## COORDINATE GEOMETRY NOTES (THE LINE & THE CIRCLE)

### THE LINE

**1) FINDING SLOPES** [the slope measures the steepness of the line]

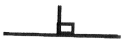
> By using formula (when you have 2 points)

> Rise / Run (when the line is drawn)

> By rearranging the equation of the line into the form $y = \underline{m}x + c$

**2) PARALLEL & PERPENDICULAR LINES**

> parallel lines have <u>equal</u> slopes

> perpendicular lines 

*if 2 lines are perpendicular, then when you multiply the 2 slopes of the lines = -1

*if one line is 3, the slope of the perpendicular line is $\frac{-1}{3}$

[flip and change sign to find the perpendicular slope]

> if the equation of a line is: $3x + 5y + 7 = 0$

then the equation of a line parallel is: $3x + 5y + k = 0$ (where k is any number)

if the equation of a line is: $3x + 5y + 7 = 0$

then the equation of a line perpendicular is: $5x - 3y + c = 0$ (where c is any number)

**3) EQUATION OF A LINE (remember tangents are just lines!)**

> need one point and the slope for formula $y - y_1 = m(x - x_1)$

**4) If a question says a "POINT IS ON A LINE"**

> **SUB IT IN!** If it satisfies the equation, i.e. L.H.S. = R.H.S., then it is <u>on</u> the line

**5) POINT OF INTERSECTION** -> solve simultaneous equations!

**6) GRAPHING A LINE**

> let $x = 0$, find y

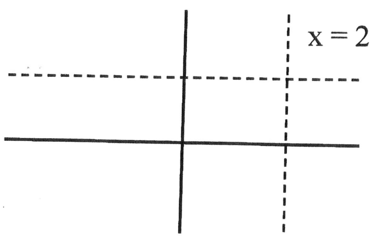
> let $y = 0$, find x

> plot those 2 points

**7)** If a line **"CUTS X-AXIS"** then you know **$y = 0$** SUB IT IN!;

if a line **"CUTS Y-AXIS"** then you know **$x = 0$** SUB IT IN!

**8)** If a line is **"PARALLEL TO AXES"**



$x = 2$ (vertical lines are x= ?)

$y = 1$(horizontal lines are y=?)

## 9) AREA OF TRIANGLE

i. A = ½ x b x h (if you have base & perpendicular height)

ii. A = ½ ab sin C (if you have an angle)

iii. $A = \frac{1}{2}|x_1 y_2 - x_2 y_1|$ (if you have 3 coordinate points of the vertices)

*need one point to be (0,0), if it's not, translate one point to (0,0) and apply same translation to the other two points.

## 10) INTERNAL DIVISION OF A LINE SEGMENT (in a ratio a:b)

$$\left(\frac{bx_1 + ax_2}{b+a}, \frac{by_1 + ay_2}{b+a}\right)$$

*sketch where possible, sometimes you can use translations to figure out answer

## 11) PERPENDICULAR DISTANCE FROM A POINT TO A LINE

$$\frac{|ax_1 + by_1 + c|}{\sqrt{(a^2 + b^2)}} = d$$

*make sure you have the line in the form, $ax_1 + by_1 + c = 0$, if not rearrange it !

**-if looking for distance between 2 parallel lines:**

> 2x + y + 5 = 0 and 2x + y = 0

-find a point on one of them

try pairs of coordinates that will work

e.g. let x = 0, 2x + y = 0

2(0) + y = 0

y = 0

-so then use (0,0) and other line equation: 2x + y + 5 = 0

a = 2, b = 1, c = 5 & sub into formula!

**-if missing information about the line, get "the form" of the line**

> e.g. say you know a point on it, sub into $y - y_1 = m(x - x_1)$ & get equation of line in terms of m, use this, then, in your formula & solve.

> e.g. say you know the slope of it, sub into y = mx + c & get equation of line in terms of c, use this, then, in your formula & solve!

**don't forget to rearrange ax + by + c = 0 to use with the formula for the distance between a point & a line!*

## 12) THE ANGLE BETWEEN 2 LINES

$$Tan\ \theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

- Always take the positive result for the acute angle and for the obtuse angle, find the acute angle first, then subtract from 180°
- Get the slopes of the 2 lines, if one line is the x-axis then its slope is 0

## 13) CENTROID, ORTHOCENTRE & CIRCUMCENTRE

- CENTROID: If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ then the centroid is:

  $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$ *this is not in your log tables

  The centroid is the point of intersection of the medians. The median line is the line containing the mid-point of 2 of the vertices and the opposite vertex. The medians of a triangle divide each other in the ratio 2:1. So find the mid-point between 2 of the vertices then do the "Internal Division of a Line Segment" formula in the ratio 2:1 with the third vertex.

- ORTHOCENTRE: The orthocentre is the point of intersection of the altitudes of the triangle. The altitude is the perpendicular line from the vertex to the opposite side.
  So, find the equations of 2 of the altitudes. To find the equation of an altitude find the slope between 2 of the points, then find the perpendicular slope by flipping & changing the sign. Use this perpendicular slope and the opposite vertex point in order to find the equation of one of the altitudes. Once you have the 2 equations of the altitudes, solve them simultaneously to find the point of intersection of those lines!

