# Evidence-Based Practices Related to Problem Solving 

PATTAN
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## Sarahpowellphd.com <br> Evidence-based mathematics resources for educators



|  |
| :---: |

8 Tickets for a play were sold on Monday, Tuesday, and Wednesday.

Tickets Sold

| Day of the Week | Number of Tickets Sold |
| :--- | :---: |
| Monday | 197 |
| Tuesday | 364 |
| Wednesday | $\boldsymbol{?}$ |

If a total of 900 tickets were sold for the play, how many tickets were sold on Wednesday?
(A) 300
(B) 339
(C) 449
(D) 461

## How would you teach this problem?

What do students need to know to solve this problem?

## What might cause difficulty for students?

## Problem Solving Difficulties



Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.


## Reading problems

## Understanding vocabulary

Identifying relevant information

## Ignoring irrelevant information

Interpreting charts and graphs
What is the total number of packages Mr. Conley delivered on Monday and Tuesday?
(A) 300
(B) 340
(c) 350
(D) 360

Identifying appropriate operation(s)

Performing the computation(s)

Word Problem Solving
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Three Takeaways About Word Problems


## Don't tie key words to operations

## Have an attack strategy

Teach word-problem schemas



Kasey made $\$ 42$, and Mandy made $\$ 37$. How much money did they make in all?
Kasey and Mandy made \$79 in all. If Kasey made \$42, how much money did Mandy make?

Kasey mowed 12 lawns on Monday. Then, she mowed 10 more on Tuesday. How many lawns has Kasey mowed?

Kasey mowed 22 lawns and Mandy mowed 7 lawns. How many more lawns did Kasey mow than Mandy?


Becky has \$70 more than Perla. If Becky has \$120, how much money does Perla have?

Becky has \$70 more than Perla. If Perla has \$50, how much money does Becky have?

Becky had 9 dinosaurs and then her sister took away 4 of them. How many dinosaurs does Becky have now?

Becky had some dinosaurs and then her sister took away 4 of them. Now Becky has 5 dinosaurs. How many dinosaurs did she start with?


Matt baked 18 cookies. His brother baked twice as many. How many cookies did his brother bake?

Matt's brother baked twice as many cookies as Matt. If Matt's brother baked 36 cookies, how many did Matt bake?

Reece has 7 bags with 3 apples in each bag. How many apples does Reece have?
Reece had 21 apples and placed 3 apples each in several bags. How many bags does Reece need?


Rachel wants to share 36 brownies with 6 friends. How many cookies will each friend receive?

Rachel shared brownies with 6 friends. Each friend ate 6 brownies. How many brownies did Rachel have to start with?

Brent made 12 cupcakes. His brother made half as many cupcakes. How many cupcakes did Brent's brother bake?

Brent made 12 cupcakes. He cut each cupcake into half. How many pieces of cupcake does Brent have?

Michelle made 17 paper airplanes. Dante made 24 paper airplanes. How many airplanes did they make altogether?

Michelle and Dante made 41 paper airplanes altogether. If Dante made 24 paper airplanes, how many did Michelle make?

Michelle made 4 paper airplanes using 2 pieces of paper for each airplane. How much paper did Michelle use altogether?

Dante and Michelle made 40 paper airplanes altogether. Dante made 24 of the paper airplanes. If Michelle gave 7 of her paper airplanes to her friend Nicole, how many planes does Michelle have now?

Key Words Used in Math Word Problems


Students need to understand key words. But, key words should not be directly tied to operations.

Name $\qquad$ Date


Read each problem. Write a number sentence and solve.

1. Mrs. Smith has 33 poodles and 18 boxers. How many more poodles does Mrs. Smith have?
2. The kennel holds 91 dogs. Mr. Glass has 67 dogs in the kennel now. How many spaces does he have left?

3. Mr. Kelly has 44 beagles. 26 of them are puppies. How many adult beagles does Mr. Kelly have?

4. There were 58 kittens at the pet shop on Friday. 29 of them were sold on Saturday. How many kittens were left?

$\qquad$
5. Pat counted 22 lizards in the tank at the pet shop. 8 were sold later that day. How many lizards were left in the tank?


