

# Optimum Noise Thresholds in Decision Directed Impulse Noise Mitigation for OFDM

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**Abstract**—Impulse noise is a significant problem in some OFDM applications including digital television broadcasting. In this paper we study the optimum threshold values for a novel decision directed impulse noise mitigation algorithm. In this algorithm the noise component in each received input sample is estimated based on preliminary decisions on the transmitted data. When the noise estimate is above a given threshold, it indicates that impulse noise is present in the sample and the estimated noise component is subtracted from the input sample before final demodulation. In this paper the optimum threshold levels for varying impulse noise parameters are calculated.

## I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) technology is used in many digital broadband communication systems. One of the advantages of OFDM compared to single carrier systems is that it is more resistant to the effects of impulse noise because of the spreading effect of the discrete Fourier transform (DFT). However impulse noise can still cause significant problems in OFDM systems. This is a major practical problem in digital video broadcasting (DVB) [1-5].

The theoretical effects of impulse noise in multicarrier systems have been analyzed [6], and a number of techniques for mitigating the effect of impulse noise have been described. One approach is to identify peaks in the received time domain signal and reduce these by either clipping or nulling the sample [5], [8], [9]. This is effective only for impulse noise with peaks larger than the wanted OFDM signal. This will be true only in very extreme cases. In high signal to noise environments such as broadcast television, the impulse noise can be well above the background Gaussian noise, yet well below the OFDM signal.

Several authors have used techniques that operate on the signal in the frequency domain [7], [10], [11]. Häring and Han Vinck [7] describe an iterative process in which information is exchanged between estimators operating in the time and frequency domains. The simulation results they present are for extreme cases with very large noise impulses. In [10], impulses are detected in the frequency domain by identifying subcarriers with extreme values. In [11] the positions of noise impulses are identified using pilot tones.

Very recently, decision directed impulse mitigation has been developed separately and independently by two groups [4], [12-14]. Some details of the techniques are slightly different, but the basic concept is the same. Preliminary decisions are made about the transmitted data and from these an estimate is made of the noise in the received signal. The estimated noise is subtracted from the original signal before final demodulation. When the input noise is impulsive, the technique substantially reduces the noise power. The technique depends on the fact that the signal appears random in the time domain and highly structured in the discrete frequency domain whereas for the impulse noise the converse is true.

In [13], [14], a more theoretical approach is taken, and an analysis of the decision and noise estimation processes is presented. Whereas [4] has a more practical emphasis with results being presented for noise captured from a real world interference source.

The decision directed estimation technique requires a non linear noise estimation function. In this paper threshold type non-linearities are investigated and the optimum thresholds found for varying impulse noise parameters.

## II. IMPULSE MITIGATION TECHNIQUE

Fig. 1 shows the block diagram of a receiver with the new mitigation technique. The received OFDM baseband signal samples are given by

$$x(l) = r(l) + n_g(l) + n_i(l) = r(l) + n_t(l) \quad (1)$$

where  $r(l)$  is the wanted OFDM signal,  $n_g(l)$  is the Gaussian noise and  $n_i(l)$  is the impulse noise.  $n_t(l) = n_g(l) + n_i(l)$  is the total noise at the input. The samples  $x(l)$  are optionally passed through a non-linearity that clips or nulls large samples. The samples at the output of the non-linearity,  $z(l)$ , are serial-to-parallel converted to form the vector of  $N$  complex samples that are input to the  $N$ -point DFT. The output of the DFT is the  $N$ -point vector  $Z$ . Preliminary decisions,  $\hat{D}_p(k)$ , about the transmitted data are made based on  $Z(k)$ . The noise component of  $Z(k)$  is  $N_t(k)$ . The observed noise,  $N_p(k)$ , is calculated using

$$N_p(k) = Z(k) - \hat{D}_p(k) \quad (2)$$

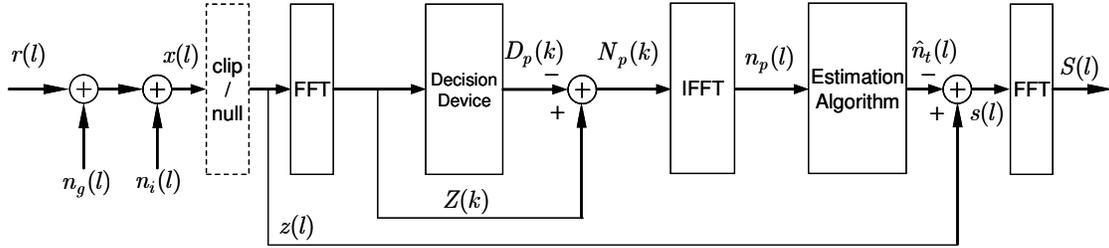


Fig. 1. Block diagram of receiver with impulse mitigation.