- CIRCUMCENTRE: The circumcentre is the point of intersection of the mediators of the triangle. A mediator of a triangle is the perpendicular bisector line of the side of the triangle. In order to find the equation of a mediator, you must first find the mid-point between 2 of the vertices. Use this mid-point as the point for the equation of the lines formula. Now you must find the slope for this formula. First find the slope between the 2 vertices then find the perpendicular slope by flipping & changing the sign. Use this perpendicular slope and the mid-point in order to find the equation of one of the perpendicular bisectors. Once you have the 2 equations of the mediators, solve them simultaneously to find the point of intersection of those lines!

## THE CIRCLE

- **$x^2 + y^2 = r^2$ centre (0,0)**

  $(x - h)^2 + (y - k)^2 = r^2$ center (h, k)

Or $x^2 + y^2 + 2gx + 2fy + c = 0$

  centre :(-g, -f) radius :$\sqrt{(g^2 + f^2 - c)}$

- **Points Inside / Outside / On circle**

  SUB IN then if L.H.S. < R.H.S. then point is INSIDE the circle

  L.H.S. > R.H.S. then point is OUTSIDE the circle

  L.H.S. = R.H.S. then the point is ON the circle

- **Intersection of a Line and a Circle**

  Simultaneous Equations- substitution method, get x= or y= with easier (linear) equation then sub this in for the x or the y in the other equation. (*if there is only one point of contact then the line is a tangent)

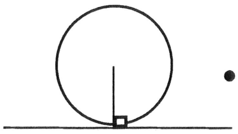
- **Intersecting Circles**
  Finding the common chord or common tangent – solve simultaneous equations with the 2 circle equations (put on underneath the other & subtract!) To find the actual point of contact, find the common line first and then use the equation of this line with one of the circle equations & find their point of intersection by simultaneous equations (substitution method)

- **Touching Circles**

  If $|c_1c_2| = r_1 + r_2$ touch externally (the distance between the centres)

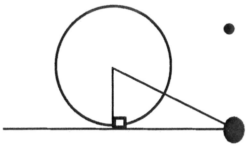
  If $|c_1c_2| = |r_1 - r_2|$ touch internally

- **Tangents and Circles!**
  - remember a tangent is just a line so use: $y - y_1 = m(x - x_1)$
    *need a point & the slope

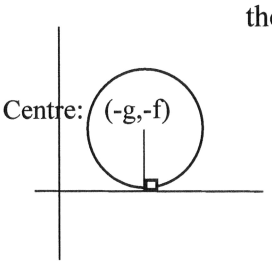


  - if you have the slope of the radius, then the slope of the tangent is the perpendicular slope [flip & change the sign]

  - the distance between the centre and the tangent is the length of the radius so using the formula for the distance between a point and a line, you can get the "form" of the tangent line using the slope; if the slope of the tangent is $\frac{3}{5}$, then the form of the tangent line is $3x + 5y + c = 0$; if slope is $-\frac{3}{5}$, then the form of the tangent line is $3x - 5y + k = 0$



  - Finding the length of a tangent to a circle from a given point- use Pythagoras !




  - if the tangents are the axes!
    then $-f = \sqrt{(g^2 + f^2 - c)}$ (the y coordinate equals the length of the radius)

    $(-f)^2 = [\sqrt{(g^2 + f^2 - c)}]^2$

    $f^2 = g^2 + f^2 - c$

    $c = g^2$ (likewise, if y-axis is a tangent, then $c = f^2$)

- **Puzzles in g, f and c**
  If a point is on the circle, SUB IT IN to $x^2 + y^2 + 2gx + 2fy + c = 0$ to get an equation in g,f and c
  If the centre is on a line SUB IN (-g, -f) INTO the equation onf the line

- **Proving a line is a Tangent to the Circle**

  Either show that the distance between the tangent line and the circle equals the radius

  Or show that there's only one point of intersection (simultaneous equations)

**PROBABILITY NOTES**

→ the measure of the probability of an event E:

$$P(E) = \frac{no.of\ successful\ outcomes}{no.of\ possible\ outcomes} \qquad 0 \leq P(E) \leq 1 \qquad \text{And } P(\text{not } E) = 1 - P(E)$$

→ the relative frequency of an event in an experiment is given by:

$$P(E) = \text{relative frequency of an event} = \frac{no.of\ successful\ trials}{no.of\ trials}$$

<u>**Mutually Exclusive Events:**</u> [events that cannot occur at the same time]

$P(A \cap B) = 0$

$P(A)$ or $P(B) = P(A) + P(B)$

$P(A)$ and $P(B) = P(A) \times P(B)$

e.g. P(King or Two) = $P(K) + P(T) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52}$

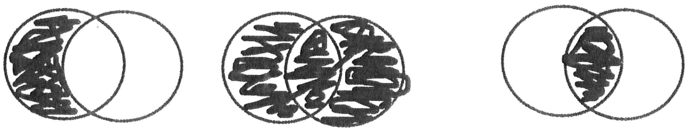
<u>**Non Mutually Exclusive Events:**</u>

e.g. P(King or Red) → can happen at same time so you must subtract overlap!

