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The Higgs Nonet Lagrangian

I. Basic Notation

I am having difficulty inventing an easy, evocative notation for all these proposed states. I feel like I need a PDG veteran to help me out.

To start out, define the nonet members as follows:

$$\Phi = \left[\begin{array}{ccc|ccc} U_u & U_c & U_t & U_d & U_s & U_b \\ C_u & C_c & C_t & C_d & C_s & C_b \\ T_u & T_c & (T_t) & T_d & T_s & (T_b) \\ \hline D_u & D_c & D_t & D_d & D_s & D_b \\ S_u & S_c & S_t & S_d & S_s & S_b \\ B_u & B_c & (B_t) & B_d & B_s & (B_b) \end{array}\right]$$

(Despite the notation, of course all members are color singlet)

Hermiticity relates all the off diagonal elements to each other. For example,

$$\overline{U}_t = T_u \qquad\qquad U_t = \overline{T}_u$$

However, the diagonal elements are not self-conjugate. Instead, for example,

$$\overline{U}_u = D_d \qquad\qquad U_u = \overline{D}_d$$

The elements in parentheses describe the vanilla Higgs h and the three Goldstone modes eaten by the W and the Z. In particular,

$$h = \frac{1}{\sqrt{2}}(T_t + B_b) = \overline{h}$$

This is an example of the main deficiency of this notation: the neutral states as defined above are not mass eigenstates . We will set this issue aside, until the Yukawa couplings to the quarks are described.

II. The Yukawa Couplings

The simplest components of the nonet are those which have electric charge, because they are in fact mass eigenstates. Of these, the T_d (along with its antiparticle D_t) is of special importance, because it is coupled at full strength to the top quark, as well as to the first-generation down-quarks in the LHC proton beams. The T_d overwhelmingly decays as follows:

$$T_d \longrightarrow t + \bar{d}$$

The final-state top quark is predominantly right-handed. The Yukawa term is, for example,

$$\mathcal{L}_{Yukawa} = \frac{\sqrt{2}}{v}\left[T_d(\bar{t}\, m_d d_R) + D_t(\bar{d}\, m_t t_R) + h.c.\right]$$

$$= \frac{\sqrt{2}}{v}\left[T_d(\bar{t}\, m_d d_R) + D_t(\bar{d}\, m_t t_R) + D_t(\bar{d}_R m_d t) + T_d(\bar{t}_R m_t d)\right]$$

$$= \frac{\sqrt{2}}{v}\left\{T_d \bar{t}\left[\frac{(m_t + m_d)}{2} + \gamma_5 \frac{(m_d - m_t)}{2}\right] d + h.c.\right\}$$

The normalization of the vev v is the standard one:

$$v \approx 246 \text{ GeV}$$

However, the neutral counterparts of the T_d , namely the T_u and B_b, mix. The mass eigenstates are

$$h^0(T_d) = \frac{1}{\sqrt{2}}(T_u + B_d) \qquad h^3(T_d) = \frac{-i}{\sqrt{2}}(T_u - B_d)$$

This notation, while clumsy, is the best I have been able to come up with. I suggest the extension of this notation to the charged members should be

$$h^+(T_d) \equiv T_d \qquad h^-(T_d) \equiv D_t$$