

Research Article

A Novel approach for designing Gear tooth Profile by Mathematical equations for three different gear types Spur, Internal, and Helical Gear and creating models using SolidWorks and CATIA

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Abstract

Gears are an important invention that is in almost every mechanical item in our life. Gears have precise calculations to draw their profile for manufacturing to handle high stresses while operating. Previous articles have complicated and time-consuming methods for drawing gear profiles lacking an appropriate method for Inner Gears (Found in Epicyclic) and Helical Gears. In this article, the author demonstrates a novel method for making a full Epicyclic gear, including Spur Gear and Inner Gear, and designing a full 3 Dimensional Computer Aided Design model. Furthermore, the article discusses the theory and equations of a Helical Gear and the method of designing a 3 Dimensional Computer Aided Design model. After outlining each Gear type method, equations are used to simulate coordinates in Excel then implemented in SolidWorks. The unique method used focuses on between teeth profile instead of casual tooth profile, which is not always applicable in unique gear parameters. The method proposed is accurate in every gear case and results in the most precise gear representation while still being simple to implement.

Keywords: Spur Gear, Helical Gear, Planetary Gear, SolidWorks, CATIA.

Introduction

Gears- defined as toothed wheels for transmitting motion- have a venerable history. Their birthplace seems to have been Alexandria, Egypt, where the Greek Kings set up the research institute for designing machinery. For example, a host of basic mechanisms- water-lifting wheels, pumps, organs, water-clocks with moving parts, and perhaps the waterwheel itself formulated the mechanic's laws. Later authors wrote of gears dating 270 B.C. However, their writings are lost; the Mechanics survive only in Arabic translation. The art of mathematical gear production passed from Byzantium to Islam [1].

The earliest gear train is a Persian astrolabe. It has five cogs to drive the three calendrical diagrams on the astrolabe's back. The entire mechanism turns by an axle attached to the rete on the front. By the early 11th century, astrolabes in the Islamic world must have been making such geared instruments; Muḥammad ibn Abī Bakr inscribed his name on the instrument [2].

Gears are known to be the simplest and most effective mechanical component in transmitting motion and power. Although much research was conducted on gear design, some fundamental gear theory basics are not thoroughly understood. This results in overdesign of systems, which leads to higher cost, and waste material. General Approach for designing gears consists of three phases. Firstly is determining the user requirements and specifications. Secondly, creating the layout, conducting the calculations, and setting the gear profile coordinates. Lastly, using the coordinates, create a prototype 3D model [1].

This research will follow a similar procedure by setting the design requirements and finding all relevant data using theoretical calculations. Then Excel is used to create XY coordinates of 4 curves surrounding the between teeth profile. Lastly, the coordinates are inputted as curves in SolidWorks to design the 3D model. Most current methods are inaccurate and are only approximations of the involute curve profile. Simple Spur gear geometry depends on three basic parameters, the Pressure Angle, the

Module, and the Number of Teeth. Unfortunately, most CAD systems have no built-in tool for designing gears. Therefore, engineers need a simple technique for gear development that is both simple and accurate [2]. The Base circle radius is the starting point for drawing the gear involute [3]. Planetary Gear Trains are used as a power transmission method in a vehicle's gearbox to match the inertias, lower the motor speed, and boost the torque. Furthermore, Planetary Gears provide a sturdy mechanical interface for pulleys, cams, drums, and other mechanical components [4].

For further knowledge about gears in general, Mechanical Engineering Design by Shigley's [3] is recommended. Many Equations used are from [3,5]. [4] Introduced an algorithm for developing gear tooth space geometry and complete gear profile using MATLAB Graphical User Interface (GUI). Then create a 3D Gear model in SolidWorks using the output data. [6] developed an involute curve using parametric technology, with an accurate involute profile. Entirely using CATIA software and built-in packages. With the possibility to make changes to gear design using parametric inputs of desired specifications. [7] Described the point to point movement for a CNC machine. The algorithms used are based upon equations of the circular head and root thickness. The article proposes two codes to test the reliability of the designed gear profile. Firstly, a Visual Basic code that can run as a macro in Excel. Secondly, a CNC Mitsubishi G code. [8] Adopted PRO/E programs method, relationships, and parameters to design various Spur Gears, analyzing their critical parameters, defining them via programming. The article also uses parameters dialog box and relationships dialog box to define basic parameters and the relationship between them step by step to complete the gear's parametric model. [9] Given the advantages planetary gears have over other types. Moreover, the article discusses the consideration taken to determine gearboxes' reduction ratios, minimum and maximum reductions per each stage of planetary gear pairs. The article [10] discusses the importance of gear tooth symmetry for proper modelling, giving a technique forming an involute curve exactly symmetrical about one of the planes. Non-symmetrical tooth profile exists and is problematic because it has two different involute profiles with different base circle

diameters. Furthermore, it does not use specific standards or any typical notation, but further research is needed. [11] Concerns with non-symmetrical tooth profile. The article proposes an algorithm for developing the geometry, Gear (NSPG) tooth, and its complete tooth profile using MATLAB Graphical User Interface (GUI) coding. During gear motion, bending stress develops at the root of the gear. The Highest Point Single Tooth Contact is the most critical loading point in any gear; it determines the gear's root fillet portion; bending stress value. Another crucial point to calculate is the Lowest Point Single Tooth Contact. [12] Develops a customized spreadsheet involving gear formula and all calculation steps. One of the earliest Gear CAD representation is [13]. The article presents an approximate and accurate method to generate solid models of involute cylindrical gears by few steps in Autodesk Inventor 3D CAD software. The steps are as follows; create a closed loop of tooth profile, Extrude the profile to generate a tooth, Extrude a root circle to generate a cylinder, lastly, circular pattern the tooth to finish the gear. The most significant work on asymmetric involute teeth is by [14], where a double involute concept determines an internal gear's profile with asymmetric involute teeth, including driving and driven gears. The article presents a mathematical model for meshing and conducts the study on a Spur and a Helical gear. Through the article's proposed double involute concept method, the required driving gear can be obtainable to a one-parameter family of a rack cutter surface. The obtained involute becomes the generating surface. Therefore, the driven gear is the involute for the family of the generating surfaces. The closest to this article is [15], where authors calculate the XYZ coordinates of an involute curve using Excel.

