

Foundational Magnetic Susceptibility

Making its debut at the March 2015 APS meeting in San Antonio

All undergraduate physics majors take electromagnetism courses, and most such courses include a discussion of the ‘magnetic properties of matter’. There, students are introduced to the topic of diamagnetic, paramagnetic, and ferromagnetic materials. Some courses go so far as to define the ‘magnetic susceptibility’ of a material, χ , via the relation $M = \chi H$. So χ tells us how much sample magnetization M arises in response to an externally-applied magnetic intensity H .

But how many of us have actually measured χ for any given material? And who has measured it from first principles (not relying on anyone else’s absolute calibration)? And who has measured it in both SI and cgs units (and understood both)? And who has measured it in a way that gives a transparent result for the *sign*, as well as the magnitude, of χ ? The answer to all these questions is: just about nobody, up until now! But this will all change, for anybody who uses TeachSpin’s new “Foundational Magnetic Susceptibility” apparatus.

We think you’ll find our new apparatus exhibits the desirable combination of being *experimentally inviting*, *conceptually accessible*, and *theoretically straightforward*.

Here’s what we mean by *experimentally inviting*: Put any sample you choose (solid, liquid, or powder) into a plastic container of 1-cm² cross section, to a depth of about 3 cm, and mount it in our apparatus. That’s about all that’s required to get a result for the susceptibility – you could scarcely imagine a less demanding sample-preparation process.

And here’s what we mean by *conceptually accessible*: Our apparatus exploits the Gouy technique, basically using a one-pan balance to detect, with high sensitivity, the vertical forces generated by the interaction of a permanent magnet and the sample material in the container. See Fig. 1 for a sketch of the geometry we use. We choose to suspend the sample, and to use the balance to ‘weigh the magnet’, because that arrangement allows an ingenious way to calibrate the magnetic field B produced by the permanent magnet – see below. There are no coils, no lock-in, no signal-averaging, no mystical calibration-factors required.

