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# Electronic transport simulations for TE power factor in nanostructures

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# General description of our group's work

## Theoretical investigations of:

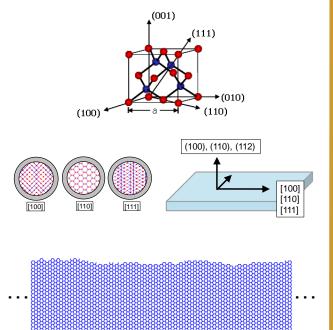
- > electronic
- > thermal
- > thermoelectric properties

in nanoscale materials and devices

# Approach - Tools

# Electronic structure (atomistic to continuum)

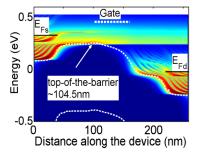
- 1) Tight-binding (sp<sup>3</sup>d<sup>5</sup>s\*)
- 2) Valence Force Fields
- 3) Force Constants
- 4) Effective mass approx.
- 5) Etc...

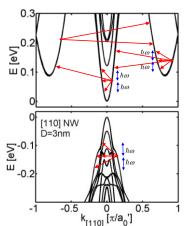


graphene nanoribbon

# <u>Transport</u> (ballistic to diffusive)

- 1) Quantum mechanical (NEGF)
- 2) Semiclassical L. Boltzmann
- 3) Monte Carlo
- 4) Landauer formalism

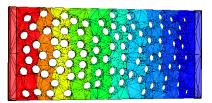


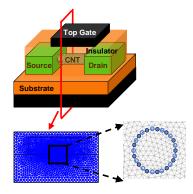


# Geometries (1D-3D, non-uniform)

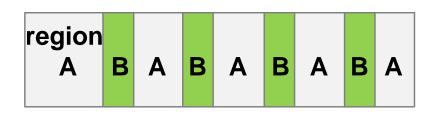
- 1) 3D geometry solvers
- 2) Nanocrystallines
- 3) Nanomeshes
- 4) Low-dimensional



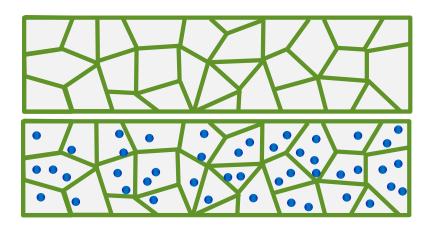




# Motivation - Very high thermoelectric power factors



**Superlattices** 



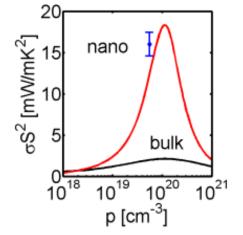
Nanocrystalline materials

Multi-phase nanocomposites

#### Very high PF:

2-phase materials: 15 mW/K<sup>2</sup>m<sup>-1</sup> 3-phase materials: 22 mW/K<sup>2</sup>m<sup>-1</sup>

(~7x compared to bulk Si)





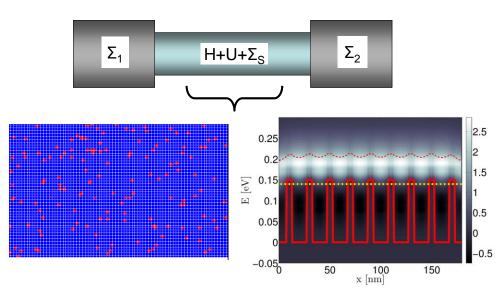
improvement in σ and S

Neophytou *et* al., Nanotechnology 2013, Lorenzi *et* al, J. Electronic Materials 2014

#### **Outline**

- Non-Equilibrium Green's Function (NEGF):
  - Method
  - Example 1: Influence of variations in SLs
  - Example 2: Filtering in 1D vs 2D
  - Example 3: Nanocomposites
- Monte Carlo semiclassical simulator development:
  - Method
  - Self-consistency
  - Scaling to large geometries
  - Inclusion of quantum effects
- Conclusions