## Don't tie key words to operations

## Have an attack strategy

Teach word-problem schemas

## For every word problem

Regardless of problem type, students need an attack strategy for working through the problem

This strategy should work for any problem type

## Routine Word Problems

24. Part A

The library has 48 fewer books about rivers than about trees.
What is the number of books the library has about rivers and what is the total number of books the library has about trees and rivers?
(4) 78 and 126
(B) 48 and 204
(C) 48 and 126
(2) 78 and 204

## Instructional Word Problems

7. Which three shapes are quadrilaterals?
(A)

©


(®)


## RIDGHSS

Read the problem.
I know statement.
Draw a picture.
Goal statement.
Fquation development. Solve the equation.


## RIDE

Read the problem.
Identify the relevant information.
Determine the operation and unit for the answer.


Enter the correct numbers and calculate, then check the answer.



## RUN!

## 1. Read the problem.

2. Underline the labels.
3. Name the problem type.


## SIGNS

Survey questions
Identify key words
Graphically draw problem
Note operations
Solve and check

## SOLVE

Study the problem.
Organize the facts. Line up the plan.
Verify the plan with computation. Examine the answer.

| S | slowly and carefully READ the problem. |
| :--- | :--- |
| H | highlight or underline key information. |
| I | identify the question by drawing a circle around it. |
| N | now solve the problem with numbers, pictures, and words. Show <br> your work. |
| E | examine your work for precision, accuracy, and clarity. |
| S | share your answer by writing a sentence. |



## Steps to Solve a Problem: UPS Check!

## UNDERSTAND

- Read the problem carefully.
- Highlight or circle necessary data, key words, and labels.
- Identify the question.


## PLAN

- Record the information needed to solve the problem.
- Draw a picture or diagram of the situation and label all parts.
- Choose the appropriate strategy, tool, or operation.


## SOLVE

- Write an appropriate equation for the situation.
- Use your plan and data to solve.
- Write your solution with units if applicable.


## CHECK

- Check your math (substitute the value(s) into your equation).
- Did you answer the question?
- Is your answer reasonable?



## Don't tie key words to operations

## Have an attack strategy

Teach word-problem schemas

## Instruction Using Schemas

A schema refers to the structure of the word problem


Additive Schemas

| Problem <br> type | Definition | Examples |  |  | Equation | Graphic organizer |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Total |  |  | Total unknown | Part unknown |  |  |

Parts put together into a total

- Emily saw 4 cardinals and 5 blue jays. How many birds did Emily see?
$-4+5=$ ?
- Emily saw 9 birds. If 4 of the birds were cardinals, how many were blue jays?

$$
\circ 4+?=9
$$

- Emily saw 9 birds. 5 of the birds were blue jays, how many were cardinals?

$$
\circ 5+?=9
$$

Total

## \& \&



Additive Word Problems

| A. <br> Mr. Jones has 8 pet crickets and 3 pet fish in his <br> classroom. He also has 6 plants. How many pets <br> does Mr. Jones have in his classroom? | B. <br> Ella and Susan planted 12 flower seeds. Ella planted <br> 3 flower seeds and 2 trees. How many flower seeds <br> did Susan plant? |
| :--- | :--- |

## Total

TMr. Jones has pet crickets and pet fish in his classroom. He also has 6 plants. How many pets does Mr. Jones have in his classroom?

$$
\begin{aligned}
& P 1+P 2=T \\
& 8+3=X \\
& 8+3=11 \\
& X=11 \text { pets }
\end{aligned}
$$

## Total

TMegan baked sugar cookies and $\mathbb{2}$ / chocolate chip cookies. Enter the total number of cookies Megan baked in all.

$$
\begin{aligned}
& P 1+P 2=7 \\
& 28+24=? \\
& 28+24=52 \\
& X=52 \text { cookies }
\end{aligned}
$$

## Total

"Are parts put together for a total?"