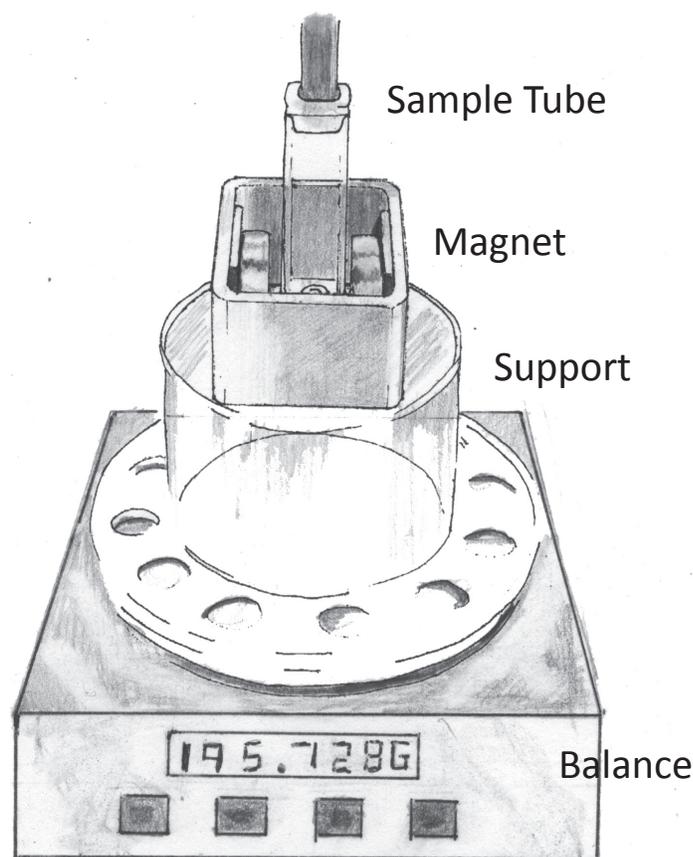


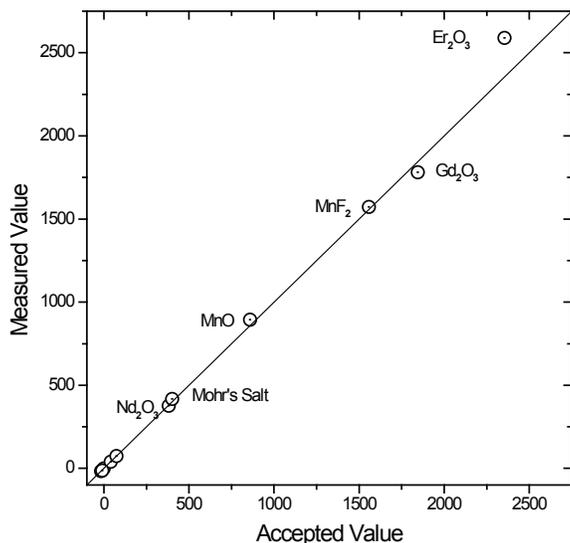
Fig. 1: A sketch of our magnetic-susceptibility apparatus. The balance supports a permanent-magnet structure, within whose high-field region a material is supported in a sample tube. The sample suspension is not shown.

Theoretically straightforward is the third attribute of our apparatus: This Gouy method for measuring the susceptibility χ is a working instance of a standard end-of-chapter homework problem, applied here to a real-world application. If we have a vertically-extended ‘log’ of solid sample material of cross-sectional area A , whose bottom end is immersed in a field of strength B (but whose top end lies outside the strong-field region), then the vertical force applied by the magnet on the sample can be shown to be:

$$F_z = \frac{\chi A}{2\mu_0} (B_{top}^2 - B_{bottom}^2)$$

You’ll note that the susceptibility χ of the sample emerges, provided the rest of the factors can be measured – and in our apparatus, they can be, since this force F_z shows up on the balance as an apparent mass change Δm , where $F_z = \Delta m \cdot g$.

You might think that forces on ferromagnetic materials are readily detected by this method, but that the forces from dia- and para-magnetic materials would be too small to detect. Actually, it’s the opposite: forces on ferromagnetic samples are typically too *large* to measure in our apparatus, which is instead ideally suited to detect much weaker forces. So, for example, the tiny diamagnetism of liquid *water* is readily detected in our apparatus – the mass indicated by the balance exhibits changes of order 10 mg in response to magnetic interactions, comfortably large compared to the 1 mg resolution and stability of the balance.



And, for the geometry shown in Fig. 1, you can readily confirm that the magnet appears to be about 10 mg *heavier* when the water sample is present (compared to when it’s removed). Clearly this shows that the water is pushing down on the magnet (so the magnet must be pushing up on the water, ie. the magnet is repelling the water from its high-field region). That gives an unambiguous determination of the *sign* of the susceptibility; it establishes that water really is diamagnetic.

Of course, the more-negative susceptibility of metallic bismuth is also easy to demonstrate. We also make it easy to detect the large (and orientation-dependent!) diamagnetic susceptibility of pyrolytic graphite. Easier still to detect are the markedly larger forces arising from *paramagnetic* samples, typically ones including transition metals and rare-earth metals. See some results in Fig. 2 below.

Now back to calibrations – the formula above shows that an absolute measurement of χ for a sample requires knowing the value of B to which the bottom end of the sample is exposed. We have included in our apparatus a method for obtaining this value, *without* requiring some expensive Hall probe, or anybody else’s calibration of it. The method is truly simple and basic. A sample tube containing no sample, but only a U-shaped ‘current hairpin’, allows dc electric current i , flowing along a length L_x , to interact with the field B_y to produce a vertical force computed from the Lorentz law, of

