Performance analysis of SC-FDMA and OFDMA systems based on SER by using ITU-pedestrian and vehicular channels with QPSK modulation

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ABSTRACT

The reason behind the transform in the wireless communication fields are back to the development in human life. High data rate is one of the important requirement that effect in the communication systems. Orthogonal Frequency Division Multiple Access (OFDMA) is considered an attractive technique to achieve this requirement. Several reasons such as high spectral efficiency and robustness to multipath phenomena made OFDMA adopted in the downlink multiple transmissions by several technologies such as Worldwide Interoperability for Microwave Access (WiMAX) and Long Term Evolution (LTE). High peak to average power ratio PAPR made Single Carrier Frequency Division Multiple Access SC-FDMA an Alternative technique as compare with OFDMA for the uplink in LTE. In this paper we study the performance analysis of OFDMA and SC-FDMA depending on Symbol Error Rate (SER). Different types of International Telecommunication Union (ITU) multipath channel were introduced to study the performance in term of SER. In the receiver systems, MMSE and ZF equalizer were introduced. MMSE equalizer appears better performance than ZF equalizer.

Keywords: SC-FDMA, OFDMA, ZF equalizer and MMSE equalizer

INTRODUCTION

Nowadays the development in the wireless communications fields become very fast. One of the important requirement in the communication systems is higher data rate. Orthogonal Frequency Division Multiple Access (OFDMA) is an extended form of OFDM system in order to achieve this requirement. OFDMA has the ability to mitigate the frequency selective fading of the wireless channel environment by transmitting data in parallel [6]. High data rate and high spectrum efficiency make OFDMA is adopted for both uplink and downlink transmission in WiMAX, while in LTE, it is adopted in the downlink transmission scheme [1][2]. In spite of these features, OFDMA is sensitive to frequency offset and high peak to average power ratio (PAPR) due to transmit data in parallel; thereby it requires high expensive linear amplifier lead to increase the cost of mobile equipment [3][4]. High PAPR considers not problem in the downlink, but it considers problem for the uplink transmission. For this reason, the third Generation Partnership Project 3GPP adopted SC-FDMA for the uplink transmission system for the LTE [5][9][25]. SC-FDMA is attractive multiple access that considers an extended technique of Single Carrier with Frequency Domain Equalizer (SC-FDE). The block diagram for SC-FDMA and overall complexity is similar to OFDMA. The difference between the two technologies is the uses of discrete Fourier transform DFT in the transmitter [15][16]. Naturally SC-FDMA system technology offers frequency diversity gain over OFDMA system due to the use of DFT process at the beginning of the transmitter system. DFT in the transmitter is inserted before the Inverse Fast Fourier Transform IFFT to transfer the time domain symbols to frequency domain symbols. For this reason SC-FDMA sometimes is called DFT spread OFDM [8][17][18]. SC-FDMA combines the low peak to average power ratio result from the use of single carrier modulation and the robustness to multipath effect and flexibility allocation of frequency such as OFDMA [23].

METHODOLOGY
In this paper we analyze the SER for both SC-FDMA and OFDMA systems and we obtained the results analysis. Four types of multipath channel (ITU-pedestrian-A & B, and ITU-vehicular-A & B). On the receiver side we considered two types of equalizer; zero forcing (ZF) and minimum mean square error (MMSE) for channel compensation to explain the influences on the performance of the two systems. Matlab program was used to achieve the performance of the mention two systems.

**OFDMA & SC-FDMA systems model**

The major functionality as well as the throughput performance of OFDMA and SC-FDMA is very similar to each other. Both systems take an advantage to mitigate the multipath signal propagation make them suitable for broadband system [13]. Figure (1) shows the main blocks of OFDMA and SC-FDMA systems. First the data symbol is mapped into a complex of multilevel sequence of symbols \( X(n) \) due to the constellation mapping type (QPSK, 16QAM or 64QAM). The multilevel complex symbols are passed through serial to parallel convertor for modulation onto \( M \) parallel sub-carriers; this processing block helps to eliminate the inter-symbol interference [1][12]. In case of SC-FDMA, the parallel complex sequence of symbols is spreading over subcarriers by using DFT as in [7]:

\[
Y(k) = DFT\{X(n)\} = \frac{1}{\sqrt{M}} \sum_{n=0}^{M-1} X(n)e^{-j2\pi nk/M} \quad (1)
\]

where \( k = 0,1,..,M-1 \).

The output from DFT is mapped according to the mapper type, localized or interleaved form. In Localized mapper (LFDMA), the output of DFT is mapped to a subset of the adjacent sub-carrier; therefore it takes only fractions of the total system bandwidth while in the interleaved mapper (IFDMA), the output complex symbols are mapped equally over the entire total system bandwidth [8][10]. The \( N \)-point IFFT used to transform the frequency domain modulated symbols from the output

\[
x(n) = IFFT\{y(k)\} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} y(k)e^{j2\pi nk/N} \quad (2)
\]

where \( n = 0,1,..,N-1, N > M \)

A cyclic prefix is added to the transmitted modulated signal as a guard interval to mitigate the inter-symbol interference through the transmission in multipath fading channel. The equation to implement cyclic prefix addition as in [23]:

\[
x(n) = x(n) - \sqrt{Ng} + 1, ..., -1 \quad (3)
\]

where Ng is the cyclic prefix period.