Then the coordinates are imported as points in SolidWorks to get graphical representation. These points represent the gear tooth outline, then by using mirror command, circular pattern a 2D profile is achieved, which extrudes to produce 3D CAD model. However, the method used was complicated and lengthy. Instead of designing the gear tooth and extruding it, this novel method designs the in-between teeth profile. Then, Extrude Cut through a generated Addendum diameter cylinder for the spur gear then the circular pattern to obtain gear model. Furthermore, literature lacks a method for

designing Internal gears (used in Epicyclic Gear), as well as Helical Gear. This research uses a novel approach for designing gears and cover two significant gaps in gear types design. This article will be split to two parts. First the design of an Epicyclic gear, and secondly a Helical Gear.

Epicyclic Gear

First case study concerns an Epicyclic gear, which is used in vehicle's gearbox for an efficient speed transmission. Values are taken

from a real vehicle gearbox design requirement. The design requirement is studied then relevant information is calculated (Table 1). An Epicyclic gear (Planetary Gear) consists of 2 gear types; Spur Gear and Inner Gear. This method unique for Spur Gears, and nearly unexplored for Inner Gears. Planetary gears has an important part as reducers where it has the advantage of small size, small mass, large transmission ratio, compact structure, and strong carrying capacity [16].

Table 1. Epicyclic Gear Design Requirements

	Symbol	Name	Sun Gear	Planet Gear	Ring Gear	unit
Given	\emptyset	Pressure angle	20	20	20	Degrees
Given	Z	teeth	16	32	80	integer
Given	m	Module	4	4	4	mm
Calculated	P	Diametral Pitch	0.25	0.25	0.25	mm
Calculated		Pressure angle	0.349	0.349	0.349	Rad
Calculated	D	Pitch Diameter	64	128	320	mm
Calculated	p	Pitch of the teeth	12.566	12.566	12.566	mm
Calculated	S	Circular tooth thickness	6.283	6.283	6.283	mm
Calculated	a	Addendum	4	4	4	mm
Calculated	b	Dedendum	5	5	5	mm
Calculated	d	Radius of pitch circle	32	64	160	mm
Calculated	ra	Radius of outer circle	36	68	164	mm
Calculated	rf	Radius of root circle	27	59	155	mm
Calculated	rb	Radius of the base circle	30.070	60.140	150.350	mm
Calculated	rc	Radius of the root concave corner	1.52	1.52	1.52	mm
Calculated	ht	Whole Depth	9	9	9	mm
Calculated	pc	Circular Pitch	12.566	12.566	12.566	mm
Calculated	t	Tooth thickness	6.283	6.283	6.283	mm
Calculated	c	Clearance	1	1	1	mm
Calculated		P angle	0.723	0.520	0.333	Rad
Calculated		Inv of P angle	0.159	0.053	0.013	Rad
Assumed	0 to 1	Sweep Parameter	0.5	0.5	0.5	

Spur Gear Terminology

Gear Terminologies are shown in figure 1. Module is the ratio between pitch diameter and number of teeth [3], pitch diameter is an imaginary circle which all calculations are based on. Circular pitch is the distance on pitch circle from point on teeth to the point on adjacent teeth. This helps us know distance between teeth. Addendum is the distance between pitch circle and tip of teeth, while dedendum is from bottom to pitch circle. Face width represents width of gear which determines stress and load calculations. W_t is the tangential force, with W_r being radial and W total force [3]. As the promptly increasing concerns on the improvement of gear product quality in recent decades, the vibration and noise problem of precision gear transmission systems have drawn

great attentions, especially for the high-speed and/or heavy load conditions [17].

Tooth Involute calculations

Equations used throughout this research.