## Total

$T$ Ella and Susan planted $\Sigma$ flower seeds. Ella planted $巴$ flower seeds and ztrees. How many flower seeds did Susan plant?

$$
\begin{array}{lrr}
P 1+P 2=7 & 3 & 12 \\
3+?=12 & +9 & -\frac{3}{9} \\
3+9=12 & \\
?=9 \text { flower seeds }
\end{array}
$$

## Total

T. A) A banana farm received a total of $C$ millimeters of rain in March and April. If millimeters of rain fell on the farm in March, how many millimeters of rain fell on the farm in April?

? = 1 millimeter

| With the total unknown |
| :--- |
| with a part unknown Word Problems |
| wotal\|| |
| DiFFERENCE |
| CHANGE (INCREASE) |
| CHANGE (DECREASE) |


| Problem <br> type | Definition | Examples |  |  | Equation | Graphic organizer |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Total |  |  | Total unknown | Part unknown |  |  |

## Difference

Greater and less amounts compared for a difference - Shinead has 9 apples. Amanda has 4 apples. How many more apples does Shinead have? (How many fewer?)
$-9-4=$ ?

- Shinead has 5 more apples than Amanda. If Amanda has 4 apples, how many does Shinead have?
- ? - 4 = 5
- Amanda has 5 fewer apples than Shinead. Shinead has

9 apples. How many apples does Amanda have?
-9 - ? = 5

## Difference

## B



## Quantity <br> 12

## Difference

D Jana has $10 /$ wooden beads and 68 glass beads. How Many more wooden beads than glass beads does Jana have?

$$
\begin{aligned}
& G-L=D \\
& 107-68=B \quad 107 \quad 68 \\
& 107-68=39 \\
& W=39 \text { more beads }
\end{aligned}
$$

Additive Word Problems

| A. <br> Mr. Jones has 8 pet crickets and 3 pet fish in his <br> classroom. He also has 6 plants. How many pets <br> does Mr. Jones have in his classroom? | B. <br> Ella and Susan planted 12 flower seeds. Ella planted <br> 3 flower seeds and 2 trees. How many flower seeds <br> did Susan plant? |
| :--- | :--- |
|  |  |
|  |  |

## Difference

Zach wrote $1 / 9$ words in his math journal. Pedro wrote $/ / 4$ words. How many fewer words did Zach write than Pedro?

$$
\begin{aligned}
& B-s=D \\
& 74-49=W \\
& 74-49=25 \quad-49 \\
& \hline \begin{array}{l}
74 \\
74
\end{array} \\
& W=25 \text { fewer words }
\end{aligned}
$$

## Total

"Are parts put together for a total?"

## Difference

"Are amounts compared for a difference?"

Farmer Hank has 6 more cows than horses. He has 4 horses. He also has 9 chickens. How many cows does he have?

## Additive Word Problems

DIFFERENCE
with the difference unknown
with a greater amount unknown
with a lesser amount unknown
CHANGE (INCREASE)

| Problem <br> type | Definition | Examples |  |  | Equation | Graphic organizer |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Total |  |  | Total unknown | Part unknown |  |  |

## Change

An amount that increases or decreases

- Shannah had $\$ 4$. Then she got $\$ 3$ for cleaning her room. How much money does Shannah have now?
- $4+3$ = ?
- Shannah has $\$ 4$. Then she earned money for cleaning her room. Now Shannah has $\$ 7$. How much money did she earn?
-4 + ? = 7
- Shannah had some money. Then she made \$3 for cleaning her room. Now she has $\$ 7$. How much money did Shannah start with?
- ? $+3=7$


## Change

## ST <br> +/- C <br> $=$ <br> E



Fuchs et al. (2008); Griffin \& Jttendra (2009)

Additive Word Problems

| E. | F. <br> A bus had 13 passengers. At the next stop, more <br> passengers got on the bus. Now, there are 28 pas- <br> sengers. How many passengers got on the bus? <br> Martina had some money. Then, she spent $\$ 42$ on <br> a sweater. Now, she has $\$ 13$. How much money did <br> she have to start with? |
| :--- | :--- |

