we introduce and study the notions of β*-open sets, β*-continuous functions and (β*, τ)-graph by utilizing the notion of β*-open sets. Also, some characterizations and properties of these notions are investigated.

Keywords: Topoloical space, Open set, β*-Open sets; β*-Continuous functions; (β*, τ)-graph.

Introduction

Studies of properties of sets and functions on topological spaces are of interest to many researchers and mathematicians (see [1-11] and the references therein). In [12-16] the authors introduced the notion of β-open sets and β-continuity in topological spaces. Moreover, in [17-21] the authors introduced δ-preopen sets and δ-almost continuity. The concepts of Z*-open set and Z*-continuity introduced by [22]. The purpose of this work is to introduce and study the notions of β*-open sets, β*-continuous functions and (β*, τ)-graph by utilizing the notion of β*-open sets. Also, some characterizations and properties of these notions are investigated. Throughout this paper (X, τ) and (Y, σ) (simply, X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A) and X\A denote the closure of A, the interior of A and the complement of A, respectively. A subset A of a topological space(X, τ) is called regular open (resp. regular closed) [23] if A=int(cl(A)) (resp. A=cl(int(A))).

A point x of X is called δ-cluster [24] point of A if int(cl(U))∩A=φ, for every open set U of X containing x. The set of all δ-cluster points of A is called δ-closure of A and is denoted clδ(A).A set A is δ-closed if and only if A=clδ(A). The complement of a δ-closed set is said to be δ-open [25]. The δ-interior of a subset A of X is the union of all δ-open sets of X contained in A. A subset A of a space X is called: (i). a-open [5] if A⊆int(cl(intδ(A))), (ii). α-open [15] if A⊆int(cl(int(A))), (iii). preopen [11] if A⊆int(cl(A)), (iv). δ-preopen [17] if A⊆int(clδ(A)), (v). δ-semiopen [16] if A⊆cl(intδ(A)), (vi). Z-open [10] if A⊆cl(intδ(A))∪int(cl(A)) (vii). γ-open [9] or b-open [3] or sp-open [4] if A⊆cl(int(A))∪int(cl(A)), (viii). e-open [6] if A⊆cl(intδ(A))∪int(clδ(A)), (ix). Z*-open [13] if A⊆cl(int(A))∪int(clδ(A)), (x). β-open [1] or semi-preopen [2] if A⊆cl(int(cl(A))) and, (xi). e*-open [7] if A⊆cl(int(clδ(A))). The complement of an a-open (resp. α-open, δ-semiopen, δ-preopen, Z-open, γ-open, e-open, Z*-open, β-open, e*-open) sets is called a-closed [5](resp. α-closed [15], δ-semi-closed [16], δ-pre-closed[17], Z-closed [10], γ-closed [3], e-closed [6], Z*-closed[13], β-closed [1], e*-closed [7]). The intersection of all δ-preclosed (resp. β-closed) set containing A is called the δ-preclosure (resp. β-closure) of A and is denoted by δ-pcl(A) (resp. β-cl(A)).

The union of all δ-preopen (resp. β-open) sets contained in A is called the δ-pre-interior (resp. β-interior) of A and is denoted by δ-pint(A) (resp. β-int(A)). The family of all δ-open (resp. δ-semiopen, δ-preopen, Z*-open, β-open, e*-open) sets is denoted by δO(X) (resp.

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δSO(X), δPO(X), Z*O(X), βO(X), e*O(X)). Let A be a subset of a topological space (X, τ). Then (i). δ-pint(A)=A∩int(clδ(A)) and δ-pcl(A)=A∪cl(intδ(A)) and (ii). β-int(A)=A∩cl(int(cl(A))) and β-cl(A)=A∪int(cl(int(A))).

Research methodology

Definition 2.1.

A subset A of a topological space(X, τ) is said to be: (i). a β*-open set if A⊆cl(int(cl(A)))∪int(clδ(A)) and (ii) a β*-closed set if int(cl(int(A)))∩cl(intδ(A))⊆A. The family of all β*-open (resp. β*-closed) subsets of a topological space (X, τ) will be as always denoted by β*O(X) (resp. β*C(X)).