$= P(K) + P(R) - P(K \cap R) = \frac{4}{52} + \frac{13}{52} - \frac{2}{52} = \frac{15}{52}$

Venn Diagrams are used for Non Mutually Exclusive Events

A \ B          A U B          A ∩ B



→ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

<u>Independent Events:</u> If proving 2 events are independent then $P(A \cap B) = P(A) \times P(B)$

<u>Dependent Events:</u> Conditional Probability $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

<u>Bernoulli Trials / Binomial Distribution</u>

$\binom{n}{r} p^r q^{n-r}$

n= no. of trials / r =no. of "successes" / p = probability of "success" / q = probability of "failure"

Criteria: **<u>F.I.T.S</u>**.: <u>F</u>inite no. of trials <u>I</u>ndependent events <u>T</u>wo outcomes <u>S</u>ame probability each time

*Permutations (arrangements): $^nP_r = \frac{n!}{(n-r)!}$      *Combinations (selections): $^nC_r = \frac{n!}{r!(n-r)!}$

**Expected Value: E(X) = $\sum xP(x)$**

Use a table and complete it with the following headings:

| **Outcome / Payout** | **Probability** | **Payout x Probability** |
|---|---|---|
| | | |
| | | |

**<u>Total:</u>** ________.

## INFERENTIAL STATISTICS NOTES

Inferential statistics -when you make predictions / inferences about populations using data drawn from populations by means of sampling.
**Population**: everyone in the group
A parameter is a numerical measurement describing some characteristic of the population.
**Sample:** a subset of members selected from the population
A statistic is a numerical measurement describing some characteristic of a sample.

### The Normal Distribution

To convert data that follows a normal distribution to standard units so that we can read probabilities from the tables we use:

$$Z = \frac{x-\mu}{\sigma}$$

x = population value

μ = population mean

σ = population standard deviation

*a z-score is the number of standard deviations a given value x is, above or below the mean of a given data set

Recap on questions involving the Normal Distribution

Convert the values to z scores.

- If it is less than, e.g. $P(Z < 1.73)$, it is a straight forward read from the tables
- If it is more than, >, e.g. $P(Z > 1.73)$, you calculate $1 - P(Z< 1.73)$
- If it is a negative value, e.g. $P(Z < -1.73)$, you also calculate $1 - P(Z< 1.73)$
- If it is more than and negative, e.g. $P(Z > -1.73)$, it is $1 - P(Z > 1.73)$ which then works out as $1 - (1 - P(Z< 1.73))$ which is basically the same as $P(Z< 1.73)$

### Empirical Rule!

-approx. **68%** of the data lies **within one standard deviation of the mean** $(\mu - \sigma, \mu + \sigma)$

-approx. **95%** of the data lies **within two standard deviations of the mean** $(\mu - 2\sigma, \mu + 2\sigma)$

-approx. **99.7%** of the data lies **within three standard deviations of the mean** $(\mu - 3\sigma, \mu + 3\sigma)$

### Confidence Interval For a Population Proportion

Notation: p = population proportion

$\hat{p}$ = sample proportion

E = margin of error / standard error

$\hat{p} - E < p < \hat{p} + E$

If p is known use population proportion

$$E = 1.96\sqrt{\frac{p(1-p)}{n}}$$

If p is unknown use $\hat{p}$ the sample proportion

$$E = 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

**Hypothesis Testing for Population Proportion**

1. State $H_0$ and $H_1$

2. Calculate the sample proportion, $\hat{p}$

3. Find the margin of error, $E = 1.96\sqrt{\frac{p(1-p)}{n}}$

4. Write down the confidence interval

$$\hat{p} - E < p < \hat{p} + E$$

5. If the value of the population proportion is

(i) **within** the confidence interval, **do not reject $H_0$**

(ii) **outside** the confidence interval, **reject $H_0$ and accept $H_1$**

**Central Limit Theorem**

-if the sample size is large ($n \geq 30$), the distribution of the sample means will approximate to a normal distribution regardless of what the population distribution is

-$\mu_{\bar{x}} = \mu$ (mean of the samples = mean of the population)

-$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ (standard deviation of the samples = standard deviation of population divide by $\sqrt{n}$)

-if the underlying population is normal, the sampling distribution of the means will always be normal even if $n < 30$

*when dealing with the sampling distribution of the mean, we convert the given units to standard units using

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

**Confidence Interval For a Population Mean / Average**

Margin of Error / Standard Error: $E = \bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$

$$\bar{x} - E < \mu < \bar{x} + E$$

**Hypothesis Testing for a Population Mean / Average**

***use this testing if the question asks anything about the mean or average!!**

1. State $H_0$ and $H_1$

2. State: the level of significance is 5%, so critical region is $z < -1.96$ or $z > 1.96$

3. Calculate test statistic: $z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$

4. Conclude

if z value is **__within__** $-1.96 < z < 1.96$, it is **__not__** in critical region; **__do not reject__** $H_0$

if z value is **__outside__** $-1.96 < z < 1.96$, it **__is__** in critical region; **__reject__** $H_0$

**__Using P-Values__**

Find the P-value that corresponds to the test statistic

P-value = 2 x P(z > |test statistic|)

= 2 x [1 – P( z < |test statistic|)]

**A P-Value tells us how likely it is to get a result like this if the null hypothesis is true!**

If P-value > **0.05**, result is **__not__** significant; we **__do not__** reject $H_0$

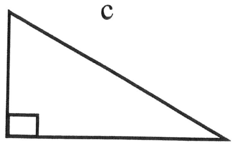
P-value ≤ **0.05**, result **__is__** significant; we **__do__** reject $H_0$