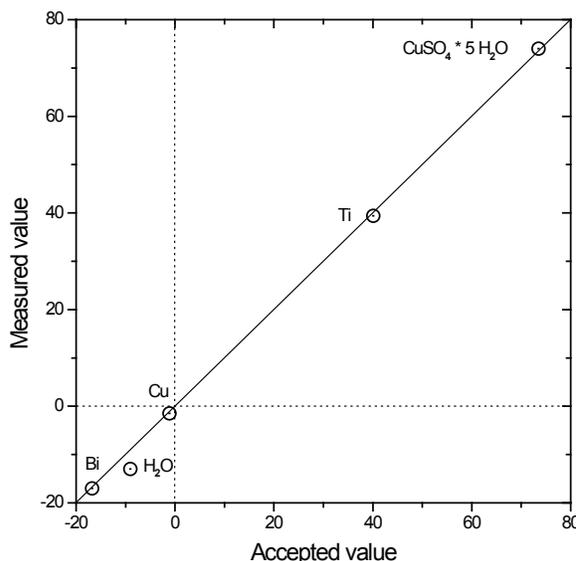


Fig. 2: In these graphs we plot (mks) mass susceptibilities, in units of $10^{-9} \text{ m}^3/\text{kg}$, for various materials. Vertical axis: the values we’ve measured in our magnetic-susceptibility apparatus; horizontal axis: accepted values gleaned from various sources. (The right-hand graph is expanded thirty-fold to show the data points nearest the origin.)

size given by $F_z = i L_x B_y$. And the 3rd-Law reaction to this force manifests itself as a force of the fixed hairpin on the weighed magnet, seen as a change of its apparent mass on the balance. With modest currents of order ± 2 A, this easily gives a signal adequate to measure B_y to a precision and accuracy of order 1% – see Fig. 3.

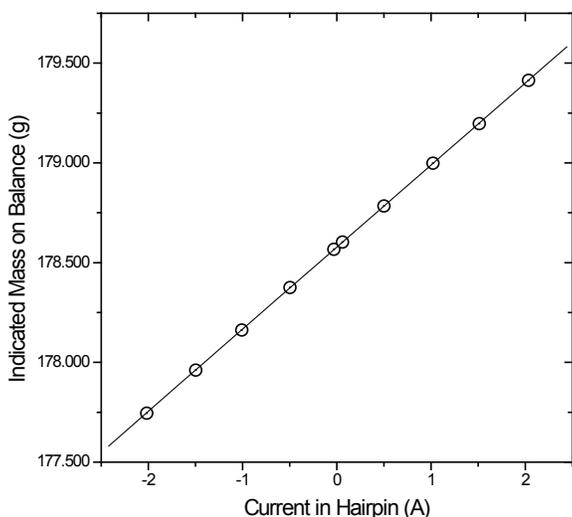


Fig. 3: Apparent mass of the magnet structure, as a function of the current i passing through the ‘hairpin’.

With the Gouy technique in hand, students can go on to measure and model susceptibilities. We include samples of graphite and bismuth, and also solid metallic samples of copper, titanium, and of (alloy) aluminum. We also include a variety of powdered chemical samples, including various paramagnetic salts and oxides. And our system keeps those samples properly labelled and capped, while providing plenty of empty sample tubes for students to fill with materials of their choice. Did you know that you can detect the recommended daily allowance of dietary iron in multivitamin pills via the paramagnetism of the ferrous fumarate they contain? We didn’t either, until we used this apparatus to measure it quantitatively!

The relevance of susceptibility

It’s all very well to determine a number for χ for every material you try, but why do we care about χ ’s value? One reason is that classical mechanics fails completely to explain magnetism at all, yet quite accessible quantum-mechanical reasoning can account for the diamagnetism and paramagnetism of simple systems. So here we have a table-top, hands-on, room-temperature experiment which measures, non-invasively and non-destructively,

a property that (only) quantum mechanics can explain. What’s more, a treatment of (for example) Curie paramagnetism involves not only the quantum mechanics of spin systems, but also the thermodynamics and statistical mechanics of many-body systems, in making predictions that can then be tested against feasible experiments.