## Change

+ C A bus had 13 passengers. At the next stop, more passengers got on the bus. Now, there are $\uparrow$ passengers. How many passengers got on the bus?

$$
\begin{aligned}
& S T+C=E \\
& 13+?=28 \quad \begin{array}{rr}
28 & 13 \\
-13 & +? \\
28
\end{array}
\end{aligned}
$$

$$
13+15=28
$$

## Total

"Are parts put together for a total?"

## Difference

"Are amounts compared for a difference?"

## Change

"Does an amount increase or decrease?"

| Additive Word Problems |
| :--- | :---: |
| TOTAL |
| WifFERENCE |
| with the start unknown |
| with the change unknown |
| CHANGE (INCREASE) |
| CHANGE (DECREASE) |

## Change

An amount that increases or decreases

- Micaela had \$9. Then she spent \$2 on candy. How much money does Micaela have now?
-9-2 = ?
- Micaela had \$9. She spent some money on candy. Now Micaela has $\$ 7$. How much money did Micaela spend on candy?
-9-? $=7$
- Micaela had some money. Then she spent \$2 on candy. Now she has $\$ 7$. How much money did Micaela start with?

$$
\circ ?-2=7
$$

## Change

## ST <br> +/- C <br> $=$ <br> E



Fuchs et al. (2008); Griffin \& Jttendra (2009)

Change

Martina had some money. Then, she spent \$42 on a sweater. Now, she has $\$ 13$. How much money did she have to start with?

| Additive Word Problems |
| :---: |
| TOTAL |
|  |  |
|  |
| CHANGE (DECREASE) |
| with the end unknown |
| with the start unknown |
| with the change unknown |


| Problem <br> type | Definition | Examples |  |  | Equation | Graphic organizer |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Total |  |  | Total unknown | Part unknown |  |  |


| Additive Word Problems |  |  |
| :--- | :--- | :---: |
| E. <br> A bus had 13 passengers. At the next stop, more <br> passengers got on the bus. Now, there are 28 pas- <br> sengers. How many passengers got on the bus? | F. <br> Martina had some money. Then, she spent $\$ 42$ on <br> a sweater. Now, she has $\$ 13$. How much money did <br> she have to start with? |  |


| Additive Word Problems |  |  |
| :--- | :--- | :---: |
| I. <br> The grocery store had 517 jars of crunchy peanut <br> butter and 434 jars of creamy peanut butter. How <br> many more jars of crunchy peanut butter were <br> there? | J. <br> The animal park has 12 zebras, 25 monkeys, <br> and some giraffes. If the total number of zebras, <br> monkeys, and giraffes at the park is 50, how many <br> giraffes are there? |  |

## Total

Ramon has a total of 815 sheep in two fields. He has 348 sheep in one of the fields. How many sheep does Ramon have in the other field?

Change

Angelina looked in her closet and saw a container of markers. She took 42 markers out of the container and counted 88 left. How many markers were in the container when she found it in the closet?

The grocery store had 517 jars of crunchy peanut butter and 434 jars of creamy peanut butter. How many more jars of crunchy peanut butter were there?

| Additive Word Problems |  |
| :--- | :--- | :--- |
| I. <br> The grocery store had 517 jars of crunchy peanut <br> butter and 434 jars of creamy peanut butter. How <br> many more jars of crunchy peanut butter were <br> there? | J. <br> The animal park has 12 zebras, 25 monkeys, <br> and some giraffes. If the total number of zebras, <br> monkeys, and giraffes at the park is 50, how many <br> giraffes are there? |

The animal park has 12 zebras, 25 monkeys, and some giraffes. If the total number of zebras, monkeys, and giraffes at the park is 50, how many giraffes are there?

$$
P 1+P 2+P 3=T
$$

Change

There were some people on a train. 19 people get off the train at the first stop. 17 people get on the train. Now there are 63 people on the train. How many people were on the train to begin with?