$$D = m \times N \quad (1)$$

$$P = \frac{N}{D} \quad (2)$$

$$S = \frac{D}{2} \quad (3)$$

$$r_b = d \times \cos(\emptyset) \quad (4)$$

$$a = m \quad (5)$$

$$b = m * 1.25 \quad (6)$$

$$r_a = d + a \quad (7)$$

$$r_f = d - b \quad (8)$$

$$pc = \frac{\pi}{P} \quad (9)$$

$$t = \frac{pc}{2} \quad (10)$$

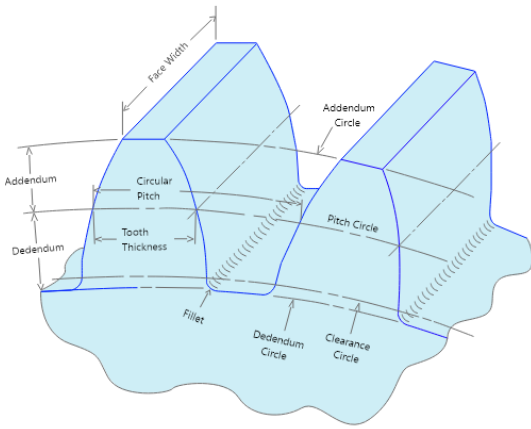


Figure 1. Gear Terminology

A capital D denotes Pitch diameter and small d for Pitch Radius, m for gear module in mm, N for Number of teeth, P for Diametral Pitch, S for Circular Tooth Thickness [3]. Furthermore, two important radiuses are r_b for Base Radius, and r_f for root radius. a represent the Addendum, and b the Dedendum [3]. Moreover, pc is circular pitch, and t for tooth thickness [3]. These are basic information that must be calculated for any gear, and as we can see in Table 1 the main difference between Spur Gears (Sun and Planet) and Ring Gear is the value of Base Radius compared to Dedendum. In gear terminology the Base Radius should be above the Dedendum and below the Pitch Radius, but this is not the case in Inner Gears as well be explained [3].

In order to model the gear, coordinates of between the teeth profile is drawn (figure 2). We need to find angle FOC which is half the circular pitch distance so it equals to 360 divided by two times the number of teeth (N), and HOC angle is half the FOC [3].

$$\angle FOC = \frac{360}{2 * N} \tag{11}$$

$$\angle HOC = \frac{\angle FOC}{2} \tag{12}$$

Next, since distance is known and angle, x and y components of point C on the pitch radius can be calculated as follows.

$$XC = d * \sin(\angle HOC) \tag{13}$$

$$YC = d * \cos(\angle HOC) \tag{14}$$

Where XC is the x-value of point C, and YC is the y-value [3]. To find the coordinates of point B concerning with the Base Circle where involute profile starts, the theory of involute must be further explained.

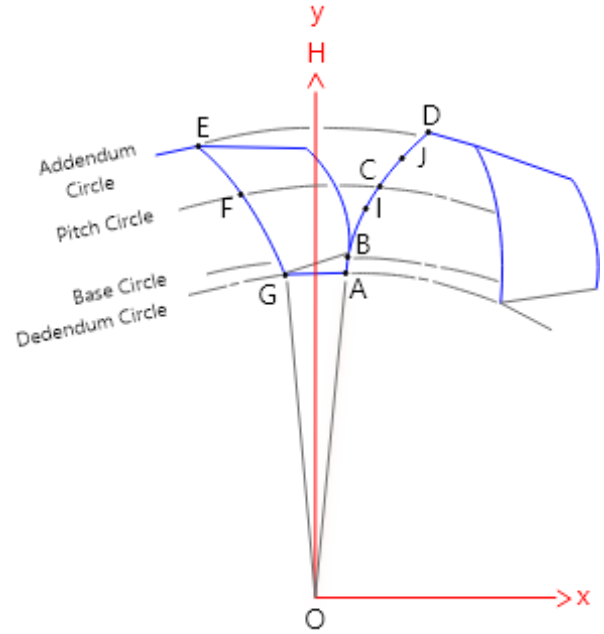


Figure 2. Between teeth profile

Gear tooth involute is outlined by a string held on the circle then un-wrapping it over while maintaining tension (Figure 3). Considering point P on the involute, and distance OP from the centre is R. Also, let angle $\angle BOP = \delta$ and angle

$\angle POQ = \lambda$. Therefore, The string length when P is at the pitch circle = $PQ = \sqrt{R_c^2 - R_b^2}$, But

$QB = PQ$ and $QB = R_b(\delta + \lambda)$. $(\delta + \lambda)$ is in Radians). Therefore $(\delta + \lambda) = \frac{PQ}{R_b}$, But $\lambda = \cos^{-1} \frac{R_b}{R_c}$,

so $\delta = (\delta + \lambda) - \lambda$. Then By knowing angle HOP

which is $\angle HOP (Rad) = \delta + \text{Base angle}$ the

coordinates of point P, $x = R_b \sin \angle HOB$ and

$y = R_b \cos \angle HOB$. Distance from AB is a straight

line so by finding point A which is Dedendum multiplied by \cos (base angle) for the Y component and \sin (base angle) for Y component.

$$PQ = \sqrt{(d)^2 - (r_b)^2} \tag{15}$$

$$(\delta + \lambda) = \frac{PQ}{r_b} \tag{16}$$

$$\lambda = \cos^{-1} \left(\frac{r_b}{d} \right) \tag{17}$$

$$\delta = (\delta + \lambda) - \lambda \tag{18}$$

$$\angle \text{Base} = \angle HOC - \delta \tag{19}$$

Where $\angle Base$ is the angle of the Base

point B (figure 3). Then points of radius above base radius till addendum are taken to find the coordinates of the involute. To calculate the coordinates of Point A (Figure 2) we take value of Dedendum radius and Base Angle with the assumption that base angle for Base is similar to Dedendum.