Except for extreme cases, most of the received subcarriers are correctly decoded and the observed noise is exactly equal to the received noise in that subcarrier. In the cases where the subcarrier is incorrectly decoded, ‘decision noise’ will be added to the observed value.

OFDM is more resistant to the effects of impulse noise than single carrier systems because of the spreading effect of the receiver DFT operation. The energy of each impulse is spread evenly across all of the subcarriers in that symbol. When there is more than one impulse in a received symbol period,  $T$ , the contributions combine linearly in each subcarrier. When there are enough impulses during  $T$  for the central limit theorem to apply,  $N_i(k)$  has a Gaussian distribution.

The vector  $\mathbf{N}_p$  is then converted back into the discrete time domain using an inverse FFT to give the vector  $\mathbf{n}_p$ . If there are no decision errors,  $n_p(l) = n_t(l)$ . However even in the presence of decision errors  $n_p(l)$  contains some information about  $n_t(l)$ .  $n_p(l)$  is then input to an estimation device to generate an estimate  $\hat{n}_t(l)$  of the total input noise. This is subtracted from  $z(l)$  to generate  $s(l)$ . The rest of the receiver is a standard OFDM receiver consisting of DFT etc.

The task of the estimation algorithm is to estimate the presence and size of noise impulses. A number of algorithms are possible. In this paper, we consider two estimation algorithms: a threshold operating on the real and imaginary components separately and an estimation algorithm operating on the amplitude of each sample. If the estimated noise component in a sample is above the threshold, then the estimated value is subtracted from the input signal before the second stage of demodulation if it is below the threshold the input sample is unchanged.

The operation of the amplitude non-linearity is described by:

$$\begin{aligned} \hat{n}_t(l) &= an_p(l) \quad \text{for } |n_p(l)| > \alpha \\ &= 0 \quad \text{for } |n_p(l)| < \alpha \end{aligned} \quad (3)$$

To describe the second non-linearity, represent  $\Re(n_p(l))$  as  $r_p$ ,  $\Re(n_t(l))$  as  $r_t$  and  $\Re(n_d(l))$  as  $r_d$ . Then the operation of the real non-linearity is described by

$$\begin{aligned} \hat{r}_t &= ar_p \quad \text{for } |r_p| > \alpha \\ &= 0 \quad \text{for } |r_p| < \alpha \end{aligned} \quad (4)$$

The operation of the imaginary non-linearity is identical.

For the technique to be effective in reducing the overall bit error rate (BER) of the system,  $n_t(l)$  must be impulsive (not stationary Gaussian) and the estimation algorithm must be non-linear. If the estimation process is linear, it will appear to improve the received constellation as each point moves towards the value  $\hat{D}_p(k)$ , but the points will move closer to both incorrect and correct decision points. However, the technique is very effective if non-linear processing is used and the noise is impulsive. This depends on the fact that for large values of  $n_t(l)$ ,  $n_p(l) \approx n_t(l)$ .

Fig. 1 shows an optional clipping or nulling function operating on the received baseband samples. This reduces the effect of very large noise impulses that are above the envelope of the OFDM signal. However simulations have shown that this improves the performance only in very extreme cases [13].

### III. IMPULSE NOISE MODELS

A number of models for impulse noise have been presented in the literature [1], [2], [15], [16]. Some characterize only the probability density function of the amplitude of the noise, whereas others also consider the time correlation of impulse events. Very recent research by the BBC, which measured a variety of impulse noise sources, has shown that many of the impulse noise sources of practical importance in OFDM applications can be modeled as gated Gaussian noise [1], [2].

In this paper we use a particular form of gated Gaussian noise, where the noise is the sum of additive white Gaussian noise (AWGN) of variance  $\sigma_n^2$  and a second higher variance Gaussian noise component which lasts for a fraction,  $\mu$ , of the time duration of each OFDM symbol and which has variance  $\sigma_i^2$  during this time. (i.e. the variance is calculated over only  $\mu T$  not over  $T$ ). In general  $\sigma_i^2 \gg \sigma_n^2$ . The total noise power is then  $\sigma^2 = \mu\sigma_i^2 + \sigma_n^2$ . Each of these variances is for the real and imaginary components taken separately. The impulsive samples are spread randomly throughout each OFDM symbol.

