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The Torsional Oscillator Again

"Magnetic Torque in Action"

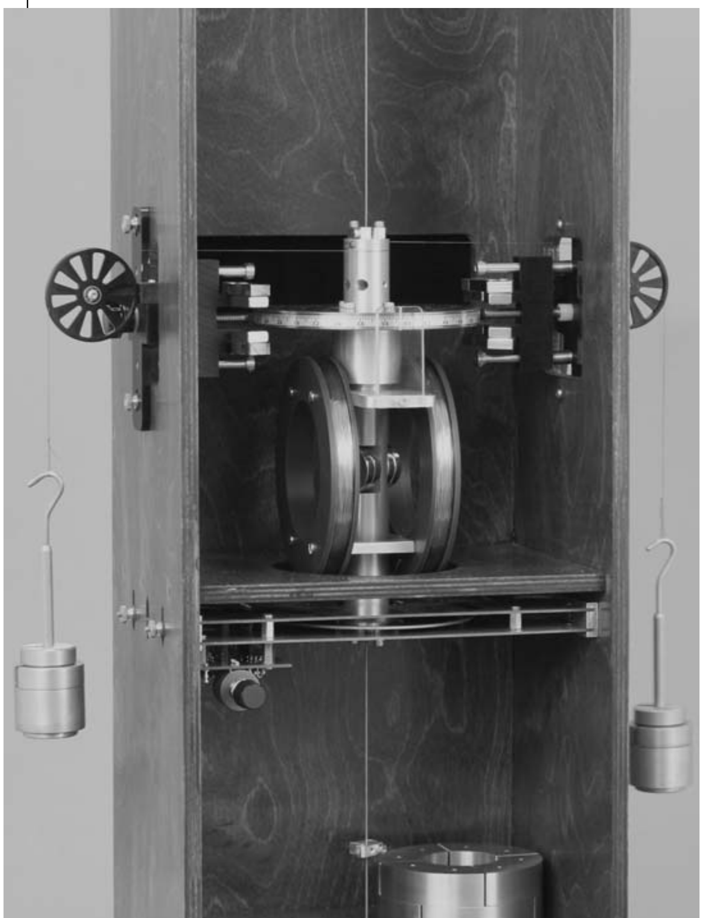

Fig 1: Center Section of the Torsional Oscillator

You'll recall that in our Torsional Oscillator, a strong taut steel fiber, anchored at its top and bottom, supports the 'rotor' at its center. In Fig. 1, you can see the rotor displaced from its equilibrium angular position. That's because of the mechanical torque applied to it by the two stretched strings and the weights they're bearing.

But notice that the rotor has a stack of four disc-shaped permanent magnets lying at the center of a pair of stationary Helmholtz coils. The cluster of magnets creates a magnetic moment, $\boldsymbol{\mu}$, whose direction in space at equilibrium is perpendicular to the axis of those coils. The coils, in turn, create a magnetic field **B** when an external steady current i is sent through them. The interaction between the field and the magnetic moment generates a torque on the rotor. The consequences of this torque are easily visible. A 'torque balance' makes an appropriate first investigation for students. Here, the student uses gravitational torque to displace the rotor, and then finds the current i which will restore the rotor to its original equilibrium position.