***__if p is low, null must go!__***

# TRIGONOMETRY NOTES

## Triangles!

### Right Angled Triangles:




Pythagoras: $c^2 = a^2 + b^2$ (*Pythagoras- used when sides given and looking for a side)

or

Trig Ratios

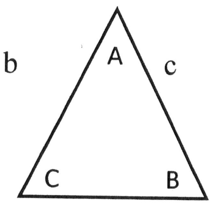
$$\text{Sin } X° = \frac{O}{H}$$

$$\text{Cos } X° = \frac{A}{H}$$

$$\text{Tan } X° = \frac{O}{A}$$

(*Trig Ratios- used when there's an angle involved)

### Non-Right Angled Triangles:




**-Sine rule:** $\frac{\text{Sin A}}{a} = \frac{\text{Sin B}}{b} = \frac{\text{Sin C}}{c}$ (*used when you can link opposites)

**-Cosine rule:** $a^2 = b^2 + c^2 - 2bc\text{CosA}$

Angle A is between these 2 sides [b and c] (*used when an angle between 2 sides is involved)

## Trig Related Formulae

-> Area of a Triangle: $A = \frac{1}{2} ab\text{Sin } C°$

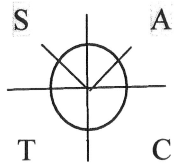
-> Area of a Sector: $A = \pi r^2\left(\frac{\theta}{360°}\right)$ (if angle is in degrees) or $A = \frac{1}{2} r^2\theta$. (if angle is in radians)

-> Length of an Arc: $l = 2\pi r\left(\frac{\theta}{360°}\right)$ (if angle is in degrees) or $l = r\,\theta$ (if angle is in radians)

## Solving Trig Equations

E.g. Sin x = $\frac{1}{2}$  $0 \le x \le 360^o$

-> where is Sin positive? (since ½ is positive)




$\text{Sin}^{-1}(\frac{1}{2}) = 30^o$

$180^o - 30^o = 150^o$ (must be in $2^{nd}$ quadrant)

*remember the spacing is always equal, in relation to the horizontal line, so if the gap is 30° into the A quadrant then it is 30° back into the S quadrant also.

E.g. Cos 2x = $\frac{-\sqrt{3}}{2}$  $0 \le x \le 360^o$

Let ■ = 2x

Cos ■ = $\frac{-\sqrt{3}}{2}$

-> where is Cos negative? (since $\frac{-\sqrt{3}}{2}$ is negative)


$\text{Cos}^{-1}(\frac{-\sqrt{3}}{2}) = 150^o$

$180^o + 30^o = 210^o$

*remember the spacing is always equal, in relation to the horizontal line, so if the gap is 30° into the S quadrant then it is 30° down into the T quadrant also.

For more solutions, +360° (because the cos curve repeats every 360°)

So: 150°; 210°; 510° (150 + 360); 570° (210 + 360)

So ■ = 150°, 210°, 510°, 570°

2x = 150°, 210°, 510°, 570°

x = 75°, 105°, 255°, 285° (dividing everything by 2)

OR

■ = 150° +360°n or y= 210 +360°n -**The General Solution** or **ALL SOLUTIONS**

2x = 150° +360°n, 210 +360°n

x = 75° + 180°n, 105° + 180°n [sub in n=0, then n=1]

**For Radians:**

Convert to radians $\pi = 180^o$

so to convert $\frac{2\pi}{3} = \frac{2(180)}{3}$  and to convert $30^o = 30 \times \frac{\pi}{180}$

If inputting cos $\frac{2\pi}{3}$ in the calculator to evaluate,
Make sure your calculator is in Radian mode!

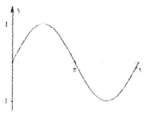
## Trig Graphs:

y = a + b(sin(cx))

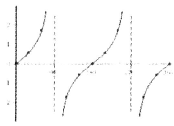
y = a + b(cos(cx)) → period = $\frac{2\pi}{c}$ or $c = \frac{2\pi}{Period}$ (how much the graph is SQUASHED)

| | |
|---|---|
| y = a | \| b \| |
| mid-line | amplitude (how much the graph is STRETCHED) |
| equation | range: [a - \| b \|, a + \| b \|] |

Key Points of Graphs:

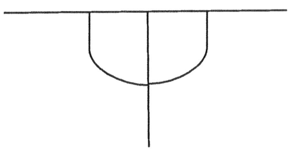
y = sin x, Shape:  and cuts the x-axis at 0°, 180°, 360° & every 180° after, Range [-1,1], Period 360°

y = cos x, Shape:  and cuts the x-axis at 90°, 270° & every 180° after, Range [-1,1], Period 360°

y = tan x, Shape:  and cuts the x-axis at 0°, 180°, 360° & every 180° after, Range [-1,1], Period 360°

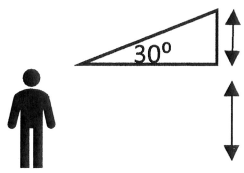
**Extra Notes**

-Note: a clinometer measures angles of elevation



however much the weight moves from 90° is the angle of elevation

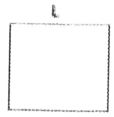
-Watch out for questions where you need to add on the height of the person to get the height you need!