The gated Gaussian model is used because it gives a good indication of the performance of OFDM systems. Here the critical factor is whether the BER for each symbol is above the threshold at which the error correcting coding will reduce the final BER to an acceptable level, rather than the BER averaged over the entire received signal. It also allows the length and power of the impulse noise to be varied in a way that makes clear the practical implications of the technique. For example, in the context of DVB, it indicates how the resistance to impulse noise can be improved by increasing the transmitter power or choosing the 8k rather than 2k mode.

#### IV. SIMULATION RESULTS

Matlab simulations were used to examine how the performance of decision directed noise mitigation technique depends on the non-linear estimation process. The performance is measured both in terms of symbol error rate (SER) and the normalized mean square error of the noise estimation. The normalized error is given by:  $E\{|\hat{n}_t - n_t|^2\} / E\{|n_t|^2\}$ .

The simulations are for a flat fading channel. For each simulation, the average power in each of the real and imaginary components of the wanted OFDM signal is unity. Figs. 3 and 4 show the resulting SER as a function of  $E_b/N_0$  where  $N_0$  is the single sided spectral density of the white Gaussian (non impulse component). The impulse noise parameters are  $\sigma_i^2 = 0$  dB and  $\mu = 0.01$ . In other words, for each plot, the impulse noise is kept constant and the effect of varying  $E_b/N_0$  is measured. 64QAM modulation and 2048 subcarriers were used. In the simulations it was assumed that all subcarriers were carrying data and no pilot tones were used. The thresholds are standardized in terms of the standard deviation of the wanted OFDM signal. Fig. 2 shows the results for amplitude thresholding, while Fig. 3 shows the results for real/imaginary thresholding. The weighting factor was set at  $a = 1$ . It is clear that for these parameters the use of an amplitude threshold give better performance than a real/imaginary threshold. Simulation for other parameters (not shown) indicate that this is true in general, so the rest of the simulation results are for the amplitude case. Amplitude thresholding gives better results because in the gated Gaussian model, both the real and imaginary components of a given sample are either impulsive or non-impulsive. This will also be true for real world impulse noise sources.

Fig. 4 shows how the normalized mean square error in the noise estimation process varies with threshold. This is of interest because calculation of mean square error rather than BER is more mathematically tractable. In [13] an analysis was presented in terms of minimum mean square error. A comparison of Figs. 2 and 4 indicates that the threshold which gives the minimum mean square error (MMSE) (0.4) is not the

same as the threshold which gives the minimum SER (0.5). This is also true for other parameters and is because the time domain noise is partially correlated.

Fig. 5 explores this relationship. From this it can be seen that the BER follows a typical curve (inverted in the x-axis because the plot is versus noise rather than SNR) for higher thresholds but not for lower thresholds.

Fig. 6 shows the normalized mean square error as a function of threshold for  $\sigma_i^2 = -12$  dB and  $\mu = 0.16$ . In other words the impulsive noise is for a longer proportion of the symbol period but at a lower level. The values are chosen so that the total noise power is the same as in the earlier figures. For these values the impulse mitigation is still effective but Fig. 6 shows (and the SER simulations confirm) that the selection of the threshold level is much more critical in this case. This is because the impulsive noise is only slightly above the background white Gaussian noise and the 'decision noise'.

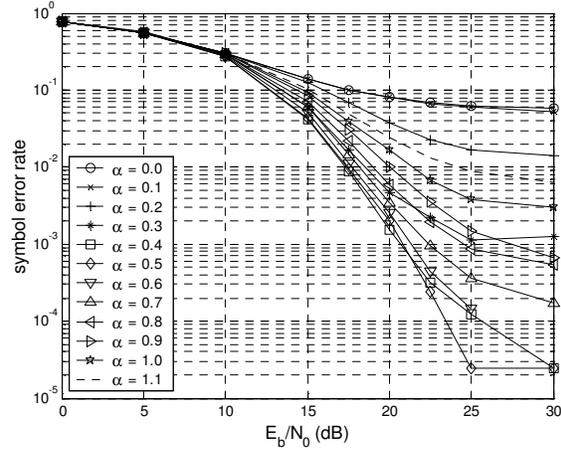


Fig. 2. SER versus  $E_b/N_0$  for amplitude threshold and varying  $\alpha$ , for  $\sigma_i^2 = 0$  dB and  $\mu = 0.01$ .