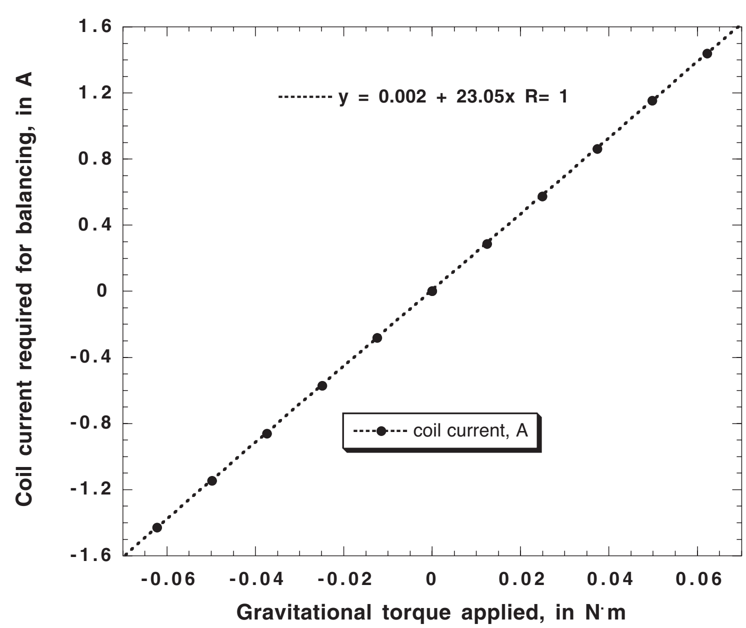

Fig 2: The current required to balance out a given gravitational torque

Last July's newsletter (find it at http://www.teach-spin.com/newsletters/) introduced our new Torsional Oscillator. It brings into the hands of students not only the dynamics of rotational motion, but all of the physics of simple harmonic oscillation. In that issue, we featured some of the fundamental *mechanical* experiments that your introductory-level students can perform on this apparatus. In this issue, we want to detail some of the rich physics that *magnetism* makes accessible to second-semester students via this highly visual and tactile apparatus. Even more advanced experiments will be featured in future newsletters.

Such data make it clear that magnetic torque exists, and that it's proportional to the current in the coils. Students can (and should!) compute, from the geometry of the coils, that the field they generate is of size B = k *i*, with coil-constant of k ≈ 3.22 mT/A. Thus the data above show quantitatively that magnetic torque τ_{mag} is linear in the field B. The best-fit line in the plot above shows that $\tau_{mag} = \mu$ B, where the coefficient $\mu \approx 13.5$ A.m^2 gives the magnitude of the magnetic moment of the permanent magnets. ***This is torque magnetometry in action.***

But there's a second set of experiments where the magnetic torque τ_{mag} is balanced instead against the restoring torque $\tau_{elastic}$ that develops when the steel fiber is twisted. Students will have already shown that the elastic torque can be modeled by $\tau_{elastic} = - \kappa \, \Delta\theta$, and they'll have measured a value for the torsion constant κ in previous experiments. Here's new data which shows the angular displacement from equilibrium which arises in response to current *i* in the coils.

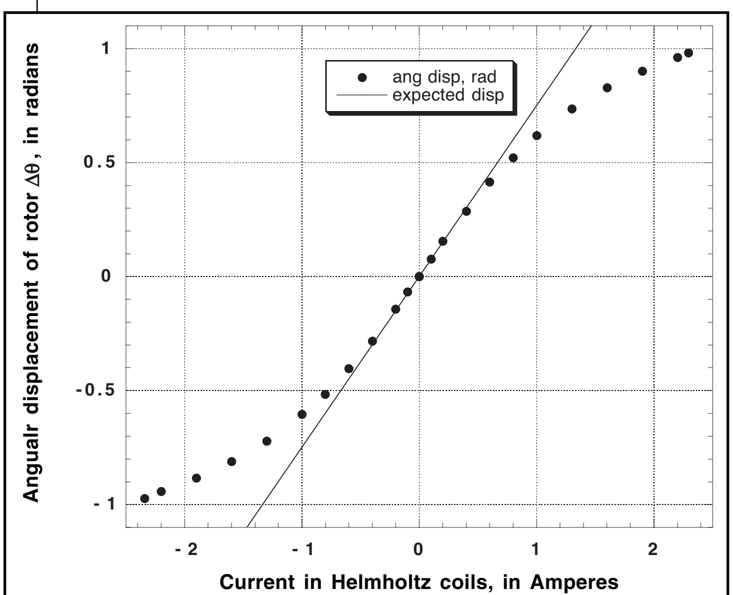

Fig 3: Angular displacement Δθ of the rotor from equilibrium, in response to magnetic torque arising from coil current *i*

In the plot, there's a line arising from an initial torque model, $\tau_{mag} + \tau_{elastic} = 0$, or

$$\mu \, B + (- \kappa \, \Delta\theta) = 0, \text{ which predicts}$$

$$\Delta\theta = (\mu/\kappa) \, B = (\mu k/\kappa) \, i.$$

That's a linear dependence of angular deflection on current, and it succeeds in fitting the i ≈ 0 data. But it's definitively *falsified* by the large-displacement data.