## Topic 16: Area/Volume

### 1) The Formulae: (Note the ones with an asterisk next to them are **NOT** in the Tables)

**Square:**




Area = $L^2$ *

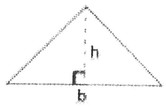
Per = 4L *

**Rectangle:**


Area = L x W *

Per = 2L + 2W *

**Triangle:**



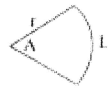

Area = ½ b x h

**Circle:**


Area = $\pi r^2$

Circum = $2\pi r$

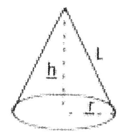
**Sector:**




Area = $\frac{A}{360°} \times \pi r^2$

$L = \frac{A}{360°} \times 2\pi r$

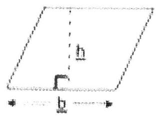
**Cone:**




Vol = $\frac{1}{3}\pi r^2 h$

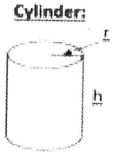
CSA = $\pi r L$

**Parallelogram:**




Area = b x h

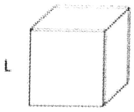
**Cylinder:**




Vol = $\pi r^2 h$

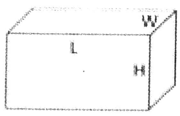
CSA = $2\pi r h$

**Cube / Cuboid:**




Vol = $L^3$ *

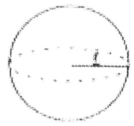
TSA = $6L^2$ *




Vol = L x W x H *

TSA = 2LW + 2WH + 2HL *

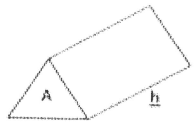
**Sphere:**




Vol = $\frac{4}{3}\pi r^3$

CSA = $4\pi r^2$

**Prism:**




Vol = Area A x h

### 2) Solving Problems:

**a) Tips for solving Area/Volume problems:**

1. Draw a good-sized diagram.
2. Label and fill in all information given.
3. Identify the shapes in the question.
4. Write down relevant formulae for those shapes.

**b) Recasting/Remoulding:**

- Melting down shapes and making new shapes.

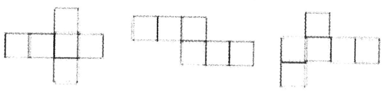
Vol of Old Shape = Vol of New Shape(s)

### 3) Nets:

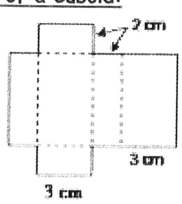
- The **net**, of a particular shape, is a flat surface that, when folded, can be made into that shape.

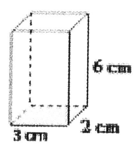
**a) Nets of Cubes:**

- There are 11 nets for a cube. Some are shown below.

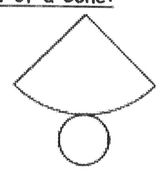


**b) Net of a Cuboid:**

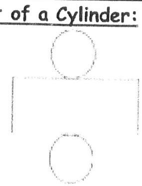


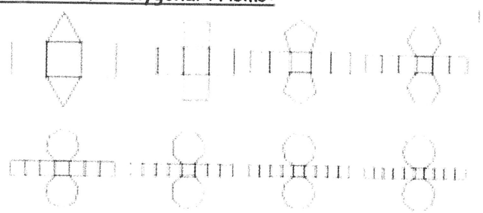

**c) Net of a Cone:**



**d) Net of a Cylinder:**



**d) Nets for some Polygonal Prisms:**



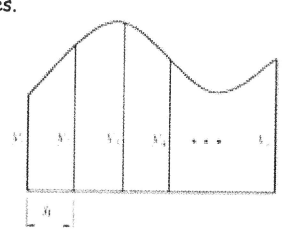
### 4) Trapezoidal Rule:

**Note:**

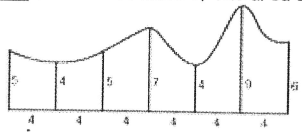
- Used to **estimate** the area of **irregular** shapes.

$$A \approx \frac{h}{2}[y_1 + y_n + 2(y_2 + y_3 + \cdots y_{n-1})]$$

1st height — Last height — Other heights



**Example:** Find an estimate of the area below:




$$A \approx \frac{h}{2}[y_1 + y_n + 2(y_2 + y_3 + \cdots y_{n-1})]$$

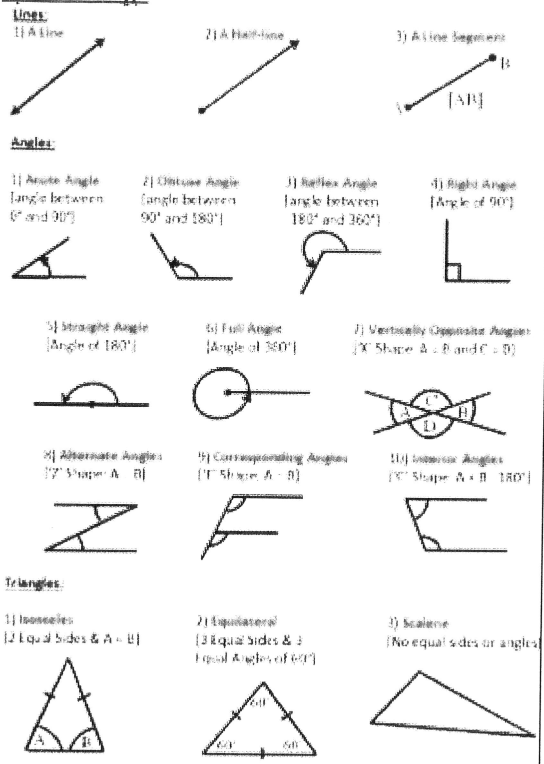
$$A \approx \frac{4}{2}[5 + 6 + 2(4 + 5 + 7 + 4 + 9)]$$