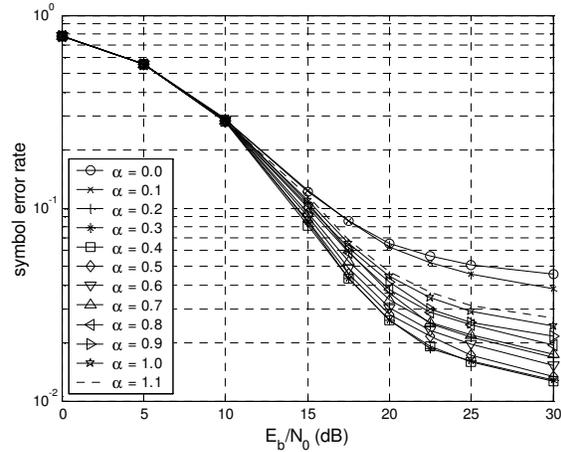


Fig. 3. SER versus  $E_b/N_0$  for real and imaginary thresholding and varying  $\alpha$ , for  $\sigma_i^2 = 0$  dB and  $\mu = 0.01$ .

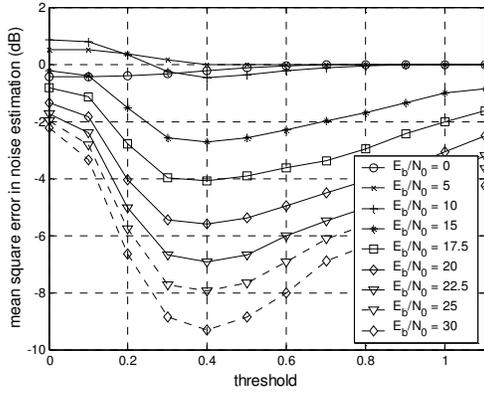


Fig. 4. Normalized mean square error versus amplitude threshold,  $\alpha$  and varying  $E_b/N_0$ , for  $\sigma_i^2 = 0$  dB and  $\mu = 0.01$ .

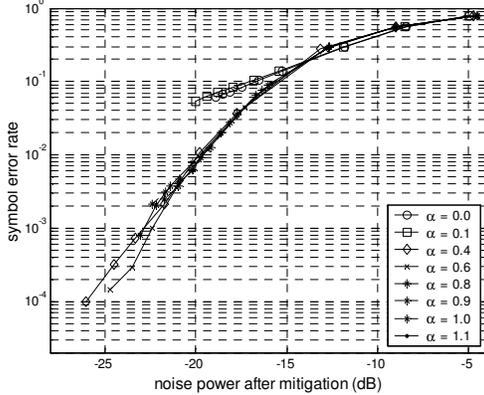


Fig. 5. SER versus normalized mean square error for varying amplitude threshold  $\alpha$  and  $\sigma_i^2 = 0$  dB and  $\mu = 0.01$ .

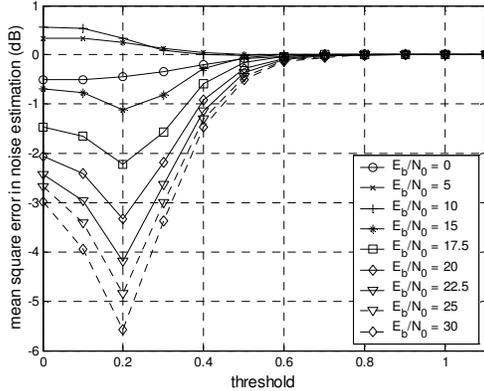


Fig. 6. Normalized mean square error versus threshold amplitude threshold,  $\alpha$  and varying  $E_b/N_0$ , for  $\sigma_i^2 = -12$  dB and  $\mu = 0.16$ .

## V. CONCLUSIONS

Simulation results have been presented to show the effect of using different non-linear estimation algorithms in decision directed impulse mitigation for OFDM. It is shown that better performance is obtained using thresholds based on the amplitude of the complex baseband signal rather than non-linearities which operate on the real and imaginary components separately.

The ultimate measure of performance is the overall SER, however mean square error is more mathematically tractable. It was found that for given simulation parameters, the threshold which results in the minimum mean square error is generally lower than the threshold which gives the minimum SER. This is because of correlation between the estimation errors in different samples. For given impulse noise parameters, the optimum threshold varies only slightly with the level of background white noise

The impulse noise mitigation gives the greatest improvement in performance when the impulsive noise energy is concentrated in a small proportion of a symbol period. For a given total impulse noise energy, the choice of threshold becomes more critical if the energy is spread over more signal samples.

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