Math 3331 – ODE's   Lect 19

Finding y_p

$$y'' - 2y' + y = 2e^x$$

$\underline{\underline{y_c}}$ $m^2 - 2m + 1 = 0$ $m = 1, 1$

$$y_c = c_1 e^x + c_2 x e^x$$

So we need to "bump" up twice

$$y_p = Ax^2 e^x$$

and find that $A = 1$ so $y = c_1 e^x + c_2 x e^x + x^2 e^x$

Way 2   Reduction of Order

using 1 sol^n of y_c

$$y = e^x u \,, \quad y' = e^x u' + e^x u$$

$$y'' = e^x u'' + 2e^x u' + e^x u$$

Sol^n

$$e^x u'' + 2e^x u' + e^x u - 2(e^x u' + e^x u) + e^x u = 2e^x$$

$$e^x u'' + 2\cancel{e^x u'} + \cancel{e^x u} - \cancel{2e^x u'} - \cancel{2e^x u} + \cancel{e^x u} = 2e^x$$

$$e^x u'' = 2e^x$$

keep const

$$\Rightarrow \quad u'' = 2 \quad \Rightarrow \quad u' = 2x + c_1$$

$$u = x^2 + c_1 x + c_2$$

$$\text{so} \quad y = (x^2 + c_1 x + c_2)e^x$$

$$= \underbrace{c_1 x + c_2 e^x}_{y_c} + \underbrace{x^2 e^x}_{y_p}$$

so this gives entire sol^n

Ex 2

$$y'' + y = \sec x$$

y_c $\quad y'' + y = 0$

$m^2 + 1 = 0 \qquad m = \pm i \qquad \alpha = 0, \ \beta = 1$

$$y_c = c_1 \sin x + c_2 \cos x$$, don't know what to guess

Set $y = \cos x \, u$

$$y' = \cos x \, u' - \sin x \, u$$

$$y'' = \cos x \, u'' - 2 \sin x \, u' - \cos x \, u$$

Sub

$$\cos x \, u'' - 2 \sin x \, u' \underbrace{\cancel{- \cos x \, u} + \cancel{\cos x \, u}}_{\text{will always cancel}} = \sec x$$

so if $u' = v$, $u'' = v'$ like befor

$$\cos x \, v' - 2 \sin x \, v = \sec x$$ linear ODE

$$v' - \frac{2 \sin x}{\cos x} v = \frac{1}{\cos^2 x}$$

$$\mu = e^{\int -\frac{2\sin x}{\cos x}\,dx} = e^{2\ln\cos x} = \cos^2 x$$

so $\frac{d}{dx}(\cos^2 x\, v) = 1$

$\cos^2 x\, v = x + c_1$ Keep const

so $v = x\sec^2 x + c_1 \sec^2 x$

$$\frac{du}{dx} = x\sec^2 x + c_1 \sec^2 x$$

$$u = \int x\sec^2 x\,dx + c_1 \overset{\tan x}{\cancel{\cos x}} + c_2$$

by parts

$$u = x\tan x + \ln|\cos x| + c_1\tan x + c_2$$

$y = \cos x\, u$

$$= \underbrace{x\sin x + \cos x\ln|\cos x|}_{y_p} + \underbrace{c_1\sin x + c_2\cos x}_{y_c}$$

Ex3 $y'' - y' - 2y = 4x^2$

$\underline{\underline{y_c}}$ $m^2 - m - 2 = 0$ $\quad y_c = c_1 e^{-x} + c_2 e^{2x}$

$(m-2)(m+1) = 0$

$m = -1, 2$ $\quad y_p = e^{-x} u$

So $y_p' = e^{-x} u' - e^{-x} u$

$y_p'' = e^{-x} u'' - e^{-x} u' - e^{-x} u' + e^{-x} u$

sub $e^{-x} u'' - 2 e^{-x} u' + \cancel{e^{-x} u} - (e^{-x} u' - \cancel{e^{-x} u}) - 2 e^{-x} u$
$= 4x^2$

$e^{-x} u'' - 3 e^{-x} u' = 4x^2$

so if $u' = v$ $u'' = v'$

$\Rightarrow$ $v' - 3v = 4x^2 e^{x}$ $\quad \mu = e^{-3x}$

$\frac{d}{dx} e^{-3x} v = 4x^2 e^{-2x}$

$e^{-3x} v = -(2x^2 + 2x + 1) e^{-2x} + c_1$

$v = -(2x^2 + 2x + 1) e^{x} + c_1 e^{3x}$

so $u' = -(2x^2+2x+1)e^{x} + c_1 e^{3x}$

$$u = -(2x^2-2x+3)e^{x} + \frac{c_1 e^{3x}}{3} + c_2$$

so $y_{\cancel{p}} = e^{-x} u$

$$= -(2x^2-2x+3)e^{-x}e^{x} + \frac{c_1 e^{-x} e^{3x}}{3} + c_2 e^{-x}$$

$$= -2x^2+2x-3 + c_1 e^{2x} + c_2 e^{-x}$$

3 absorbed into c_1

would have been much easier if

$$y_p = Ax^2+Bx+C$$