$$
S T-C+C=E
$$

Change

Mrs. Lanier saved \$617 in January. In February, she spent $\$ 249$ of the money she saved. She saved \$291 more in March. How much has Mrs. Lanier saved by the end of March?

$$
S T-C+C=E
$$

| Additive Word Problems |  |
| :--- | :--- |
| TOTAL |  |
| DIFFERENCE |  |
| CHANGE (INCREASE) |  |
| CHANGE (DECREASE) |  |

## Let's Review

Why is an attack strategy helpful?
What's a Total problem?
What's a Difference problem?
What's a Change problem?
Why should students learn schemas?

## Total

A baker has 159 cups of brown sugar and 264 cups of white sugar. How many total cups of sugar does the baker have?

## Change

Pablo goes to a stamp show where he can share, buy, and sell stamps.

## 26. Part A

The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.

What is the total number of stamps that Pablo has after the first day of the stamp show?

## Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.

Recycled Plastic


Based on the graph, how many more pounds of plastic did the family recycle in July than in April?

## Total

Roland's family drove $4 \frac{6}{10}$ kilometers from their home to the gas station.
They drove $2 \frac{30}{100}$ kilometers from the gas station to the store.

Which expression can be used to determine the number of kilometers Roland's family drove altogether?

## Change

At the beginning of June, a bean plant was $3 \frac{4}{5}$ inches tall. By the beginning of July, the plant was $6 \frac{2}{5}$ inches tall. How many inches did the plant grow during June? Enter your answer in the response box.

Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.

10. Part A

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?
(4) 300
(B) 340
(c) 350
(D) 360

Farmer Hank has 6 more cows than horses. He has 4 horses. He also has 9 chickens. How many cows does he have?

## Change

Shelby's teacher gives out gold stars for great math work. Yesterday, Shelby earned 4 gold stars. Today, she earned 3 more. How many gold stars did Shelby earn in all?

## Difference

Mrs. Taylor's class is measuring the wingspans of butterflies, in inches.
They record their results in this table.

| Butterfly | Wingspan (in) |
| :--- | :---: |
| Monarch | $3 \frac{2}{4}$ |
| Zebra | $2 \frac{3}{4}$ |
| Checkered White | $1 \frac{2}{4}$ |
| Tiger | $?$ |

The wingspan of the Tiger Butterfly is $\frac{3}{4}$ inch longer than that of the Monarch Butterfly.

What is the difference, in inches, between the longest and shortest wingspans?

## Total

Jana gets a sticker for every 5 minutes she spends on her chores each day. She puts them on a picture graph as shown.

## Jana's Chores

| Day | Minutes of Chores |
| :---: | :---: |
| Monday | $\hat{\sim}$ |
| Tuesday |  |
| Wednesday |  |
| Thursday | $\hat{\sim} \dot{\sim}$ |
| Friday |  |


| KEY |
| :---: |
| $\sim=5$ minutes |

Jana spends a total of 130 minutes doing chores during the week. How many stickers should Jana get on Friday?

## Change

You have 18 marbles but you gave your friend 9. Now how many marbles do you have?

## Change

Ana starts eating lunch at 12:15 p.m. She finishes eating lunch 40 minutes later.

Which clock shows the time that Ana finishes eating lunch?
(4)

(B)

©

©


## Difference

Mr. Thompson sold 247 meals on Tuesday at his restaurant. He sold 516 meals on Wednesday. What is the difference between the numbers of meals Mr. Thompson sold on these two days?

Change
There were 58 kittens at the pet shop on Friday. 29 of them were sold on Saturday. How many kittens were left?

## Total

The table shows the number of pennies Nolan saved each week for four weeks.

## Pennies Saved Each Week

| Week | Number of Pennies |
| :---: | :---: |
| Week 1 | 18 |
| Week 2 | 40 |
| Week 3 | 32 |
| Week 4 | 25 |

What is the total number of pennies Nolan saved during the four weeks? Show your work.