Additive White Gaussian Noise (AWGN) was added to the transmitting signal as channel modeling. The spectral density of AWGN is constant with the amplitude is Gaussian distribution. It is expressed in watt per hertz [13]. The received signal can be expressed as in [7][23]:

\[
y(n) = \sum_{l=0}^{L-1} h(n,l)x(n-l) + w(n) \quad (4)
\]

Where \( L \) is the total number of paths of the frequency selective channel, \( w(n) \) is the Additive White Gaussian Noise (AWGN) with zero mean and variance \( E[|w(n)|^2] = \sigma_w^2 \), and \( h(n,l) \) is the sample spaced channel response. After removing the cyclic prefix, the received signal \( y(n) \) is transformed into the frequency domain by using Fast Fourier Transform FFT whereas the output signal \( y(k) \) is implemented as in [7][23] is:

\[
Y(k) = FFT\{y(n)\} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y(n)e^{-j2\pi nk/N} \quad (5)
\]

where \( k = 0,1,..,N-1 \).

If the cyclic prefix is assumed to be longer than the length of the channel impulse response, there is no inter-symbol interference.
will occur, the output signal can be implemented as in [7][14][23].

\[ Y(k) = X(k)H(k) + W(k) \]

Where \( H(k) \) represented the channel frequency response,

\[ H(k) = \sum_{n=0}^{N-1} H(k, n) \]  \hspace{1cm} (7)

, and \( W(k) \) represented the Fourier transform of the vector \( w(m) \), where the equation is as in [24]:

\[ W(k) = \sum_{m=0}^{N-1} w(m)e^{-j2\pi mk/N} \]  \hspace{1cm} (8)

### ITU channel Model

Two terrestrial tests of multipath channel models were proposed by the International Telecommunication Union (ITU) for developing 3G (IMT-2000) in which the structure is similar to the 3GPP multipath channel model, pedestrian and vehicular channel models for different environments according to user mobility. For each terrestrial test there are two types of channel models A for low spread delay and B for medium spread delay [19]. Tables (1) and (2) show the average power and the relative delay for A and B pedestrian and vehicular channel models as in [19]:

<table>
<thead>
<tr>
<th>Table 1 ITU - Pedestrian channel model</th>
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<tbody>
<tr>
<td>Tap No.</td>
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<tr>
<td></td>
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<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
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<tr>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2 ITU - Vehicular channel model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tap No.</td>
</tr>
<tr>
<td></td>
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<tr>
<td>1</td>
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<tr>
<td>4</td>
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Vehicular-A channel models have relatively long delay spread as compared with pedestrian-A channel; thereby it is more severe to frequency selective. The terrestrial pedestrian channel model is categorized by small cells and low transmitted power while vehicular terrestrial test is categorized by large cells and high transmitted power [19].

### Equalization

The cyclic prefix insertion not enough to mitigate the fading caused by the multipath propagation channel especially in case of SC-FDMA due to sequential transmission. Essentially channel equalization is the received signal filtering to mitigate the effect of inter-symbol interference (ISI) introduced by the channel impulse response variation in order to decrease the probability of error. In the receiver system of SC-FDMA the frequency domain equalizer simply divided the output signal from Fast Fourier Transform point by point by the frequency response of the estimate channel [5][21][22].

#### Zero Forcing (ZF) equalizer

Zero Forcing (ZF) is one tap linear equalizer used in communication systems. The concept of ZF equalizer is based on inverting the frequency response of the channel. It is simply used to restore the original signal by applying the invert channel response to the received signal at the output of the FFT process in order to remove the effect from the received signal. The equation for implementing ZF equalizer as in [16][20] is:

\[ H_{zf}(i, k) = H^{-1}(i, k) \]  \hspace{1cm} (9)

Where \( H_{zf}(i, k) \) is the channel frequency response and \( H^{-1}(i, k) \) is the inverse channel response for each symbol i and sub-carrier k.

If the output signal from the Fast Fourier Transform (FFT) is \( Y(i, k) \) then the output signal from ZF equalizer is:

\[ Z(i, k) = Y(i, k)H_{zf}(i, k) \]  \hspace{1cm} (10)

### Minimum Mean Square Error (MMSE) equalizer

Minimum Mean Square Error (MMSE) is another type of equalization process used to minimize the Mean Square Error (MSE) and it is considered wide approach for implementing
equalizer in communication systems. A higher advantage of using MMSE is not usually reduced the ISI, but it reduced the total power of noise and ISI component in the output. This feature of the MMSE equalizer of enhanced noise performance makes it more preferable than ZF equalizer which is poor toward the noise performance [20][22].

