Three-dimensional finite-difference magnetotelluric forward modeling in the presence of magnetic permeable heterogeneity

Libin Lu[1], Handong Tan[1] and Kunpeng Wang[1]
[1]China University of Geosciences, Beijing, China

Summary

Allowing magnetic permeability to be heterogeneous in the subspace, we have implemented an algorithm for the three-dimensional forward modeling in magnetotelluric method (MT) based on finite difference scheme. The finite difference equations were deduced based on integral forms of Maxwell's equations. The inverse of magnetic permeability was parameterized in preference to its original form. To improve the efficiency of our algorithm, the divergence correction process was incorporated into our implementation and the algorithm has also been parallelized using OpenMP. The forward responses calculated by our algorithm were quite consistent with that generated through comsol multiphysics software package in the validation.

We tested our algorithm on two synthetic models, either one of which consisted of a conductive or resistive body in a homogeneous half space. Through synthetic studies we found that magnetic permeability had a quasi-resistive impact on the forward responses for both conductive and resistive model cases. Specifically, the apparent resistivities came to be more higher and the phases decreased. For the conductive model case, the resistivity differences as seen from forward responses (i.e. apparent resistivity and phase) were lessened if the magnetic permeability of the anomalous body was higher than the background. Meanwhile, The resistive body became more resistive due to the elevated magnetic permeability. Moreover, the resistive anomalies could be more easily influenced by magnetic permeability than the conductive ones.

Keywords: magnetotelluric, forward modeling, magnetic permeability, finite difference, staggered grid

Introduction

Rapid improvements have been attained in three-dimensional magnetotelluric forward modeling in recent decades. Numerical simulations based on integral equation, finite difference and finite element methods have the capabilities of modeling geo-electrical structures which have the properties of electrical conductivity σ and magnetic permeability μ et al. However, most work ignored magnetic permeability and accepted the assumption that the magnetic permeability of the medium from the subsurface is the same as that of free space. When concerning strongly-magnetic substance, such as magnetite, it is questionable to retain the supposition. Sometimes, the magnetic permeability of the medium might be several times of μ_0, the magnetic permeability of free space. In some cases, The elevated magnetic permeability could be so high as to affect the estimated electrical conductivity from electromagnetic data, like ZTEM, a magnetotelluric-variant method (Sattel & Witherly, 2015).

Model tests from finite element implementation suggested that with high-valued magnetic permeability the effects on forward responses should be accounted for (Ren, Kalscheuer, Greenhalgh, & Maurer, 2014). As a supplementary research, we have implemented an algorithm using finite difference method for three-dimensional forward modeling in magnetotellurics considering heterogeneous magnetic permeability. And more model tests are studied to evaluate and illustrate the effects of magnetic permeability.

Methods

The electromagnetic fields in magnetotellurics are governed by the quasi-static forms of Maxwell's equations with the displacement current neglected and assuming a time dependency of $e^{-i\omega t}$, which have the integral forms of

$$\oint \mathbf{E} \cdot dl = \iint i\omega\mu\mathbf{H} \cdot dS \tag{1}$$

$$\oint \mathbf{H} \cdot dl = \iint \sigma\mathbf{E} \cdot dS \tag{2}$$

where **E** and **H** are electric and magnetic fields respectively. ω is angular frequency. σ and μ are the electrical conductivity and magnetic permeability of the materials. Here we choose the integral forms of Maxwell's equations rather than the differential ones because Mackie (1993) indicated that they led to the same difference equations as those from the differential forms and clarified better understanding of the coupling between those two equations. Usually one kind of the electromagnetic fields (**E** or **H**) will be resolved and the other should be derived using Eq. (1) or Eq. (2). Our algorithm solves for electric fields firstly and later obtains magnetic fields, by which more flexibility will be added to the mesh designation (Siripunvaraporn, Egbert, & Lenbury, 2002).

Staggered grid is often employed to solve electromagnetic (e.g. magnetotelluric) forward problem that uses finite difference method, as shown in Fig. 1.

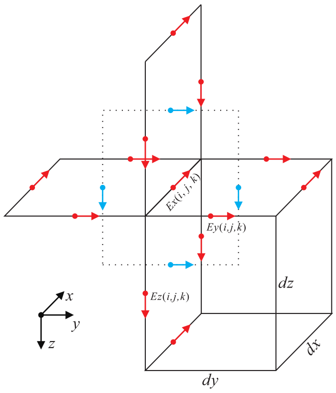

Figure 1. Demonstration for the coupling Maxwell's equations on staggered grid in finite difference method for 3-D magnetotelluric forward modeling. The electric fields are sampled along the cell edges and presented as red arrows. The magnetic fields are sampled across the center of the cell faces and presented as light blue arrows. The magnetic fields are across the discontinuities, thus requiring averages of the magnetic permeability of the corresponding adjacent cells.