Enter your answer and your work in the space provided.

## Change

Adyssen started with $\$ 87$ in her bank account. She put $\$ 213$ into her account last week and another $\$ 137$ this week. What is the total amount Adyssen now has in her bank account?

## Difference

Carla buys apples and peaches at the store. The mass of the apples is 724 grams. The mass of the peaches is 471 grams.

How much greater is the mass, in grams, of the apples than the mass of the peaches?

## Change

## Mathematics Year 2

## Set A: Paper 2

There were some people on a train.
19 people get off the train at the first stop. 17 people get on the train.
Now there are 63 people on the train. How many people were on the train to begin with?

Show your method.

## Total

Mrs. Green has 60 terriers. 25 of them are boys. How many terriers are girls?

## Change

Mary's cat had kittens. She gave 3 to her friends.
She now has 6 kittens left. How many kittens did she have to start with?

## Change

Last month Jim drove his car 2,718.3 miles. That brought the car's total mileage to 87,416 miles. What was the car's total mileage before last month?

## Total

Vicente hung three posters in his bedroom.

- The first poster had a length of 59 centimeters.
- The second poster had a length of 92 centimeters.
- The third poster had a length of 127 centimeters.

What is the best estimate of the total length of these three posters in centimeters?

A 260 cm
B 350 cm
C 240 cm
D 280 cm

Nolan has 16 pennies in one jar and 94 pennies in another jar.
He uses some of the pennies to buy a pencil that costs 25 cents. What is the total number of pennies Nolan has left after he buys the pencil? Show your work.

## Let's Look Back

8 Tickets for a play were sold on Monday, Tuesday, and Wednesday.

Tickets Sold

| Day of the Week | Number of Tickets Sold |
| :--- | :---: |
| Monday | 197 |
| Tuesday | 364 |
| Wednesday | $\boldsymbol{?}$ |

If a total of 900 tickets were sold for the play, how many tickets were sold on Wednesday?
(A) 300
(B) 339
(C) 449
(D) 461

## Don't tie key words to operations

## Have an attack strategy

Teach word-problem schemas

Multiplicative Schemas


## Instruction Using Schemas

A schema refers to the structure of the word problem

## Equal Groups

## Comparison

## Ratios/Proportions

## Equal Groups

## Groups multiplied by number in each group for a product

- Mark has 2 bags of apples. There are 6 apples in each bag. How many apples does Mark have altogether?
- $2 \times 6=$ ?
- Mark has 12 apples. He wants to share them equally among his 2 friends. How many apples will each friend receive?
- $2 \times$ ? $=12$
- Mark has 12 apples. He put them into bags containing 6 apples each. How many bags did Mark use?
- ? $\times 6=12$


## Equal Groups

## G <br>  <br> P



| Multiplicative Word Problems |  |
| :--- | :--- | :--- |
| A. <br> Ms. Thompson sold 6 cartons of cherries at the <br> Farmers'Market. Each carton holds 25 cherries. How <br> many cherries did she sell? | B. <br> If a scuba diver could carry 36 coins to the surface in <br> one trip, how many trips would it take to carry 108 <br> coins? |

A.

Ms. Thompson sold $6 /$ cartons of cherries at the Farmers' Market. Each carton holds 25 cherries. How many cherries did she sell?

$$
\begin{aligned}
& G \times N=P \\
& 6 \times 25=? \\
& 6 \times 25=150
\end{aligned}
$$

$$
?=150 \text { cherries }
$$

## Equal Groups

"Are there groups with an equal number in each group?"

## Equal Groups

Cade hasef boxes. He puts 9 model cars in each box.
What is the total number of model cars Cade put in these boxes?


$$
\begin{aligned}
4 \times 9 & =? \\
? & =36 \text { model cars }
\end{aligned}
$$

## Equal Groups

Jane bought 24 light bulbs. The light bulbs come in packs of 4 \& How many packs of light bulbs did Jane buy?