This is 'cognitive dissonance' in action -- a torque model that worked well in Fig. 2 fails systematically in Fig. 3. At small angles, the torque model $\tau_{mag} = \mu$ B works, but goes astray at larger angles. This is the hands-on experience that can lead students from a model $\tau_{mag} = \mu$ B to the cross-product model, $\boldsymbol{\tau}_{mag} = \boldsymbol{\mu} \times \mathbf{B}$. The magnitude of the correctly-predicted torque is $\tau_{mag} = \mu \, B \sin \phi$, where ϕ is the angle between $\boldsymbol{\mu}$ and $\mathbf{B}$. In the Torsional Oscillator, ϕ starts at 90°, but it *changes* with the angular displacement of the rotor. The improved torque-balance model is:

$$\mu \, B \sin (90° + \Delta\theta) + (- \kappa \, \Delta\theta) = 0, \text{ which gives}$$

$$\mu \, k \, i \cos (\Delta\theta) - \kappa \, \Delta\theta = 0 \text{ , or}$$

$$\Delta\theta / \cos (\Delta\theta) = (\mu \, k/\kappa) \, i \, .$$

Hence it's not Δθ, but instead Δθ/cos(Δθ), that is predicted to be linear in the current *i*. So the data of Fig. 3 is re-plotted in this fashion in Fig. 4, and now the data fall right along the predicted straight line.

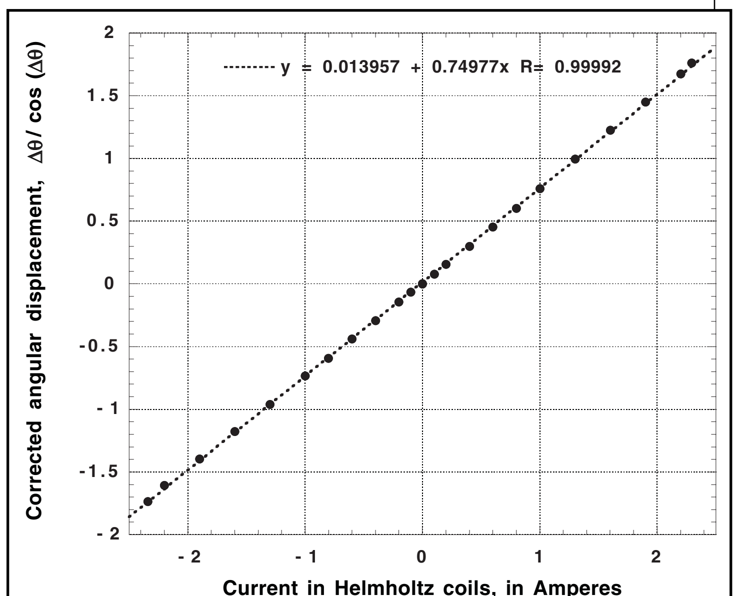

Fig 4: The data of Fig. 3 re-plotted, showing the linear dependence of Δθ/cos (Δθ) on coil current *i*

Now students can have confidence in a physically-motivated and experimentally-validated model for the magnetic torque they can exert on the rotor,

$\tau_{mag} = \mu \, B \sin \phi = \mu \, k \, i \cos (\Delta\theta)$. For motions of the rotor confined to small angular displacements, where $\cos (\Delta\theta) \approx 1$, the torque will be given by $\tau_{mag}(t) = \mu \, k \, i(t)$, even if the current is a function of time, $i(t)$. And the torque-per-unit-current of the torque drive is not some manufacturer-supplied or guessed number, but the value $\mu \, k \approx 0.0434$ N.m/A deduced right from the plots above. Future newsletters will display the wonderful resonant-excitation data that can be obtained using drive currents of sinusoidal, and other, waveforms in the Helmholtz coils.