# Non-Equilibrium Green's Function (NEGF)



nano-inclusions

superlattices

- Very powerful approach
- Can include scattering (decoherence)
- Can be computationally very expensive
- Captures the exact geometry and disorder

- Device Green's function:

$$G(E) = [(E+i0^{+})I - H - \Sigma_{1} - \Sigma_{2}]^{-1}$$

- Transmission:

$$T(E) = Trace(\Gamma_1 G \Gamma_2 G^+)$$

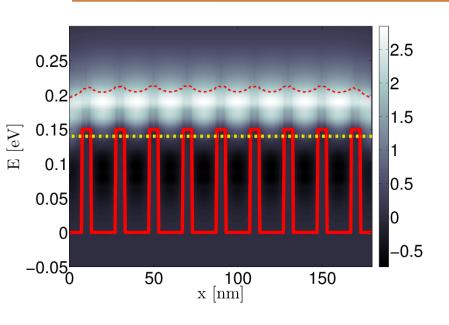
- TE coefficients:

$$I^{(j)} = \int_{-\infty}^{+\infty} \left( \frac{E - E_F}{k_B T} \right)^j T(E) \left( -\frac{\partial f}{\partial E} \right) dE$$

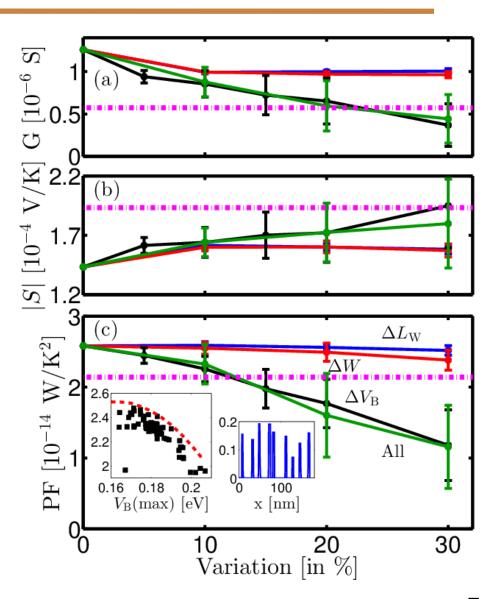
$$G = \left(\frac{2q^2}{h}\right)I^{(0)} \qquad [1/\Omega]$$

$$S = \left(-\frac{k_B}{q}\right) \frac{I^{(1)}}{I^{(0)}} \qquad [V/K]$$

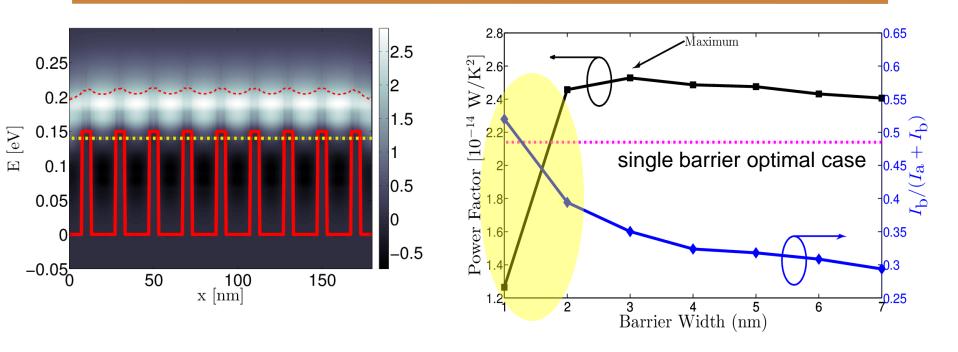
# Example 1a: Variation study in superlattices



- (1) Variation in  $V_B$  reduces PF
- (2) Variations in wells size, barrier width do not affect the PF