Definition 2.2.

Let (X, τ) be a topological space. Then (i). The union of all β*-open sets of contained in A is called the β*-interior of A and is denoted by β*-int(A) and (ii). The intersection of all β*-closed sets of X containing A is called the β*-closure of A and is denoted byβ*-cl(A).

Definition 2.3.

A function f: (X, τ) → (Y, σ) is called β*-continuous if $f^{-1}(V)$ is β*-open in X, for each V ∈ σ.

Definition 2.4.

A function f: (X, τ) → (Y, σ) is called super-continuous [14] (resp. a-continuous [5], α-continuous [12], pre-continuous [11], δ-semi-continuous[8], Z-continuous [10], γ-continuous [9], e-continuous[6], Z*-continuous [13], β-continuous [1], e*-continuous[7]) if $f^{-1}(V)$ is δ-open (resp. a-open, α-open, per open,δ-semiopen, Z-open, γ-open, e-open, Z*-open, β-open,e*-open) in X, for each V ∈ σ.

Example 2.5.

Let X={a, b, c, d} with topology τ = {φ, {a}, {c}, {a, b}, {a, c}, {a, b, c}, {a, c, d}, X}. Then the function f: (X, τ) → (X, τ) defined by f(a) = a, f(b) = f(c) = c and f(d) = d is β*-continuous but it is not β-continuous. The function f: (X, τ) → (X, τ) defined by f(a) = d, f(b) = a, f(c) = c and f(d) = b is e*-continuous but it is not β*-continuous.

Example 2.6.

Let X = {a, b, c, d, e} with topology τ = {φ, {a, b}, {c, d}, {a, b, c, d}, X}. Then the function f: (X, τ) → (X, τ) defined by f(a) = a, f(b) = e, f(c) = c, f(d) = d and f(e) = b is β*-continuous but it is not Z*-continuous.

Remark 2.7

(i). If A∈δO(X) and B∈β*O(X), then A∩B∈β*O(X), (ii). Let A and B be two subsets of a space (X,τ). If A∈δO(X) and B∈β*O(X), then A∩B∈β*O(A) and A∩B∈β*O(X).

Definition 2.8.

The β*-frontier of a subset A of X, denoted by β*-Fr(A), is defined by β*-Fr(A)=β*-cl(A)∩β*-cl(X\A) equivalently β*-Fr(A)=β*-cl(A)\β*-int(A).

Definition 2.9.

A function f: X → Y has a (β*, τ)-graph if for each (x, y)∈(X×Y)\G(f), there exist a β*-open U of X containing x and an open set V of Y containing y such that (U×V)∩G(f) =φ.

Definition 2.10.

A topological space (X, τ) is said to be β*-connected if it is not the union of two nonempty disjoint β*-open sets.

Definition 2.11.

A space X is said to be β*-compact if every β*-open cover of X has a finite subcover.

Results and discussion

In this section we give the results of our study. We begin with characterizations of β*****-Open sets.

Theorem 3.1.

Let (X, τ) be a topological space. Then the following hold. (i). The arbitrary union of β*-open sets is β*-open. (ii). The arbitrary intersection of β*-closed sets is β*-closed.

Proof.

(i). Let {A_i: i∈I} be a family of β*-open sets. Then A_i⊆cl(int(cl(A_i)))∪int(clδ(A_i)) and hence ∪$_i$$A_i$⊆∪$_i$(cl(int(cl($A_i$)))∪int(clδ($A_i$)))⊆cl(int(cl(∪$_i$$A_i$)))∪int(clδ(∪$_i$$A_i$)), for all i∈I. Thus, ∪$_i$$A_i$ is β*-open. The proof of (ii) follows from (i).

Remark 3.2.

By the following next example we show that the intersection of any two β*-open sets is not β*-open.

Example 3.3.

Let X = {a, b, c} with topology τ = {φ, {a}, {b}, {a, b}, X}. Then A = {a, c} and B = {b, c} are β*-open sets. But, A∩B = {c} is not β*-open.

Theorem 3.4.