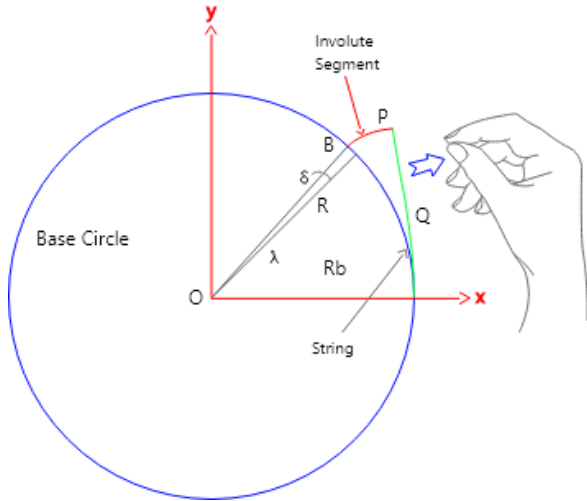


Figure 3. Drawing Involute Profile

$$x_A = b * \sin(r_b) \tag{20}$$

$$y_A = b * \cos(\angle Base) \tag{21}$$

Then to find the XY coordinates for 4 curves; Involute, Dedendum (b), Addendum (a), and Mirror Involute the following method is used in Excel Worksheet. With taking Radius

Point values (RP) as desired starting from Base Circle to Addendum value.

$$PQ = \sqrt{RP^2 - r_b^2} \tag{22}$$

$$(\delta + \lambda) = \frac{PQ}{r_b} \tag{23}$$

$$\lambda = \cos^{-1} \frac{r_b}{RP} \tag{24}$$

$$\delta = (\lambda + \delta) - \lambda \tag{25}$$

$$\angle HOP = \angle Base + \delta \tag{26}$$

It should be noted that all angle values are in Rad. For each curve there is an X coordinate and a Y coordinate calculations.

Curve 1: Involute XY Coordinates:

$$X_{coordinate} = RP \times \sin(\angle HOP) \tag{27}$$

$$Y_{coordinate} = RP \times \cos(\angle HOP) \tag{28}$$

Curve 2: Dedendum Curve:

$X_{coordinate}$ is from negative Dedendum X Coordinate to Positive value (taking each step as desired)

$$Y_{coordinate} = \sqrt{b^2 - X_{coordinate}^2}$$

Curve 3: Addendum Curve:

$$Y_{coordinate} = \sqrt{a^2 - X_{coordinate}^2}$$

$X_{coordinate}$ is from negative Addendum X Coordinate to Positive value (taking each step as desired)

Curve 4: Mirror Involute Curve:

Mirror the involute's Y Coordinates with Negative x-coordinates (33)

Then plot the 4 curves to create the between teeth profile. (As shown in Figure 4, 5)

