Math 4315 – PDE's

Last class

$$u_x - u_y = 0$$

start with $u_s = u_x x_s + u_y y_s$

choose $x_s = 1, \; y_s = -1$

so $u_s = u_x - u_y = 0$ / from PDE)

so we solve

$x_s = 1 \;\Rightarrow\; x = s + a(r)$

$y_s = -1 \;\Rightarrow\; y = -s + b(r)$

$u_s = 0 \qquad u = c(r)$

so $x + y = a(r) + b(r) = A(r)$

$r = A^{-1}(x+y)$

$u = c\left(A^{-1}(x+y)\right) = f(x+y)$ soln

Constant Coefficient 1st order PDE

$$a u_x + b u_y = c u + d \qquad a-d \text{ const.}$$

ex 1 $\quad u_x - 3u_y = 2$

ex 2 $\quad 2u_x + u_y = u$

we will always start with

$$u_s = u_x x_s + u_y y_s$$

& pick x_s & y_s to give part of our PDE

ex 1 $\quad u_x - 3u_y = 2$

pick $x_s = 1$, $y_s = -3$

$$u_s = u_x - 3u_y = 2 \quad (\text{from PDE}$$

so we need to solve

$x_s = 1 \Rightarrow x = s + a(v)$

$y_s = -3 \Rightarrow y = -3s + b(v)$

$u_s = 2 \Rightarrow u = 2s + c(v)$

so $\quad 3x+y = 3a(r)+b(r) = A(r)$

$$2x-u = 2a(r)-c(r) = B(r)$$

$r = A^{-1}(3x+y)$

$$\Rightarrow \quad 2x-u = B\left(A^{-1}(3x+y)\right)$$

$$u = 2x - B\left(A^{-1}(3x+y)\right)$$

$$\boxed{u = 2x + f(3x+y) \quad \text{soln}}$$

what does this mean

if we calc. u_x & u_y from

then $\quad u_x - 3u_y = 2$

check $\quad u_x = 2 + 3f'(3x+y)$

$\quad u_y = \quad f'(3x+y)$

$\Rightarrow$

$$u_x - 3u_y = 2+3f' - 3f' = 2 \checkmark$$

Ex 3 $\quad 2u_x + u_y = u$

$$u_s = u_x x_s + u_y y_s$$

pick $x_s = 2$, $y_s = 1$

so $\quad u_s = 2u_x + u_y = u$

so we need to solve

$$x_s = 2 \quad \Rightarrow \quad x = 2s + a(r)$$

$$y_s = 1 \quad \Rightarrow \quad y = s + b(r)$$

$$u_s = u \quad \Leftarrow \text{ this one is different!}$$

ODE $\quad \frac{dy}{dx} = ky \quad$ sep $\quad \frac{dy}{y} = k\,dx$

$$\ln|y| = kx + \ln c \quad \Rightarrow \quad y = c\,e^{kx}$$

so $\quad u_s = u \qquad u = c(r)\,e^{s}$

$(k=1)$

Now eliminate s

$$x = 2s + a(r)$$
$$y = s + b(r)$$

$$x - 2y = a(r) - 2b(r) = A(r)$$

$$y = s + b(r) \Rightarrow s = y - b(r)$$

$$u = c(r)\,e^{s}$$

$$u = c(r)\,e^{y - b(r)} = c(r)\,e^{y} \cdot e^{-b(r)}$$

let $c(r)\,e^{-b(r)} = B(r)$

so $u = B(r)\,e^{y}$ $\quad r = A^{-1}(x - 2y)$

$$u = B(A^{-1}(x - 2y))\,e^{y}$$

$$\boxed{u = e^{y} f(x - 2y) \quad \text{Sol}^{n}}$$

Check.

$$u = e^{y} f(x-2y)$$

$$u_x = e^{y} f'(x-2y)(1)$$

$$u_y = e^{y} f(x-2y) + e^{y} f'(x-2y)(-2)$$

L.S. $2u_x + u_y = u$

$$2\cancel{e^{y} f'(x-2y)} + e^{y} f(x-2y) - 2\cancel{e^{y} f'(x-2y)}$$

R.S. $u = e^{y} f(x-2y)$ same

So yes it's a sol^n

In both examples we have an arbitrary function f. How does this get set?

Ex

$$3u_x - 5u_y = 6 \qquad u(x,0) = 7x$$

How does this fit in?