Let A, B be two subsets of a topological space (X, τ). Then the following hold: (i). β*-cl(X) = X and β*-cl(φ) = φ, (ii). A ⊆ β*-cl(A), (iii). If A ⊆ B, then β*-cl(A) ⊆ β*-cl(B), (iv) x ∈ β*-cl(A) if and only if for each a β*-open set U containing x, U ∩ A /= φ, (v). A is β*-closed set if and only if A = β*-cl(A), (vi). β*-cl(β*-cl(A)) = β*-cl(A), (vii). β*-cl(A) ∪ β*-cl(B) ⊆ β*-cl(A ∪ B), (viii). β*-cl(A ∩ B) ⊆ β*-cl(A) ∩ β*-cl(B).

Proof.

The other conditions hold by definition. To prove (vi), by using (ii) and A ⊆ β*-cl(A), we have β*-cl(A) ⊆ β*-cl(β*-cl(A)). Let x ∈ β*-cl(β*-cl(A)). Then, for every β*-open set V containing x, V ∩ β*-cl(A) /= φ.

Example 3.5.

Let X = {a, b, c, d} with topology τ = {φ, {a}, {c}, {a, b}, {a, c}, {a, b, c}, {a, c, d}, X} and consider y ∈ V ∩ β*-cl(A). Then, for every β*-open set G containing y, A ∩ G /= φ. Since V is a β*-open set, y ∈ V and A ∩ V /=φ, then x ∈ β*-cl(A). Therefore, β*-cl(β*-cl(A)) ⊆ β*-cl(A).

Theorem 3.6.

For a subset A in a topological space (X, τ), the following statements are true: (i). β*-cl(X\A) = X\β*-int(A) and (ii). β*-int(X\A) = X\β*-cl(A).

Proof.

Follows from the fact the complement of β*-open set is a β*-closed and $\cap_i(X\setminus A_i) = X\setminus\cup_i A_i$.

Theorem 3.7.

Let A be a subset of a topological space(X, τ). Then the following are equivalent: (i). A is a β*-open set and (ii). A = β-int(A) ∪ pintδ(A).

Proof.

(i)⇒(ii). Let A be a β*-open set. ThenA ⊆ cl(int(cl(A))) ∪ int(clδ(A)) and hence, A ⊆ (A ∩ cl(int(cl(A)))) ∪ (A ∩ int(clδ(A))) = β-int(A) ∪ pintδ(A) ⊆ A.
(ii)⇒(i). Trivial.

Theorem 3.8.

For a subset A of space (X, τ).Then thefollowing are equivalent: (i). A is a β*-closed set and (ii) A = β-cl(A) ∩ pclδ(A).

Proof.

Follows from Theorem 3.7.

Theorem 3.9.

Let f: (X, τ) →(Y, σ) be a function. Then the following statements are equivalent: (1) f is β*-continuous, (2) For each x ∈ X and V ∈ σ containing f(x), there exists U ∈ β*O(X) containing x such that f(U) ⊆ V, (3) The inverse image of each closed set in Y is β*-closed in X, (4) int(cl(int(f^{-1}(B)))) ∩ cl(intδ(f^{-1}(B))) ⊆ f^{-1}(cl(B)),for each B ⊆ Y, (5) f^{-1}(int(B)) ⊆ cl(int(cl(f^{-1}(B)))) ∪ int(clδ(f^{-1}(B))),for each B ⊆ Y, (6) β*-cl(f^{-1}(B)) ⊆ f^{-1}(cl(B)), for each B ⊆ Y,(7) f(β*-cl(A)) ⊆ cl(f(A)), for each A ⊆ X,(8) f^{-1}(int(B)) ⊆ β*-int(f^{-1}(B)), for each B ⊆ Y.

Proof.