## Comparison

## Set multiplied by a number of times for a product

- Jill picked 6 apples. Mark picked 2 times as many apples as Jill. How many apples did Mark pick?
- $6 \times 2=$ ?
- Mark picked 12 apples. He picked 2 times as many apples as Jill. How many apples did Jill pick?
- ? $\times 2$ = 12
- Mark picked 12 apples, and Jill picked 6 apples. How many times as many apples did Mark pick as Jill did?
- $6 \times$ ? $=12$


## Comparison

## S $x$ <br> T <br> p

| Multiplicative Word Problems |  |
| :--- | :--- | :--- |
| A. <br> Ms. Thompson sold 6 cartons of cherries at the <br> Farmers'Market. Each carton holds 25 cherries. How <br> many cherries did she sell? | B. <br> If a scuba diver could carry 36 coins to the surface in <br> one trip, how many trips would it take to carry 108 <br> coins? |

If a scuba diver could carry 36 coins to the surface in one trip, how many trips would it take to carry 108 coins?

## Comparison

Isabella has 2 times as many DVDs as Emma. Emma has 6 DVDs.

$2 \times 6=?$
$?=12 \mathrm{DVDS}$

## Equal Groups

"Are there groups with an equal number in each group?"

## Comparison

## "Is a set compared a number of times?"

## Combinations

Set multiplied by a set for a product

Examples

- Sam bought 4 pairs of pants and 3 jackets, and they can all be worn together. How many different outfits consisting of a pair of pants and a jacket does Sam have?
$\circ 4 \times 3=$ ?


## Ratios/Proportions

Description of relationships among quantities


## Ratios/Proportions



Sylvia used $/$ cans of icing to ice $2 /$ cupcakes. How many cupcakes can she ice with $\mathcal{Z}$ cans of icing?

## 2 cans $=3$ cans <br> 24 cupcakes ? cupcakes ? = 36 cupcakes

| Multiplicative Word Problems |  |
| :--- | :--- | :--- |
| A. <br> Ms. Thompson sold 6 cartons of cherries at the <br> Farmers'Market. Each carton holds 25 cherries. How <br> many cherries did she sell? | B. <br> If a scuba diver could carry 36 coins to the surface in <br> one trip, how many trips would it take to carry 108 <br> coins? |

There are 176 slices of bread in 8 loaves. If there are the same number of slices in each loaf, how many slices of bread are in 5 loaves?


A sea turtle made 460 dives in 12 hours. At this rate, how many dives did the sea turtle make in 3 hours?


## Equal Groups

## "Are there groups with an equal number in each group?"

## Comparison

## "Is a set compared a number of times?"

## Ratios/Proportions

"Are there relationships among quantities if this thenthis?"

## Multiplicative Word Problems

E.
F.

At the store, lollipops are sold in packages of 4. How Isaiah put 301 floor tiles in 7 rows. Each row had the many packages would you need to buy to have 24 same number of tiles. How many tiles did Isaiah put lollipops? in each row?
G.

There are 27 large t-shirts and 15 small t-shirts in a box. Each t -shirt costs $\$ 9$. What is the cost of all the t-shirts in the box?

Zenobia put 3 large pictures and 4 small pictures on each page of a photo album. What is the total number of pictures on 9 pages of the album?
E.

At the store, lollipops are sold in packages of 4. How many packages would you need to buy to have 24 lollipops?
F.

Isaiah put 301 floor tiles in 7 rows. Each row had the same number of tiles. How many tiles did Isaiah put in each row?
G.

There are 27 large t-shirts and 15 small t-shirts in a box. Each $t$-shirt costs $\$ 9$. What is the cost of all the $t$-shirts in the box?
H.

Zenobia put 3 large pictures and 4 small pictures on each page of a photo album. What is the total number of pictures on 9 pages of the album?

Multiplicative Word Problems

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I. J.
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On average, thunder is heard in Tororo, Uganda, 251 days each year. What is the probability that thunder will be heard in Tororo on any day?

