Math 1496 - Calc I

Chapter 5 - Integration

we now know how to take derivatives

so if

$$f(x) = x^2 \qquad f'(x) = 2x$$

so what if I tell you that

$$f'(x) = 2x$$

then what is $f(x)$?

well $f(x) = x^2,\ x^2+1,\ x^2-5,\ x^2+c$

where c is any constant.

If $F(x)$ is such that

$$F'(x) = f(x) \quad \text{given}$$

we say $F(x)$ is the antiderivative of f

If $F'(x) = e^x$

then $F(x) = e^x + c$

If $F'(x) = \sin x$

$F(x) = -\cos x + c$

The process of finding the anti derivative is called integration & symbolically

$$\int e^x dx = e^x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int F'(x)dx = F(x) + c$$ (in general)

we have a whole list of standard integrals

Derivatives	Integrals
$\frac{d}{dx}(c) = 0$	$\int 0\,dx = c$
$\frac{d}{dx} x^n = n x^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$
$\frac{d}{dx} \sin x = \cos x$	$\int \frac{dx}{x} = \ln\lvert x\rvert + c$
$\frac{d}{dx} \cos x = -\sin x$	$\int \sin x\,dx = -\cos x + c$
$\frac{d}{dx} \tan x = \sec^2 x$	$\int \cos x\,dx = \sin x + c$
$\frac{d}{dx} \sec x = \sec x \tan x$	$\int \sec^2 x\,dx = \tan x + c$
$\frac{d}{dx} \cot x = -\csc^2 x$	$\int \sec x \tan x\,dx = \sec x + c$
$\frac{d}{dx} \csc x = -\csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + c$
$\frac{d}{dx} e^x = e^x$	$\int e^x dx = e^x + c$
$\frac{d}{dx} a^x = a^x \ln a$	$\int a^x dx = \frac{a^x}{\ln a} + c$
$\frac{d}{dx} \ln\lvert x\rvert = \frac{1}{x}, \quad x \neq 0$	$\int \tan x\,dx$?
$\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$	$\int \sec x\,dx$?

Ex $\int x\,dx = \frac{x^2}{2} + c$

$$\int \frac{dx}{x^2} = \int x^{-2}\,dx = \frac{x^{-2+1}}{-2+1} + c = \frac{-1}{x} + c$$

Also $\int (f+g)\,dx = \int f\,dx + \int g\,dx$

$\int k f\,dx = k \int f\,dx$

So $\int \frac{x+2}{\sqrt{x}}\,dx = \int x^{1/2} + 2x^{-1/2}\,dx$

$$= \frac{x^{3/2}}{3/2} + 2\frac{x^{1/2}}{1/2} + c = \frac{2}{3}x^{3/2} + 4x^{1/2} + c$$

So what about the c? we need more info.

If $F'(x) = 3x^2$ find $F(x)$ such that $F(1) = 2$

so $F(x) = x^3 + c$ Now $F(1) = 2 \Rightarrow 1^3 + c = 2 \Rightarrow c = 1$

So $F(x) = x^3 + 1$.