(1)⇔(2) and (1)⇔(3) are obvious.
(3)⇒(4). Let B ⊆ Y. Then by (3) f^{-1}(cl(B)) is β*-closed.
This means f^{-1}(cl(B)) ⊇ int(cl(int(f^{-1}(cl(B)))))∩ cl(intδ(f^{-1}(cl(B)))) ⊇ int(cl(int(f^{-1}(B)))) ∩cl(intδ(f^{-1}(B))).
(4)⇒(5). By replacing Y\B instead of B in (4), we have
int(cl(int(f^{-1}(Y\B)))) ∩cl(intδ(f^{-1}(Y\B))) ⊆ f^{-1}(cl(Y\B)).
Therefore, f^{-1}(int(B)) ⊆ cl(int(cl(f^{-1}(B))))∪ int(clδ(f^{-1}(B))), for each B ⊆ Y.
(5)⇒(1). Obvious.
(3)⇒(6). Let B ⊆ Y and f^{-1}(cl(B)) be β*-closed in X. Then β*-cl(f^{-1}(B)) ⊆ β*-cl(f^{-1}(cl(B))) = f^{-1}(cl(B)).
(6)⇒(7). Let A ⊆ X. Then f(A) ⊆ Y. By (6), we have f^{-1}(cl(f(A))) ⊇ β*-cl(f^{-1}(f(A))) ⊇ β*-cl(A). Therefore, cl(f(A)) ⊇ f^{-1}(cl(f(A))) ⊇ f(β*-cl(A)).
(7)⇒(3). Let F ⊆ Y be a closed set. Then,f^{-1}(F) = f^{-1}(cl(F)). Hence by (7),f(β*-cl(f^{-1}(F))) ⊆ cl(f (f^{-1}(F))) ⊆ (F) = F, thus, β*-cl(f^{-1}(F)) ⊆ f^{-1}(F), so, f^{-1}(F) = β*-cl(f^{-1}(F)). There-fore, f^{-1}(F) ∈ β*C(X).
(1)⇒(8). Let B ⊆ Y. Then f^{-1}(int(B)) is β*-open in X. Thus, f^{-1}(int(B)) = β*-int(f^{-1}(int(B))) ⊆ β*-

int(f^{-1}(B)).Therefore, f^{-1}(int(B)) ⊆ β*-int(f^{-1}(B)).

(8)⇒(1). Let U ⊆ Y be an open set. Thenf^{-1}(U) = f^{-1}(int(U)) ⊆ β*-int(f^{-1}(U)). Hence, f^{-1}(U) isβ*-open in X. Therefore, f is β*-continuous.

Remark 3.10.

If f: X → Y is a β*-continuous and g: Y → Z is a continuous, then the composition g ∘ f: X → Z is β*-continuous.

Next, we consider some properties and separation axioms. We state the following propositions.

Proposition 3.11.

If f: (X, τ) → (Y, σ) is a β*-continuous function and A is δ-open in X, then the restriction given by f\A:(A, τA) → (Y, σ) is β*-continuous.

Proof.

Let V be an open set of Y. Then by hypothesis f^{-1}(V) is β*-open in X. Hence, we have $(f\backslash A)^{-1}$(V) = f^{-1} (V) ∩ A β* ∈ O(A). Thus, it follows that f\A is β*-continuous.

Proposition 3.12.

Let (X, τ) → (Y, σ) be a function and {G_i: i ∈ I} be a cover of X by δ-open sets of (X, τ).

If f\G_i: (G_i,τ_{Gi})→(Y, σ) is β*-continuous for each i ∈ I, then f is β*-continuous.

Proof.

Let V be an open set of (Y, σ). Then by hypothesis

f^{-1}(V) = X ∩ f^{-1}(V) = ∪ {G_i∩ f^{-1} (V):i ∈ I} = ∪ {$(f\backslash G_i)^{-1}$(V): i ∈ I}.

Since f\G_i is β*-continuous for each i ∈ I, then $(f\backslash G_i)^{-1}$(V) ∈ YO(G_i)for each i ∈ I. By Proposition 3.11, we have $(f\backslash G_i)^{-1}$(V) isβ*-continuous in X. Therefore, f is β*-continuous in(X, τ).

Theorem 3.13.

The set of all points x of X at which a function f: (X, τ) → (Y, σ) is not β*-continuous is identical with the union of the β*-frontiers of the inverse images of open sets containing f(x).