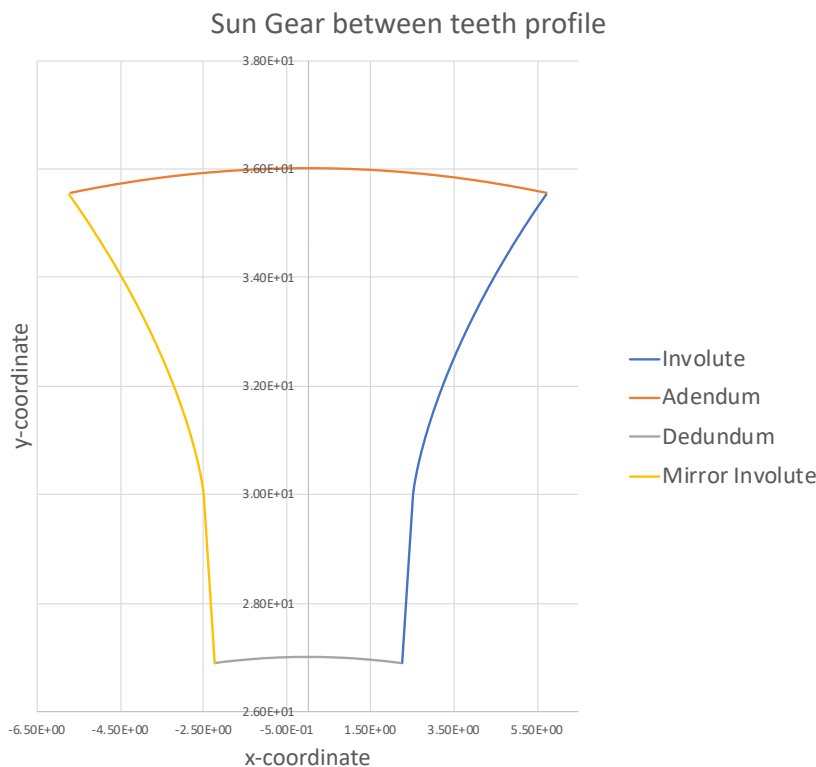


Figure 4. Sun Gear between Teeth Profile

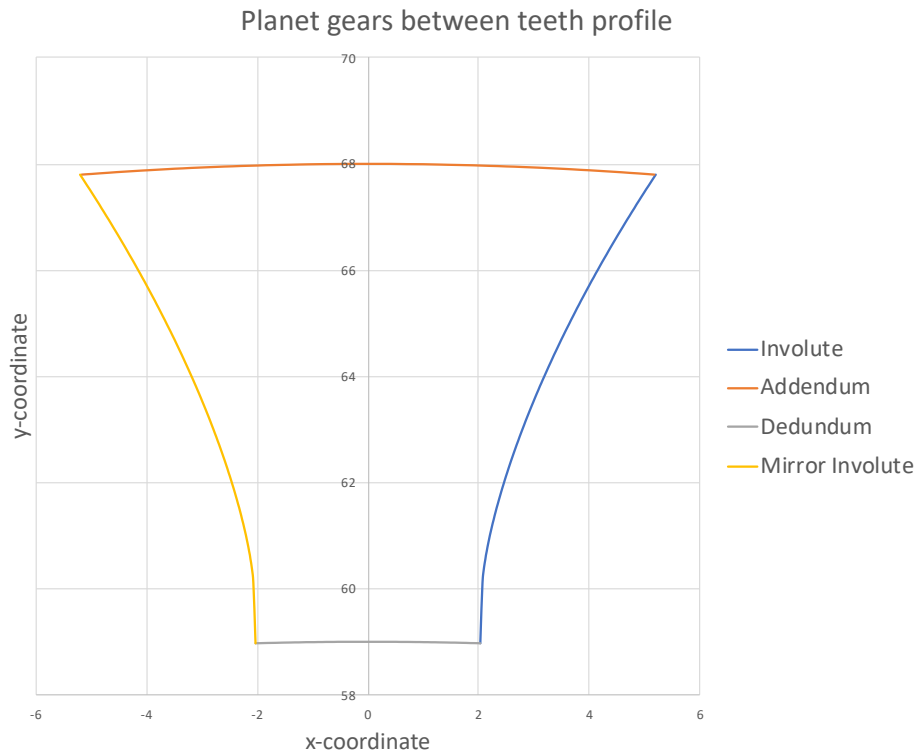


Figure 5. Planet Gear between Teeth Profile

Internal Gear Terminology

As shown in figure 6 both gears have their centre of rotation on the same side of the pitch point. Therefore, the positions of the Addendum and Dedendum circles are reversed. So for Spur gears going from top to bottom order is; Addendum, Pitch circle, and Dedendum. However, for Inner Gear the order is reversed with the Addendum being the smallest and Dedendum the largest circle. Furthermore, the Base circle for the Internal Gear lies inside the pitch circle near the Addendum circle [9].

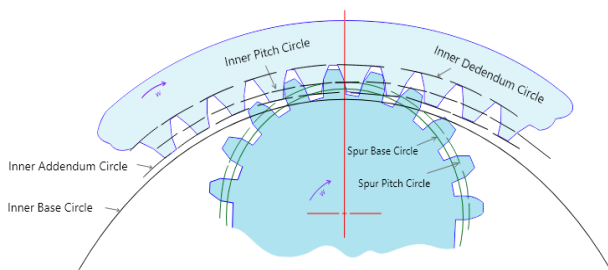


Figure 6. Ring Gear

Inner teeth profile

Unlike outer teeth where between teeth profile is found, in inner teeth the shape of tooth is drawn, by following similar steps to outer teeth but with minor changes.

$$a_{radius} = d - a \tag{34}$$

$$b_{radius} = d \times b \tag{35}$$

While a_{radius} , b_{radius} stands for

Addendum radius and Dedendum radius respectively and d is pitch radius. While pitch and base circle is calculated similar to Spur Gear, as well as angle values and with the base circle considered a starting point for the Involute profile. Taking radius values from base circle to Dedendum radius and finding coordinates of involute at each radius as done in Spur gears. However, for the involute and mirror involute the coordinates start from the Addendum radius till the Dedendum. Thus calculating the XY coordinates for each curve as discussed previously. As for mirror involute take negative values of X coordinate, while keeping Y the same. Lastly, drawing the curves in excel (figure 7).

Design in SolidWorks

Spur Gear Model

Taking the coordinates obtained in excel for each curve of Gear, and inputting them in SolidWorks. By going to Tools > Curve > Curve through XYZ Points, and choosing the .txt file with each curve coordinate for a single curve. So in our case repeat process four times. After inputting the curves create a sketch and use ‘Convert Entities’ to project the curve on that plane. After preparing the between teeth profile

‘Extrude Cut’ it through a cylinder with an Addendum Radius. Lastly, use ‘Circular Pattern’ according to number of teeth (figure 8, 9).

Internal Gear Model

Internal gear model procedure is similar to Spur gear, but instead of ‘Extrude Cut’ we use Extrude of the gear tooth profile around a ring Cylinder (figure 10). The Ring Cylinder inner

diameter is equal to the Dedendum and Outer diameter does not matter in our case so assumed 10 mm more than inner diameter. Lastly, as mentioned in Spur Gear use ‘Circular Pattern’ according to number of teeth. Lastly is assembly of the three gear types, using one Sun Gear, and three Planet Gears, and one Ring Gear (figure 11).