Applying Eqs. (1) and (2) on staggered grid all over the modeling subsurface will establish a linear system of equations (Tan, Yu, Booker, & Wei, 2003),

$$A\mathbf{E} = \mathbf{b} \tag{3}$$

where A is a complex, sparse and symmetric matrix which can has 13 elements at most in every row. **E** is the unknown vector of the discretized electric fields. **b** depends on boundary condition, which can be imposed by a dirichlet boundary condition using two-dimensional modeling result (Mackie, Smith, & Madden, 1994). We notice that the magnetic fields are across the discontinuities since we allow magnetic permeability to be variable over cells, plus we should also decide the approximation of the conductivity related to the centered electric field, i.e., $E_x(i, j, k)$ in Fig. 1. Straightforwardly, we use volume-weighted conductivities of the four corresponding cells to get the approximated conductivity. And the averaging can be done on magnetic permeability (Alumbaugh, Newman, Prevost, & Shadid, 1996), but we prefer the reformation based on its inverse,

$$\tau_{avg} = \frac{\Delta L_1}{\Delta L_1 + \Delta L_2}\tau_1 + \frac{\Delta L_2}{\Delta L_1 + \Delta L_2}\tau_2 \tag{4}$$

Here τ is the inverse of magnetic permeability. ΔL_1 and ΔL_2 are the lengths of two adjacent cells along the discontinuous direction.

The linear system of equations arisen from finite difference method can be well solved by Krylov method, such as QMR, an efficient and stable method for solving the sparse and non-hermitian linear system (Freund & Nachtigal, 1991, 1996), which has been used by several codes of electromagnetic forward modeling (Siripunvaraporn, Egbert, Lenbury, & Uyeshima, 2005; Kelbert, Meqbel, Egbert, & Tandon, 2014). The iterative process will be terminated when the relative residual norm, $||\mathbf{b} - A\mathbf{x}_n||/||\mathbf{b}||$, has decreased below 10^{-10}. To reduce the computation time, we parallelize our code using OpenMP over frequencies being used in magnetotelluric forward modeling.

We have also incorporated divergence correction, which is proven to be useful to accelerate the solutions. In our algorithm divergence correction is applied every 50 iterates which should be moderate for the computation time and accuracy of the solutions. (Farquharson & Miensopust, 2011; Wang, Tan, Zhang, Li, & Cao, 2016)

To validate our algorithm, a conductive and magnetic permeable prism embedded in a resistive homogeneous half space served as a test model, as shown in Fig. 2.

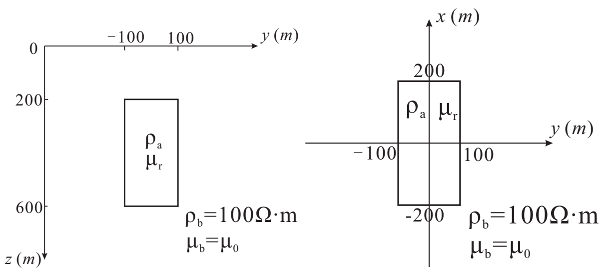

Figure 2. Cross section (left panel) and plane view (right panel) of the tested model used by validation and synthetic studies.

The resistivity of the prism, ρ_a, is 10 $\Omega{\cdot}m$ and relative magnetic permeability, μ_r, is 2.0 in the validation. The forward responses are measured along y axis at $x = 0$. We compared the responses at the frequency of 100 Hz calculated by our algorithm (denoted as FDMT3DM) with that by comsol multiphysics software package which is capable of modeling geophysical electromagnetic methods (Butler & Zhang, 2016). The apparent resistivities and phases are plotted in Fig. 3. Acceptable agreements have been obtained by those two kinds of approaches.

Model tests

The tested model is shown in Fig. 2. Both the conductive anomaly and resistive anomaly were examined.

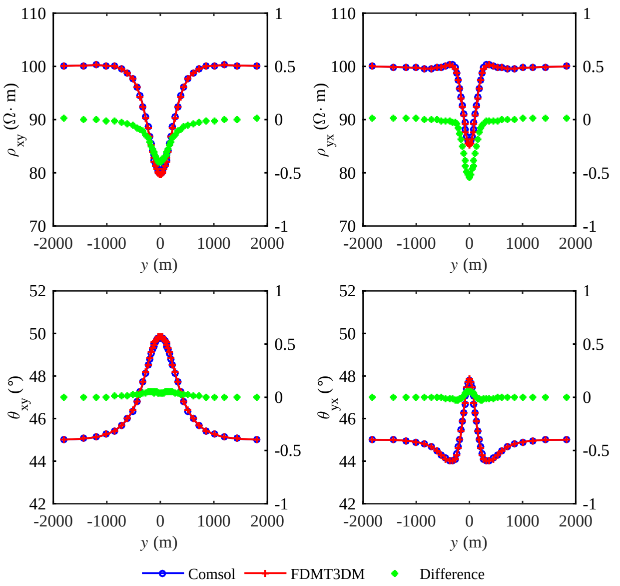

Figure 3. Comparing forward responses computed by our algorithm (FDMT3DM) with that by comsol multiphysics. The differences between them are plotted in green dots and controlled by the right axis.