$$A \approx 2[69] = 138 \text{ units}^2$$

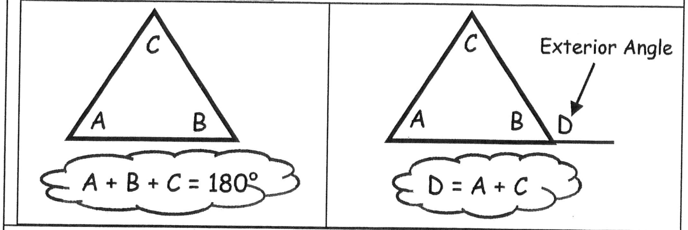
Topic 12: ***Geometry***

## 1) The Basics:

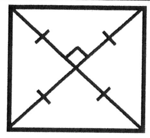
### a) Terminology:



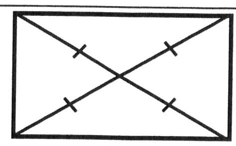

### b) Properties of Triangles:



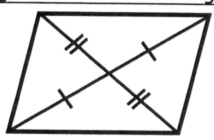

### c) Properties of Quadrilaterals:



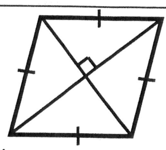
Square
- 4 equal sides
- 4 equal angles of 90°
- Opp sides are parallel
- Diagonals bisect at 90° angles



Rectangle
- Opp sides are equal & parallel
- 4 equal angles of 90°
- Diagonals bisect each other



Parallelogram
- Opp sides are equal & parallel
- Opp angles are equal
- Diagonals bisect each other

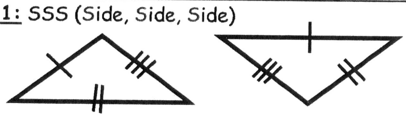


Rhombus
- 4 equal sides & opp sides are parallel
- Opp angles are equal
- Diagonals bisect at 90° angles

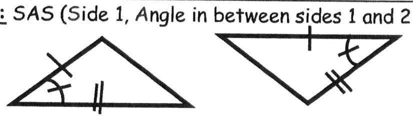
### d) Conguent Triangles

- Triangles that sit exactly on top of each other
- Matching sides are called **corresponding sides**

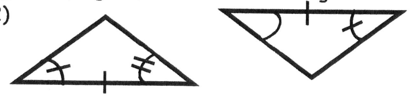
**Type 1:** SSS (Side, Side, Side)



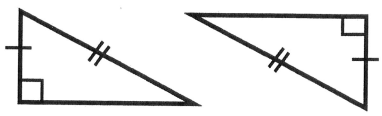
**Type 2:** SAS (Side 1, Angle in between sides 1 and 2, Side 2)



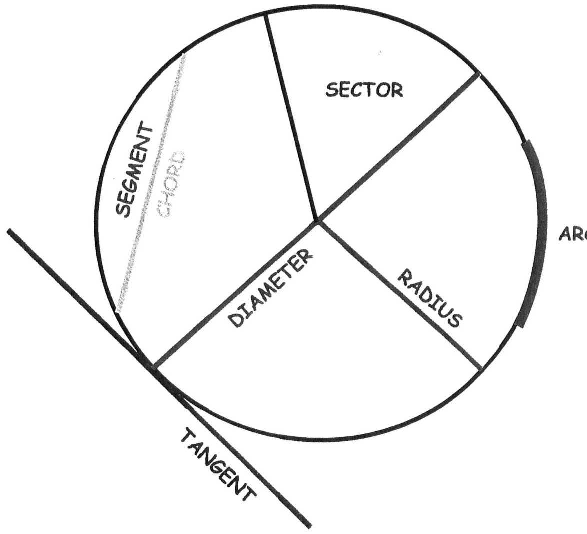
**Type 3:** ASA (Angle 1, Side in between angles 1 and 2, Angle 2)



**Type 4:** RHS (Right Angle, Hypotenuse, One other side)



### e) Circle Terminology:

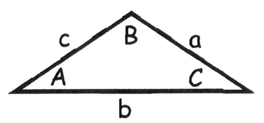


## 2) Theorems: (Note that the formal proofs of Theorems 11, 12 and 13 need to be known)

### a) Theorems:

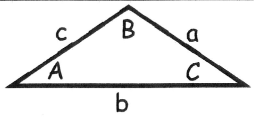
1. Vertically opposite angles are equal in measure.
2. In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.
3. If a transversal makes equal alternate angles on two lines then the lines are parallel, (and converse).
4. The angles in any triangle add to 180°.
5. Two lines are parallel if and only if, for any transversal, the corresponding angles are equal.
6. Each exterior angle of a triangle is equal to the sum of the interior opposite angles.

7. In a triangle, the largest angle is opposite the largest side, and the smallest angle is opposite the smallest side.




If a is the largest side, then A is the largest angle, and vice versa.