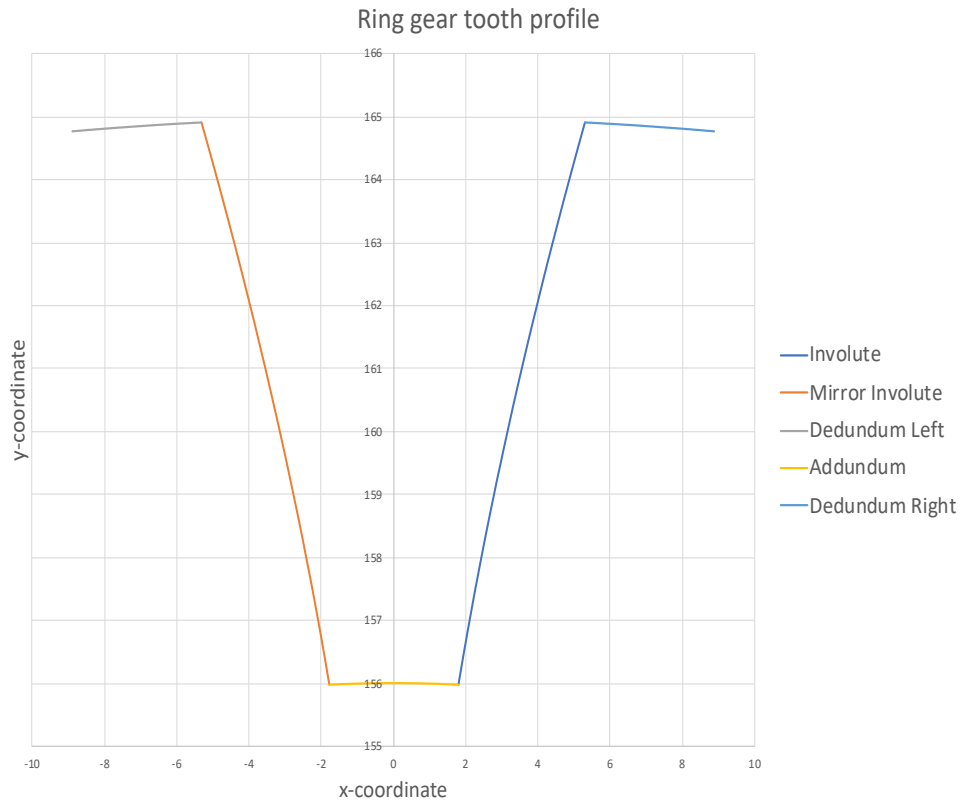


Figure 7. Ring Gear tooth profile from Excel

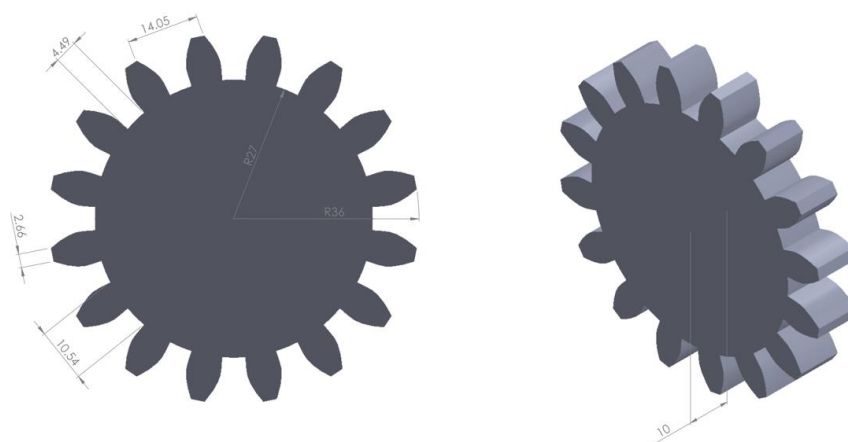


Figure 8. SolidWorks Spur Gear

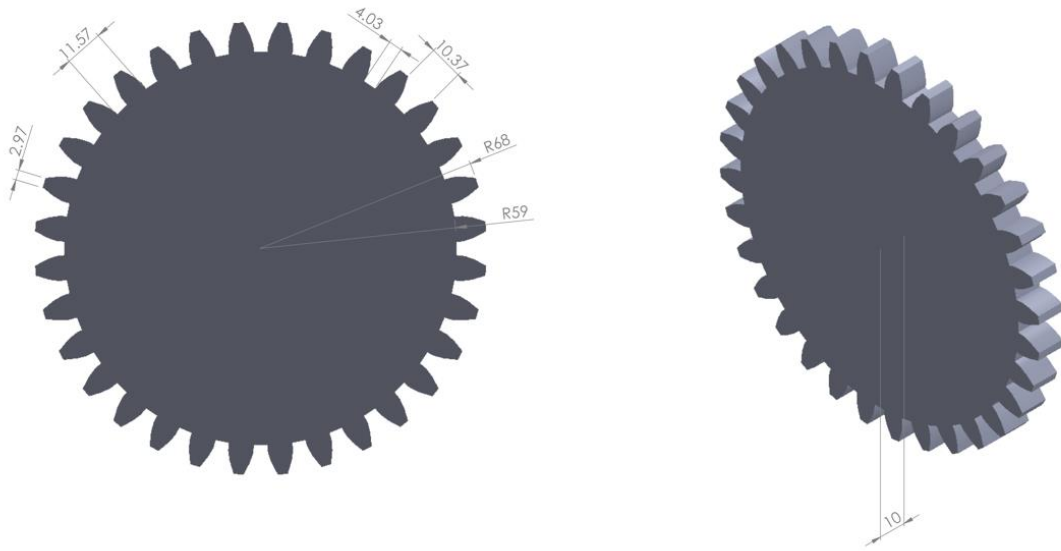


Figure 9. SolidWorks Planet Gear

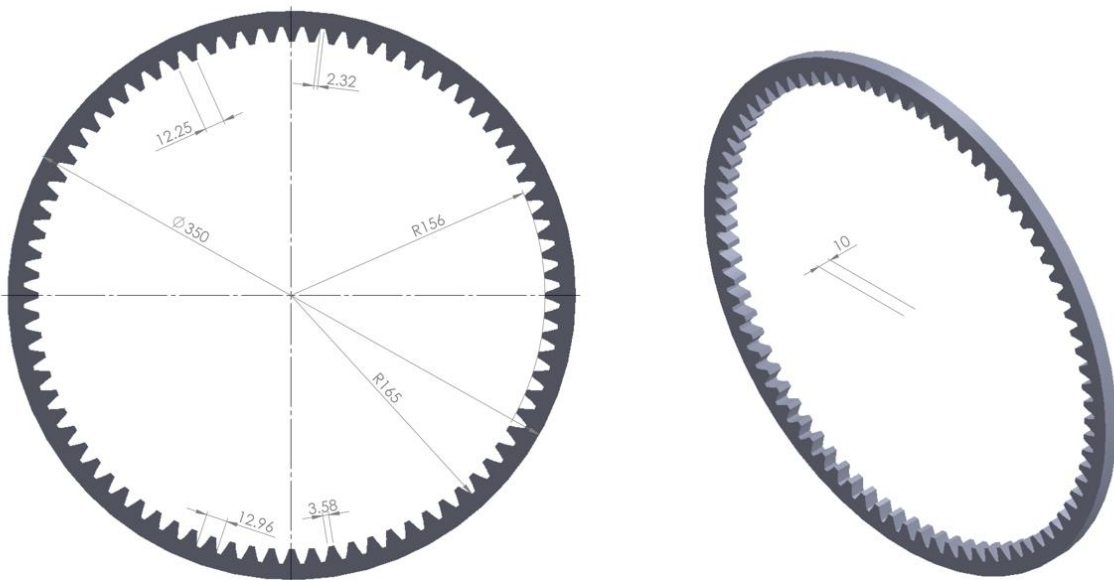


Figure 10. Inner Gear SolidWorks Model

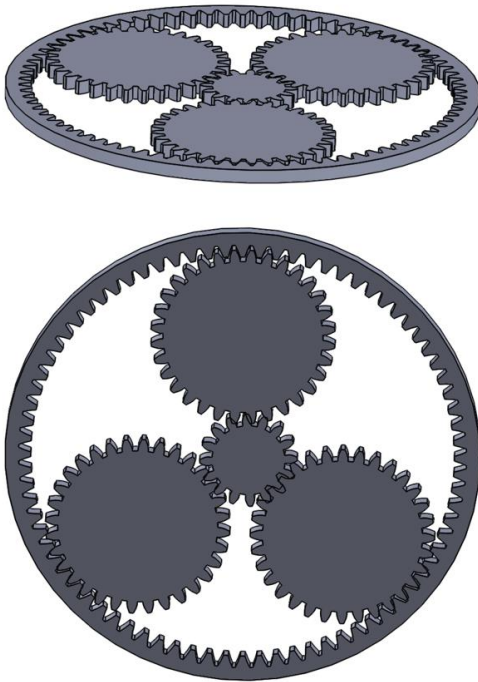


Figure 11. Epicyclic Gear Assembly

Helical Gear

The second case study is on x-axis gantry motion helical gear. Where two side opposite gears are used to eliminate stresses and torsion. The design requirement are taken and this article’s method is used to design the gear. Latest research on helical gears has been done by Komal Bagadi to enhance the AODV protocol for rural and urban areas, and improvement on good put, instantaneous delay, packet delivery radio, and packet loss [18].

Helical Gear Terminology

Helical gears have a slanted tooth trace. They have many advantages over Spur gears. For example, they have a larger contact ratio, quieter, less vibration, and capable of transmitting larger force. A pair of helical gears have opposite helix directions, and both combined a double helical gear eliminates thrust force. Due to their twisted teeth, their manufacturing is more complicated.

For the Helical Gear requirement, one extra information must be given; the Helix angle, which determines teeth twist angle along the cylinder (figure 12).