What can our Manual do for your students?

In addition to building the apparatus required, we have brought together into one place the necessary lore and training required for describing the magnetic properties of matter, and we have illustrated concepts, where possible, with concrete numbers taken from actual experiments.

- Our Manual will define, illustrate, and motivate for students the vector fields M and H , which are essential for understanding even the definition of the magnetic susceptibility χ .
- The Manual illustrates the first-principles calculation of χ for simple systems, including the derivation of the diamagnetic susceptibility of helium gas right from the Schrödinger Equation, and the derivation of Curie susceptibility of model paramagnetic systems.
- It relates and motivates the connections between ordinary (volume) susceptibility, and the related mass, molar, and molecular susceptibilities, and it illustrates concretely which of them is useful in various situations.
- It works out the connections between the cgs units used in the historical magnetism literature, and the SI representation of the same physics.
- It derives the connections between the susceptibility of a substance, and what can be measured by the Gouy method using a powdered sample of that material.

Thus, users of our Magnetic Susceptibility system will become proficient not only in generating numerical results, but also in understanding the computational manipulations that lie behind them.

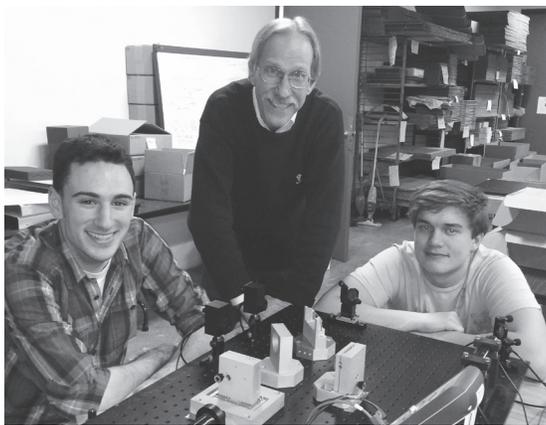
Just as an example of the empowerment these capabilities will bring, we mention that the kit includes all that is needed to measure the (single large) diamagnetic susceptibility of (non-isotropic) pyrolytic graphite, and the Manual then illustrates how to compute straightforwardly the criterion for the strength of a magnetic-field gradient that will diamagnetically *levitate* such a graphite slab. And just for fun, we include the ingredients and instructions for making a working demonstration of this form of levitation. Any number of hobbyists have built such a demonstration, but users of this apparatus will join the very select fraternity of those who can say, qualitatively *and* quantitatively, why it works.



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Introducing Foundational Magnetic Susceptibility



TeachSpin's **Winter Interns** Michael Rinkus (Rochester Institute of Technology, left) and Max Stein (SUNY Binghamton, right) spent three weeks in January at our factory in Buffalo, performing a host of experiments on our 'Modern Interferometry' apparatus. Here's David Van Baak looking over their set-up for the interferometric detection of magnetostriction in nickel. Michael and Max also detected Ångström-scale motion due to piezo-electricity, and did feasibility studies for Sagnac detection of absolute rotation. Teachspin will offer 8-10 week internship possibilities to two students during summer 2015 – do you know students who ought to hear about this?

Recruitment is in progress for a position that's been advertised in *Physics Today*:

A small tight-knit company (that's us) is looking to hire an experimental physicist, preferably one with teaching experience, who enjoys people, playing with apparatus, travel, and schmoozing with faculty about teaching. As sales and marketing manager, this person will be expected to operate, understand, and be able to explain to both faculty and students every instrument in our catalog. We expect this person to make a significant long-term contribution to the development of our company. A minimum of an M.S., but preferably a Ph.D. in experimental physics is required. Send resume to JFReichert@teachspin.com.

**Visit TeachSpin's Exhibit at the APS meeting, in San Antonio, TX
March 2-5, Booth #617-619**