8. In a triangle, two sides of a triangle are always greater than the third.

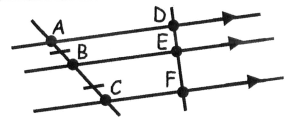



$a + b > c$
$b + c > a$
$a + c > b$

9. In a parallelogram, opposite sides are equal and opposite angles are equal (and converses).

10. The diagonals of a parallelogram bisect each other.

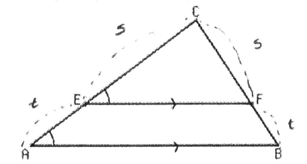
11. If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.
**(Proof)**




If |AB| = |BC|

=> |DE| = |EF|

12. Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio s:t, then it also cuts [AC] in the same ratio.
**(Proof)**

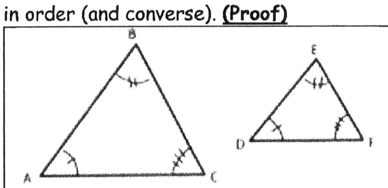



If EF is parallel to AB

=> $\frac{|AE|}{|CE|} = \frac{|BF|}{|CF|}$

or $\frac{|AE|}{|AC|} = \frac{|BF|}{|BC|}$

13. If two triangles are similar, then their sides are proportional, in order (and converse). **(Proof)**




$\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$

OR

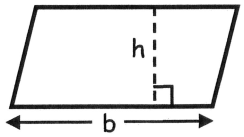
$\frac{|DE|}{|AB|} = \frac{|EF|}{|BC|} = \frac{|DF|}{|AC|}$

14. [Theorem of Pythagoras] In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.

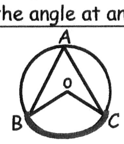
15. If the square of one side of a triangle is the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

16. For a triangle, base x height doesn't depend on the choice of base.
17. A diagonal of a parallelogram bisects the area.

18. The area of a parallelogram is base x height.

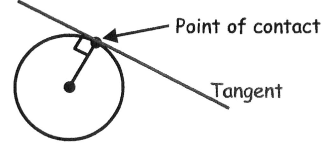



19. The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on that arc.

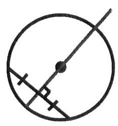



$|\angle BOC| = 2(|\angle BAC|)$

20. Each tangent to a circle is perpendicular to the radius at the point of contact.




21. The perpendicular from the centre of a circle to a chord bisects the chord.

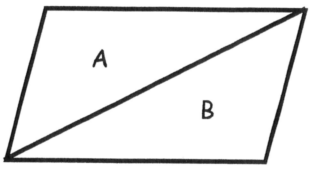


### b) Other Theorem Terminology:

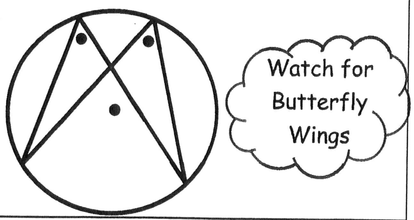
1. **Collinear** points are points that lie on the same line.
2. An **axiom** is a statement that we accept without any proof: e.g. There is exactly one line through any two given points.
3. A **theorem** is a rule that you can prove by following a certain number of logical steps or using a previous theorem or axiom. E.g. Pythagoras' Theorem
4. A **proof** is a series of logical steps that we use to show a theorem is true.
5. A **corollary** is a statement that follows readily from a previous theorem.
6. The **converse** of a statement is formed by reversing the order in which the statement is made.
   e.g. Statement: If P, then Q    Converse: If Q, then P.
7. '**Implies**' is a term we can use in a proof when we write down a fact or conclusion that follows from previous statements. The symbol is for implies is: =>

## 3) Corollaries: (The 6 results below follow on from the 21 theorems above)

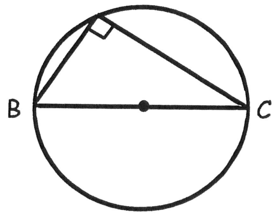
1. A diagonal divides a parallelogram into 2 congruent triangles i.e. triangles A and B below are congruent.



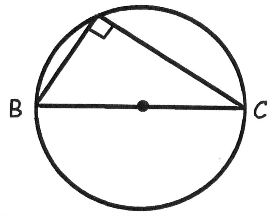

2. All angles at points of a circle, standing on the same arc, are equal, (and converse).



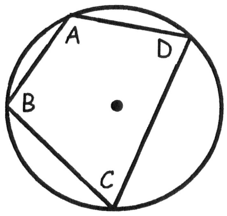

3. Each angle in a semi-circle is a right angle.




4. If the angle standing on a chord [BC] at some point, of the circle is a right-angle, then [BC] is a diameter.

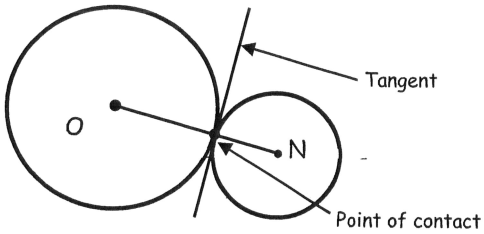



5. In a cyclic quadrilateral, then opposite angles sum to 180, (and converse).




$A + C = 180°$
$B + D = 180°$

6. If two circles intersect at one point, their centres and the point of contact are collinear.




## 4) Constructions:

**General Tips:**
1. Keep your work neat and tidy.
2. Choose an appropriate pencil to draw the construction, not too dark and not too light.
3. Draw rough sketches of construction first, especially for triangles and rectangles.
4. Show all your construction lines & label your construction.