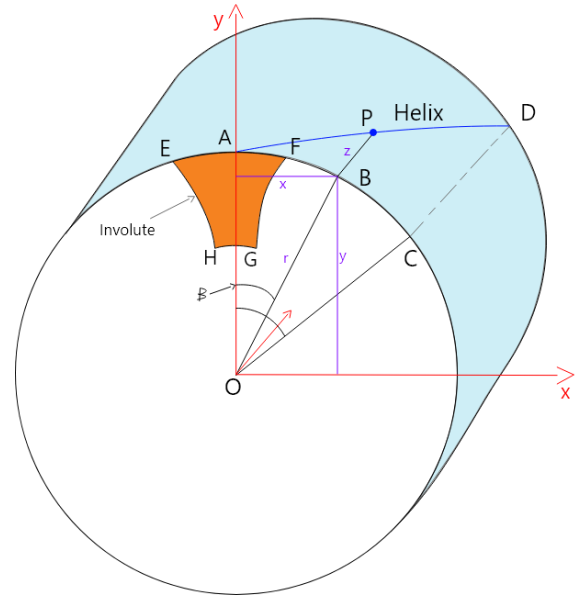


Figure 12. Helical Gear Terminology

Table 2. Helical Gear Design Requirement

	Symbol	Name	Value	Unit
Given	ψ	Helix angle	25	Deg
Given	ϕ_n	Normal Pressure angle	20	Deg
Given	m	Normal Module	5	mm
Given	N	Number of teeth	16	Integer
Given	Th	Thickness	25	mm
Calculated	ϕ_t	Transverse Pressure angle	0.38	Rad
Calculated	m_t	Transverse module	5.52	mm
Calculated	a	Addendum	5.52	mm
Calculated	b	Dedendum	6.90	mm
Calculated	d	Pitch circle Radius	44.14	mm
Calculated	a_{radius}	Addendum Radius	49.65	mm
Calculated	b_{radius}	Dedendum Radius	37.24	mm
Calculated	r_b	Base circle Radius	40.96	mm

Helical Gear Curve Equations

$$\phi_t = \tan^{-1} \left[\frac{\tan(\phi_n)}{\cos(\psi)} \right] \tag{36}$$

$$m_t = \frac{m}{\cos(\psi)} \tag{37}$$

$$D = N \times m_t \tag{38}$$

$$r_b = d \times \cos(\phi_t) \tag{39}$$

$$a_n = m_t \tag{39}$$

$$b_n = m_t * 1.25 \tag{40}$$

$$r_a = d + a_n \tag{41}$$

$$r_f = d - b_n \tag{42}$$

Where ψ is the Helix angle, ϕ_n is Normal Pressure angle, ϕ_t is Transverse Pressure angle, m_t is Transverse module. Therefore, a_h, b_h Helix Addendum, and Helix Dedendum equations include Transverse module. The strategy is to generate the space between two adjacent teeth similar to previous discussed method, then slotting on the Helix curve on the cylinder. Then pattern it according to number of teeth.