But there's more that can be done, even at this level, with the magnet-in-coil system. The July newsletter showed that the permanent-magnet stack, moving within the Helmholtz coils, generates an emf which is a surrogate for the angular *velocity* of the rotor. Below is a graph of the output of the angular-position signal (from the Torsional Oscillator's electronic angular-position transducer) and this angular-velocity signal, both obtained as functions of time during simple harmonic motion of free torsional oscillations.

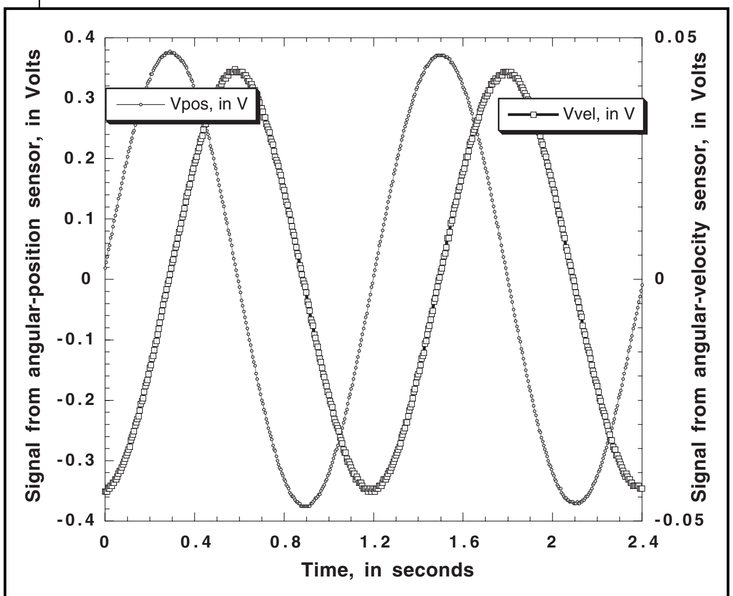

Fig 5: Signals from the position sensor (left scale) and the velocity sensor (right scale) during simple harmonic motion

The position signal is visibly sinusoidal, with period T ≈ 1.21 s, and with an amplitude of 0.373V. Previous calibration of the angular-position transducer allows the amplitude of the torsional oscillation, A ≈ 0.190 radians, to be deduced too. Students can now write the angular position as a function of time via the equation

$$\theta(t) = A \sin(2\pi\, t/T),$$

and use differentiation to predict an angular-velocity function of the form

$$d\theta(t)/dt = A\,(2\pi/T) \cos(2\pi\, t/T).$$

Thus, the peak angular velocity is predicted to have value $2\pi A/T = 0.989$ rad/s. Comparing that to the peak emf of the velocity signal shown in Fig. 5 permits a *calibration* of the velocity sensor. The result is that

(emf out) = (*s*) (angular velocity, dθ/dt),

where the sensitivity *s* has the numerical value of

s ≈ 43.6 mV/(rad/s).

Students will be surprised to find that the units of torque-per-unit-current, (for the action of the Helmholtz coils on the rotor) come out the *same* as those of emf-per-unit-angular-velocity (for the action of the magnets on the coil): namely, N.m/A = V/(rad/s). And even their instructors may be surprised to find that the value of the Oscillator's measured torque-per-unit-current, μ k ≈ 0.0434 N.m/A, comes out the *same* as the (separately) measured sensitivity of

s ≈ 43.6 mV/(rad/s).