Rima and Eric have earned a total of 135 tokens to buy items at the school store. The ratio of the num-
ber of tokens that Rima has to the number of tokens that Eric has is 8 to 7 . How many tokens does Rima have?
K.

The length of a photograph is 5 inches and its width is 3 inches. The photograph is enlarged proportionally. The length of the enlarged photograph is 10 inches. What is the width of the enlarged photograph?

Stacie rides her bike 3 miles in 12 minues. At this rate, how long will it take her to ride her bike 7 miles?
I.

On average, thunder is heard in Tororo, Uganda, 251 days each year. What is the probability that thunder will be heard in Tororo on any day? ( 1 year $=365$
days)
J.

Rima and Eric have earned a total of 135 tokens to buy items at the school store. The ratio of the number of tokens that Rima has to the number of tokens that Eric has is 8 to 7 . How many tokens does Rima have?
K.

The length of a photograph is 5 inches and its width is 3 inches. The photograph is enlarged proportionally. The length of the enlarged photograph is 10 inches. What is the width of the enlarged photograph?

Stacie rides her bike 3 miles in 12 minutes. At this rate, how long will it take her to ride her bike 7 miles?

## Equal Groups

Mr. Kowolski ordered 35 boxes of granola bars. Each box contained 24 granola bars.

What is the total number of granola bars Mr. Kowolski ordered?

## Comparison

Susan has 3 times as many books as Mary. Mary has 18 books. Which equation can be solved to figure out how many books Susan has?

## Equal Groups

There are 1,092 people who work in an office building. The building has 4 floors, and the same number of people work on each floor. How many people work on each floor?

## Ratios/Proportions

Ethan correctly answers $80 \%$ of the total questions on his history test. He correctly answers 32 questions.

## Equal Groups

The members of a gym use 98 towels every day. How many towels are used in 7 days?

## Equal Groups

Ryan makes 6 backpacks. He uses $\frac{3}{4}$ yard of cloth to make each backpack. What is the total amount of cloth, in yards, Ryan uses to make all 6 backpacks?
A. $1 \frac{1}{2}$
B. $2 \frac{1}{4}$
C. $4 \frac{1}{2}$
D. $6 \frac{3}{4}$

## Comparison

A basketball team scored a total of 747 points for the season. This was 9 times the number of points scored in the first game. How many points were scored during the first game?

## Ratios/Proportions

A company makes 625 cell phone cases each day. How many cell phone cases does the company make in 31 days?

## Equal Groups

Eric has 158 action figures to put in display cases. Each display case can hold 8 action figures. How many cases does Eric need to hold all his action figures?

## Ratios/Proportions

There are 7 math folders on a classroom shelf. This is $\frac{1}{3}$ of the total number of math folders in the classroom.

## Ratios/Proportions

A bus travels 36 miles in 45 minutes.
Enter the number of miles the bus travels in 60 minutes at this rate.

## Comparison

Danielle's full-grown dog weighs 10 times as much as her puppy. The puppy weighs 9 pounds.

Enter the number of pounds the full-grown dog weighs.

## Ratios/Proportions

The number of blueberry muffins that a baker makes each day is $40 \%$ of the total number of muffins she makes.

## 36. Part A

On Monday, the baker makes 36 blueberry muffins.
What is the total number of muffins that the baker makes on Monday?
Enter your answer in the box.

## Equal Groups

A basketball team plays 82 games each year. How many games will the team play in 25 years?

## Let's Review

What's an Equal Groups problem?
What's a Comparison problem?
What's a Ratio/Proportions problem?

## There are 9 cherry trees.

- Kim picks 8 cherries from each tree. - Kim eats 14 of the cherries she picked.

Hayley has 272 beads. She buys 38 more beads. She will use 89 beads to make bracelets and the rest to make necklaces. She will use 9 beads for each necklace.

What is the greatest number of necklaces Hayley can make?

## Don't tie key words to operations

## Have an attack strategy

Teach word-problem schemas

## Contact Information

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Evidence-based mathematics resources for educators