The profile located where it did in the validation. And the responses at the frequency of 100 Hz were calculated for the two different cases. We compared the forward responses of a conductive and magnetic permeable anomaly with a identically conductive but non-magnetic anomaly to see the effects of magnetic permeability. The conductive anomaly case is shown in Fig. 4 while the resistive anomaly case is shown in Fig. 5.

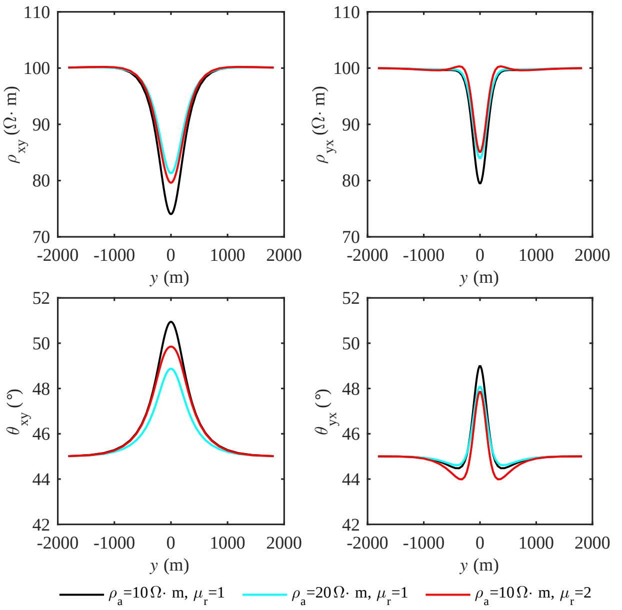

Figure 4. Magnetic effects on magnetotelluric forward responses of a conductive anomaly.

An extra more resistive anomaly was appended to the comparison list to evaluate the extend of the magnetic effects.

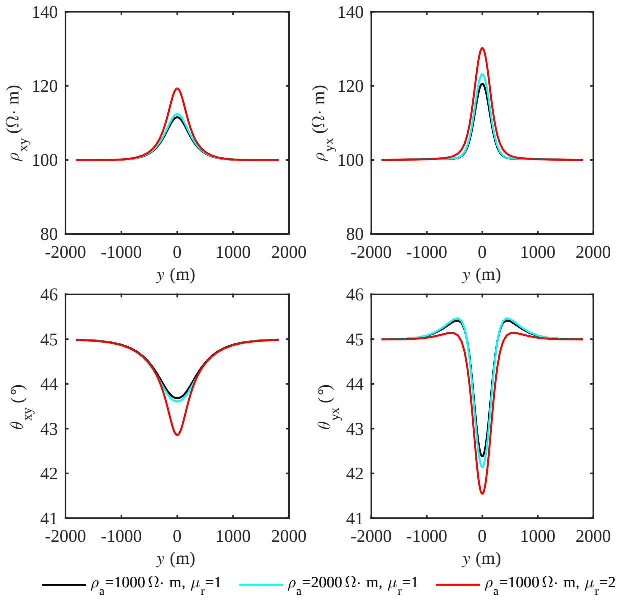

Figure 5. Magnetic effects on magnetotelluric forward responses of a resistive anomaly.

The magnetotelluric data were acquired within several minutes.

Discussion

For both conductive and resistive cases, we can see the increases of the apparent resistivities and the decreases in phases due to the magnetic permeable heterogeneity. So in some sense, the magnetic permeability act as resistive features. Through theoretical analysis of a magnetic permeable half space, we can easily realize that the apparent resistivity results from multiplying the background resistivity by the relative magnetic permeability. Consequently, the responses over the anomalies turned to be more resistive in some degree. That is also how we determined the extra model for comparison.

Comparing the conductive and permeable anomalies with the corresponding extra model showed a sign that the magnetic permeability affected differently between the conductive anomalies and the resistive ones. This discrepancy might be owing to the complicated coupling between the conductive and magnetic permeable anomalies (Noh, Oh, Seol, Lee, & Byun, 2016). For the resistive anomaly case, the forward responses were characterized to be significantly more resistive. And the data of different polarizations was quite consistent with each other. However, for the conductive anomaly case, the data of two different polarizations became inconsistent compared with the extra resistive model. Therefore, we inferred that the coupling effects of resistive and magnetic permeable anomalies got strengthened because the effects of them were very much alike. On the contrary, the coupling between conductive and magnetic permeable anomalies were weakened by the opposite effects of them and might depend on model geometry and the polarization of magnetotelluric sources.

Conclusions

In this work, we have developed an algorithm for 3-D magnetotelluric forward modeling that can simulate conductive and magnetic permeable anomalies. The staggered-grid finite difference method was employed to accomplish the algorithm. Synthetic model tests indicated that magnetic permeability could have significant impact on the forward responses of magnetotelluric method, especially for the resistive anomalies. So accounting for magnetic permeability in magnetotellurics is of consideration.

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