This agreement is not an accident, but is instead a revelation of the deep principle of *reciprocity* governing the interaction of the magnets and the coils. In one case the coils act on the magnet to produce a torque; in separate experiments, the magnets act on the coils to produce an emf -- but the magnetic interaction between the two is governed by a common constant.

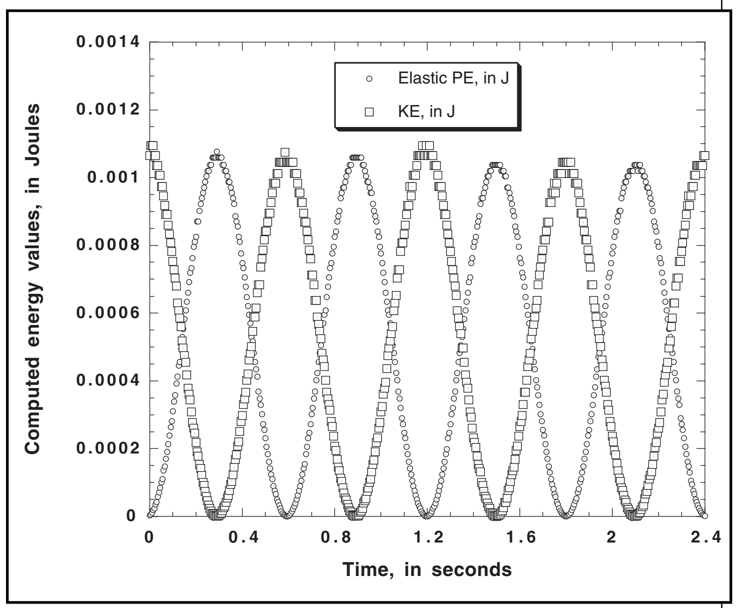

Fig 6: Waveforms for kinetic and potential energy as functions of time, computed from the data of Fig. 5

Given independent, and independently calibrated, sensors for angular position and angular velocity, it's irresistible to transform the two voltage waveforms of Fig. 5 into graphs of angular position (θ, in rad) and angular velocity (dθ/dt , in rad/s), respectively. Since previous mechanical experiments will have given good values for the torsional constant, κ, and the rotational inertia, I, of the system, it is also possible to graph the time history of the system's elastic potential energy, $U(t) = \kappa\,[\theta]^2/2$, and also its kinetic energy, $K(t) = I\,[d\theta/dt]^2/2$. These are plotted in Figure 6. The constancy of the sum, K(t) + U(t), calculated from the data, is a direct test of the conservation of energy in this mechanical system.

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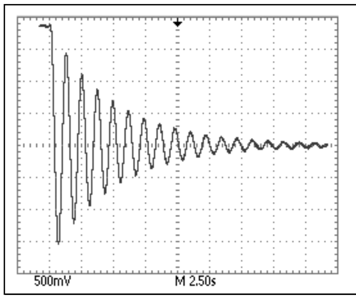

More Explorations with the Torsional Oscillator

Taking Real Harmonic Oscillators
out of the hands of theorists to put them
Into the Hands of Your Students.

continued from page 3

There are yet more experiments that can be done at this level to understand further details of the electro-magnetic and mechanical physics of this system, but perhaps you can already think ahead to lots more higher-level experiments you'd like to perform. Future newsletters will show you how time-dependent currents can resonantly, or off-resonantly, excite the mechanical motion of this oscillator. They'll also show you the three independent kinds of damping that the oscillator can experience. And we've lately discovered even more advanced-level experiments, including parametric excitation and coupled oscillations, that can be studied using these Torsional Oscillators.

For more information about the work-in-progress on this new instrument (the first deliveries have already occurred), see our website **www.teachspin.com**. Take advantage of our very low introductory price of $1,995/unit because it will end February 1, 2009. Low introductory prices are a tradition at Teachspin, designed to help our customers with tight budgets and help us get new instruments into the teaching community. ***We've finally gotten real harmonic oscillators out of the hands of theorists, now you can get them into the hands of your students.***

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