Proof.

Necessity. Let x be a point of X at which f is not β*-continuous. Then, there is an open set V of Y containing f(x) such that U ∩ (X\$f^{-1}$(V)) is not φ, for every U ∈ β*O(X) containing x. Thus, we have x ∈ β*-cl(X\$f^{-1}$(V)) = X\β*-int($f^{-1}$(V)) and x ∈ f^{-1}(V).Therefore, we have x ∈ β*-Fr(f^{-1}(V)) is open set containing f(x). *Sufficiency*. We assume that f is β*-continuous at x ∈ X. Then there exists U ∈ β*O(X) containing x such that f(U) ⊆ V. Therefore, we have x ∈ U ⊆ f^{-1}(V) and hence x ∈ β*-int(f^{-1}(V)) ⊆ X\β*-Fr(f^{-1}(V)). This is a contradiction. This means that f is not β*-continuous at x.

The following implications are hold fora topological space X.

Lemma 3.14.

A function f: X → Y has a has a (β*, τ)-graph if and only if for each (x, y) ∈ X × Y such that y is not equal to f(x), there exist a β*-open set U and an open set V containing x and y, respectively, such that f(U) ∩ V = φ.

Proof.

Trivially follows readily from the above definition.

Theorem 3.15.

If f: X → Y is a β*-continuous function and Y is Hausdorff, then f has a (β*, τ)-graph.

Proof.

Let (x, y) ∈ X × Y such that y is not equal to f(x). Then there exist open sets U and V such that y ∈ U, f(x) ∈ V and V ∩ U = φ. Since f is β*-continuous, there exists β*-open W containing x such that f(W) ⊆ V. This implies that f(W) ∩ U ⊆ V ∩ U = φ. Therefore, f has a (β*, τ)-graph.

Theorem 3.16.

If f: (X, τ) → (Y, σ) has a (β*, τ)-graph, then f(K) is closed in (Y, σ) for each subset K which is β*-compact relative to (X, τ).

Proof.

Suppose that y is not in f(K). Then (x, y) is not in G(f) for each x ∈ K. Since G(f) is (β*, τ)-graph, there exist a β*-open set U containing x and an open set V of Y containing y such that f(U) ∩ V= φ. The family {U_x: x ∈ K} is a cover of K by β*-open sets. Since K is β*-compact relative to (X, τ), there exists a finite subset K_0 of K such that f(K) is closed in (Y, σ).

Theorem 3.17.

If f: (X, τ) →(Y, σ) is a β*-continuous injection and (Y, σ) is T_i, then (X, τ) is β*-T_i, where i = 0, 1, 2.

Proof.

We prove that the theorem for i = 1. Let Y be T_1 and x, y be distinct points in X. There exist open subsets U, V in Y such that f(x) ∈ U, f(y) is not in U, f(x) is not in V and f(y) ∈ V. Since f is β*-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are β*-open subsets of X such that x ∈ $f^{-1}(U)$, y is not in $f^{-1}(U)$, x is not in $f^{-1}(V)$ and y ∈ $f^{-1}(V)$. Hence, X is β*-T_1. K ⊆ ∪ {U_x: x ∈ K0}. Let V = ∩ {V_x: x ∈ K_0}. Then V is an open set in Y containing y.

Therefore, we have f (K) ∩V ⊆ ($\cup_{x \in K0}$ f (U_x)) ∩ V ⊆ $\cup_{x \in K0}$(f (U_x)) ∩ V = φ. It follows that, y is not in cl(f(K)). Therefore, f(K) is closed in(Y, σ).

Corollary 3.18.

If f: (X, τ) → (Y, σ) is β*-continuous function and Y is Hausdorff, then f(K) is closed in (Y, σ)for each subset K which is β*-compact relative to (X, τ).

Theorem 3.19.

If f: X → Y is a β*-continuous function and Y is a Hausdorff space, then f has a (β*, τ)-graph.

Proof.