- There are 21 constructions on the course for Leaving Cert Ordinary Level. (**See Booklet from class for step by step instructions)**

**Constructions List:**
1. Bisector of a given angle, using only compass and straight edge.
2. Perpendicular bisector of a segment, using only compass and straight edge.
3. Line perpendicular to a given line l, passing through a given point not on l.
4. Line perpendicular to a given line l, passing through a given point on l.
5. Line parallel to a given line, through a given point.
6. Division of a line segment into 2 or 3 equal segments, without measuring it.
7. Division of a line segment into any number of equal segments, without measuring it.
8. Line segment of a given length on a given ray.
9. Angle of a given number of degrees with a given ray as one arm.

10 - 12. Triangle, given i) SSS ii) SAS or iii) ASA data

13. Right-angled triangle, given the length of the hypotenuse and one other side.
14. Right-angled triangle, given one side and one of the acute angles (several cases).
15. Rectangle, given side lengths.
16. Circumcentre and circumcircle of a given triangle, using ruler and compass.
17. Incentre and incircle of a given triangle, using ruler and compass.
18. Angle of 60°, without using a protractor or set square.
19. Tangent to a given circle at a given point on it.
20. Parallelogram, given the length of the sides and the measure of the angles.
21. The centroid of a triangle.
22. The orthocentre of a triangle.

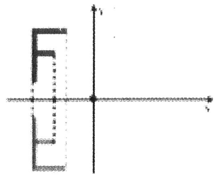
## 5) Transformations/Symmetries/Enlargements:

### a) Transformations:

**Note:** In each of the pictures below, the red shape is the **object** and the second coloured shape is the **image**.

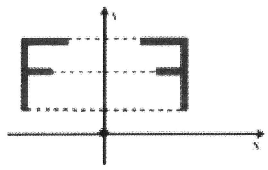
**Axial Symmetry in the X-axis: ($S_x$)**

- Shapes are mirrored/reflected in the X-axis. See example below.



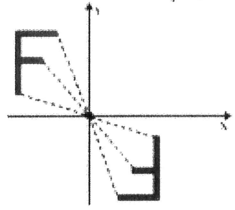
**Axial Symmetry in the Y-axis: ($S_y$)**

- Shapes are mirrored / reflected in the Y-axis. See example below.



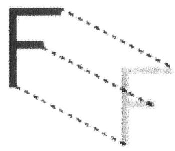
**Central Symmetry in the Origin: ($S_o$)**

- Shapes end up flipped and rotated as shown below.
- Central symmetry in a point other than the origin would have the same effect on the shape i.e. flipped and rotated



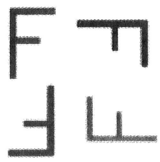
**Translation:**

- Note that shapes don't change when translated as the shape just '**slides**' to another position



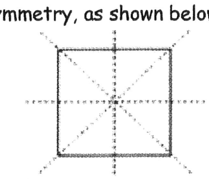
**Rotations:**

- The shape in blue below is a rotation of the red shape 90° clockwise. The green is a rotation of 180°. Note that it looks similar to the central symmetry in a point image from above. The orange is a rotation of 270° clockwise.

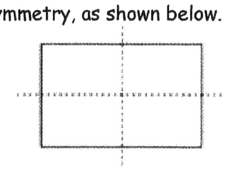


### b) Axes of Symmetries of Shapes:

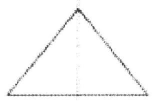
**Square:**
A square has 4 axes of symmetry, as shown below.



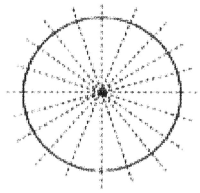
**Rectangle:**
A rectangle has 2 axes of symmetry, as shown below.



**Triangle:**
An isosceles triangle has 1 axis of symmetry, as shown below. If the triangle was an equilateral triangle it would have two more axes of symmetry from the other two vertices of the triangle.



**Circle:**
A circle has an infinite number of axes of symmetry, as shown below.



### c) Enlargements:

**Notes:**

- An **enlargement** is a scaled up/down version of an object.
- The **scale factor 'k'** tells by how much the image has been scaled.
  - If $k > 1$ => scaled up
  - If $0 < k < 1$ => scaled down
- To find the scale factor from a given enlargement, divide a side of the image by its corresponding side in the object.
- The area of the image can be found by using:

$$Area\ Image = k^2 \times Area\ of\ Object$$

Not in Tables

- The **centre of enlargement** is the point where the object is being enlarged from.

**Steps for constructing an enlargement:**

1. Using a ruler, draw dashed construction lines from the centre of enlargement o, out through some of the main points of the object.
2. Measure the length of |OA|.
3. Multiply |OA| by the scale factor k and then measure out that distance from o along the dashed line to find location of A'.
4. Repeat for the other key points B, C, D, E etc.
5. Join up A', B', C', D' etc. to form image.
6. Check |A'B'| ÷ |AB| should be = the scale factor k.

Example: Enlargement for k = 3 of small L shape is shown below.