The Helix Curve

Design an Excel sheet with the following columns to get the XYZ coordinates of the Helix Curve, which will guide the between teeth profile to make the slot in SolidWorks. First of all, Circle Angle is required, which is calculated as follows; consider the top surface of the

cylinder to be a rectangle, then set a starting point and with the Helix angle from y-axis move up till end of rectangle width (which is cylinder thickness). Using Trigonometry calculate opposite side value. Revert rectangle to Cylindrical shape and use equation of an arc: $S = radius \times \theta$ to find the radius in Rad. Then

StepAngle Value is Circle Angle divided by Number of Z values chosen, use this value as a constant increment.

$$PZ: \text{choose a point on the length of the cylinder with constant increment} \quad (43)$$

$$Y_{coordinate} = a_h \times \cos(\text{StepAngle}) \quad (44)$$

$$X_{coordinate} = a_h \times \sin(\text{StepAngle}) \quad (45)$$

$$Z_{coordinate} = \frac{PZ}{\cos(\psi)} \quad (46)$$

Excel table representation is shown in Table 3, The resulted curve will be three dimensional.

Table 3. Helix Curve Coordinates

Helical Angle	25 in Deg	0.44 in Rad	Circle Angle	13.44 in Deg	0.2346 in Rad
Z when Rectangular	Y value	X Value	Z Actual	Step Angle	Rad
1	49.650	0.466	1.103	0.538	0.009
5	49.597	2.329	5.517	2.688	0.047
10	49.434	4.652	11.034	5.376	0.093
20	48.780	9.263	22.068	10.752	0.188
25	48.292	11.540	27.584	13.440	0.235

Concerning the between teeth profile the method is exactly same as spur gear as discussed before (figure 13).

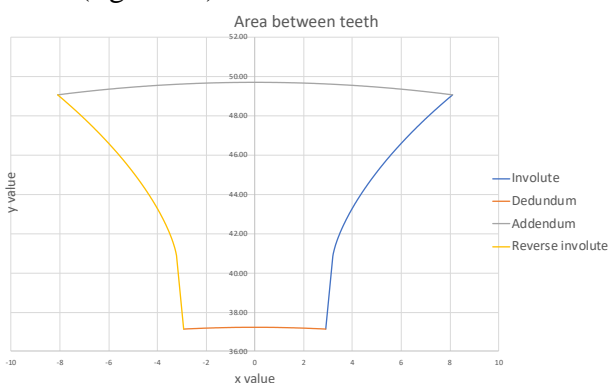


Figure 13. Helical Gear between Teeth Profile

Helical Gear Design in CATIA

When designing Helical Gears, it is easier to use CATIA instead of SolidWorks because SolidWorks enters the coordinates as curves but not sketches, which need to be cast on a Sketch on one of the planes. The Helix Curve is not on any plane, so it is hard to make it a usable curve.

However, in CATIA, a file exists in its program files called GSD_PointSplineLoftFromExcel.xls, which has macros and the XYZ coordinates included. When running the Macros, this results in the curve/Spline in CATIA part file. After making the four curves of the between teeth profile, input the Helix Curve, then use command 'Slot' to create the Helix. Lastly, the 'Circular Pattern' command around the cylinder according to the number of teeth. The generated 3D model is precise and accurate as shown (figure 14).

Conclusions

A method for designing the gear tooth profile for three different gears was developed in this research. The unique method used concerns with designing the in-between teeth profile with the mathematical procedure given. Then, using Excel and discussed mathematical equations to list profile coordinates. The coordinates are used as input curves in either SolidWorks or CATIA; then, the profile is 'Extruded Cut' through a

cylinder with Addendum diameter for the spur gears to generate a 3D Gear model. In the case of Internal Gears, which are opposite to Spur, as the Addendum is less than the Dedendum Diameter with the Base Diameter the least. In internal gear, the tooth profile is generated. After generating a single profile, the circular pattern command is used according to the number of teeth to obtain the gear model. The method proposed for Internal Gear is novel and straightforward and yields accurate Ring Gear representation. Finally, a method for designing a Helical Gear is discussed, which previous research lacked in this area. A method for calculating the Helix Curve is discussed with mathematical equations and inputted into Excel to generate XYZ coordinates of the Helix curve for the Gear profile to use as a guiding slot.

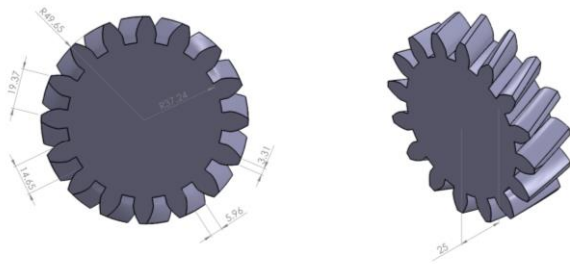


Figure 14. Helical Gear Designed in CATIA, Drawing from SolidWorks

Conflict of interest

Authors declared no conflict of interests.

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