The equation to describe the function of MMSE as in [16][20]

\[
H_{\text{mmse}}(i, k) = \frac{H(i, k)}{(\|H(i, k)\|^2 + \sigma^2 p)}
\]

(11)

where (‘) mean the conjugate operation, \(\sigma^2\) is the variance of noise and \(p\) is the average power of the transmitted signal. If MMSE equalizer was used as equalization process, then the output signal [16][20] is:

\[Z(i, k) = Y(i, k)H_{\text{mmse}}(i, k)\]

(12)

The output signal \(Y(n)\) from the equalizer is transformed to time domain before going to the detection part (hard decision detection) to recover the original signal according to the following relationship [16][20].

\[Y(n) = \sqrt{M} \sum_{k=0}^{M-1} Z(k)e^{j2\pi nk/M}\]

(13)

**Simulation & Results**

In this part we analyze the SER for both SC-FDMA and OFDMA systems and we obtained the results analysis. The system parameters are shown in the table (3). To complete the simulation of the two systems we considered four types of multipath channel (ITU-pedestrian-A &B, and ITU- vehicular-A &B) with additive white Gaussian noise (AGWN). On the receiver side we considered two types of equalizer; zero forcing (ZF) and minimum mean square error (MMSE) for channel compensation to explain the influences on the performance of the two systems.

**Table 3 System parameters for simulation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>System bandwidth</td>
<td>5MHZ</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>3.84MHZ</td>
</tr>
<tr>
<td>IFFT in the transmitter &amp;FFT in the receiver</td>
<td>512</td>
</tr>
<tr>
<td>DFT in the transmitter &amp;IDFT in the receiver</td>
<td>128</td>
</tr>
<tr>
<td>Cyclic prefix size</td>
<td>64</td>
</tr>
<tr>
<td>Modulation type</td>
<td>QPSK</td>
</tr>
<tr>
<td>Channel estimation</td>
<td>Perfect</td>
</tr>
<tr>
<td>Equalizer type</td>
<td>ZF &amp; MMSE</td>
</tr>
<tr>
<td>Detection type</td>
<td>Hard decision</td>
</tr>
<tr>
<td>System iteration</td>
<td>(10^3)</td>
</tr>
</tbody>
</table>

In figures (2), (3), (4), (5) below with pedestrian-A, pedestrian-B, vehicular-A and vehicular-B respectively use both MMSE and ZF equalizer show the performance of SC-FDMA for the two sub-carrier mapper (LFDMA & IFDMA). It is clear that the performance of LFDMA is better than IFDMA for all types of ITU- channels in term of SER. We can also observe that MMSE equalizer gives a better performance to noise than ZF equalizer for all channel environments.
In figure (7) the comparison is between pedestrian-A & vehicular-A channel to explain the performance of SC-FDMA (LFDMA and IFDMA) using MMSE equalizer.

It is clear that the performance of the two types of mapper with pedestrian-A channel is better than with vehicular-A, and this is normal (pedestrian-A channel used in small environments and vehicular-A channel used for large environments.

In figures (8), (9), (10), (11) with pedestrian-A, pedestrian-B channel, vehicular-A and vehicular-B channels respectively using MMSE to show the performance of OFDMA. The performance of localized mapper is better than interleaved mapper in term of SER for all types of channel environments as well as the same result obtained if we use a Zero Forcing equalizer.
In figure (12) the performance of the two OFDMA mappers with both pedestrian (A & B) using MMSE equalizer. We can observe that the performance of the two mappers (localized & interleaved) with pedestrian-A is better than with the pedestrian - B channel.

In figure (13) the comparison is between pedestrian-A & vehicular-A. It is clear that the performance of the OFDMA system mapper in case of pedestrian-A is better than in case of a vehicular-A channel (pedestrian-A is considered for small environment with four taps whereas vehicular-A is considered for large environments with six taps).

In figures (14), (15) the comparison is between SC-FDMA & OFDMA mapper using vehicular-A &B channel environment with ZF &MMSE. It is clear that from the result shown the performance of SC-FDMA system is better than OFDMA systems for both types of sub-carrier mapper.
Conclusion

In this paper we have focused on the performance analysis of SC-FDMA and OFDMA systems in terms of SER, four types of ITU-channel (pedestrian and vehicular) were introduced with two types of equalizer; MMSE and ZF. For all types of ITU-channel, LFDMA sub-carrier mapper shows better SER performance than IFDMA sub-carrier mapper. Furthermore MMSE equalizer gives better performance as compared with ZF equalizer. IFDMA and LFDMA sub-carrier mapper with pedestrian-A channel gives better performance in SER as compared with pedestrian-B and vehicle-A (No. of taps is less in case of pedestrian-A channel). For all types of ITU-channel, the SER performance of SC-FDMA is better than OFDMA systems.

References


Figure(14) SER comparison of both OFDMA & SC-FDMA mappers for vehicular-A channel with MMSE equalizer

Figure(15) SER comparison of both OFDMA & SC-FDMA mappers for vehicular-A channel with MMSE equalizer