Let (x, y) ∈ X × Y such that y is not in f(x) and Y be a Hausdorff space. Then there exist two open sets U and V such that y ∈ U, f(x) ∈ V and V ∩ U = φ. Since f is β*-continuous, there exists a β*-open set W containing x such that f(W) ⊆ V. This implies that f(W) ∩ U ⊆ V ∩ U = φ. Therefore, f has a (β*, τ)-graph.

Corollary 3.20.

If f: X → Y is β*-continuous and Y is Hausdorff, then G(f) is β*-closed in X × Y.

Theorem 3.21.

If f: X → Y has a (β*, τ)-graph and g: Y → Z is a β*-continuous function, then the set {(x, y):f(x) = g(y)} is β*-closed in X × Y.

Proof.

Let A = {(x, y): f(x) = g(y)} and (x, y) is not in A. We have f(x) is not equal to g(y) and then (x, g(y)) ∈ (X × Z)\G(f). Since f has a (β*, τ)-graph, then there exist a β*-open set U and an open set V containing x and g(y), respectively such that f(U) ∩ V = φ. Since g is a β*-continuous function, then there exist an β*-open set G containing y such that g(G) ⊆ V. We have f(U) ∩ g(G) = φ. This implies that (U × G) ∩ A = φ. Since U × G is β*-open, then (x, y) /∈ β*-cl(A). Therefore, A is β*-closed in X × Y.

Theorem 3.22.

If f: X → Y is a β*-continuous function and Y is Hausdorff, then the set {(x, y) ∈ X × X: f(x) = f(y)} is β*-closed in X × X.

Proof.

Let A = {(x, y): f(x) = f(y)} and let(x, y) ∈ (X × X)\A. Then f(x) is not equal to f(y). Since Y is Hausdorff, then there exist open sets U and V containing f(x) and f(y), respectively, such that U ∩ V = φ. But, f isβ*-continuous, then there exist β*-open sets H and Gin X containing x and y, respectively, such that f(H) ⊆ U and f(G) ⊆ V. This implies (H × G) ∩ A = φ. By Theorem 3.21, we have H × G is a β*-open set in X × X containing(x, y). Hence, A is β*-closed in X × X.

Theorem 3.23.

If f: (X, τ) → (Y, σ) is β*-continuous and S is closed in X × Y, then υ_x(S ∩ G(f)) is β*-closed in X, where υ_x represents the projection of X × Y onto X.

Proof.

Let S be a closed subset of X × Y and x ∈ β*-cl (υ_x(S ∩ G(f))). Let U ∈ τ containing x and V ∈ σ containing f(x). Since f is β*-continuous, by Theorem 3.21, x ∈ $f^{-1}(V)$ ⊆ β*-int($f^{-1}(V)$). Then U ∩ β*-int($f^{-1}(V)$) ∩ υ_x(S ∩ G(f)) contains some point z of X. This implies that (z, f(z)) ∈ S and f(z) ∈ V. Thus we have (U × V) ∩ S /= φ and hence (x, f(x)) ∈ cl(S). Since A is closed, then (x, f(x)) ∈ S ∩ G(f) and x ∈ υ_x(S ∩ G(f)).Therefore υ_x(S ∩ G(f)) is β*-closed in (X, τ).

Theorem 3.24.

If (X, τ) is a β*-connected space and f: (X, τ) → (Y, σ) has a (β*, τ)-graph and β*-continuous function, then f is constant.

Proof.

Suppose that f is not constant. There exist disjoint points x, y ∈ X such that f(x) = f(y). Since (x, f(x)) is not in G(f), then y /= f(x), hence, there exist open sets U and V containing x and f(x) respectively such that f(U) ∩ V = φ. Since f is β*-continuous, there exist a β*-open sets G containing y such that f(G) ⊆ V. U and V are disjoint β*-open sets of (X, τ), it follows that (X, τ) is not β*-connected. Therefore, f is constant.

Conclusions

In the present paper, we have studied various notions of continuity in general topological spaces. We have introduced and studied the notions of β*-open sets, β*-continuous functions and (β*, τ)-graph by utilizing the notion of β*-open sets. Also, some characterizations and properties of these notions have been investigated.

Conflicts of interest

Authors declare no conflict of interest.

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