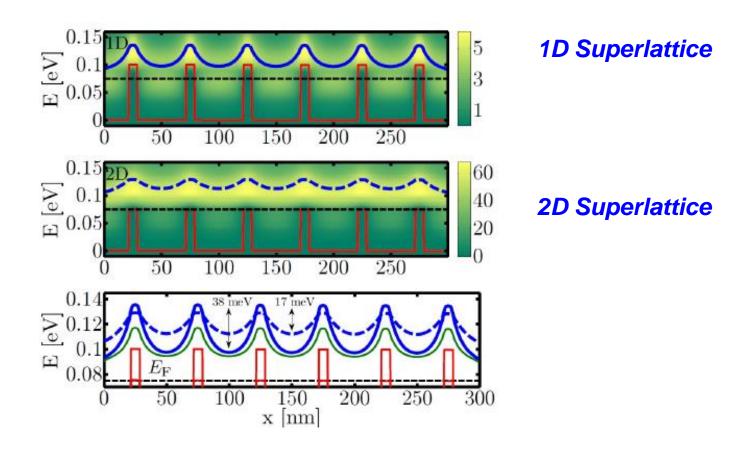


# Example 1: Detrimental effect of tunneling



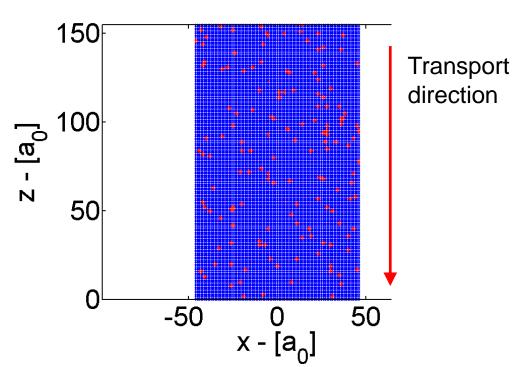
Quantum tunneling is detrimental to S and to the PF

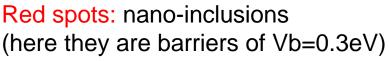
# Example 2: Filtering in 1D vs 2D



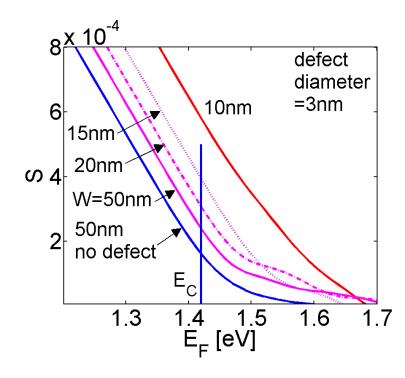
- (1) Variation in energy of current is larger in 1D
- (2) 1D Utilizes S of barriers and σ of wells better
- (3) 1D Utilizes energy filtering more effectively

# Example 3: Nanocomposites – increase in S





Blue region: channel

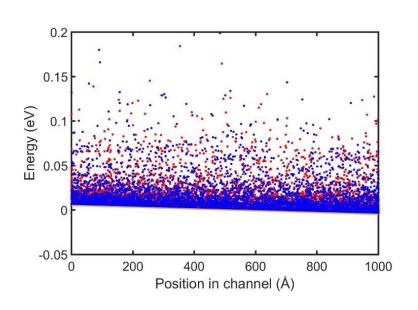


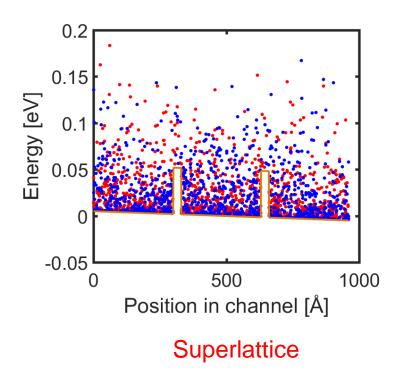
- (1) Nano-inclusions improve S
- (2) As the domain size decreases, the increase in S is larger

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### Monte Carlo method

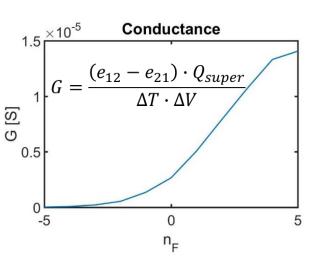


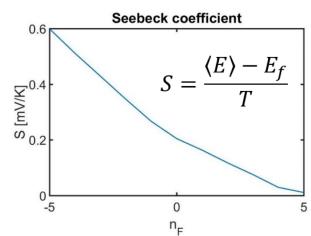


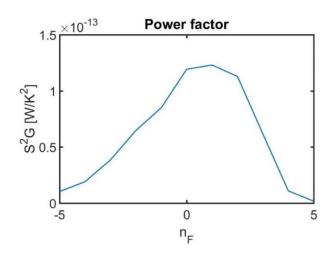
#### Uniform channel

- ➤ Electrons distributed in the channel according to the Fermi distribution and the Density of States
- Allowed to disperse under the influence of the potential
- Scattering by acoustic and optical phonons, ionized impurities, etc.

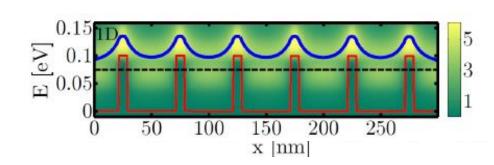
## Thermoelectric coefficients from MC



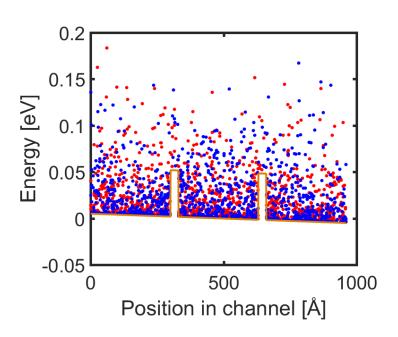




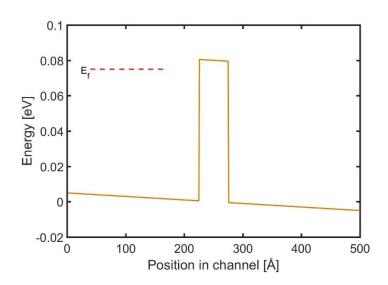
- Calculate the average energy of the current
- 2.  $S = \frac{-I_{\Delta T} \cdot \Delta V}{I_{\Delta V} \cdot \Delta T}$  for arbitrary  $\Delta V$  and  $\Delta T$
- 3.  $S = \frac{-\Delta V}{\Delta T}$  for  $I_{\Delta T} = I_{\Delta V}$

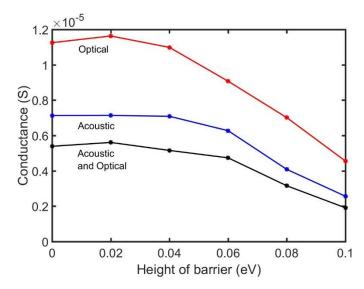


# Simulations of superlattices in MC



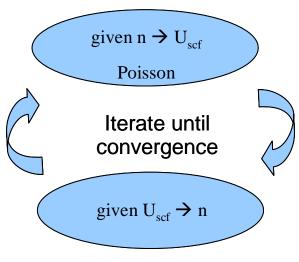
Include all relevant scattering parameters (next Ionised Impurities)





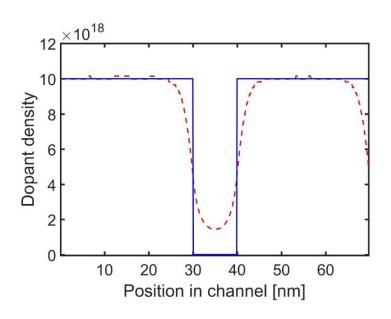
### Include self-consistent electrostatics

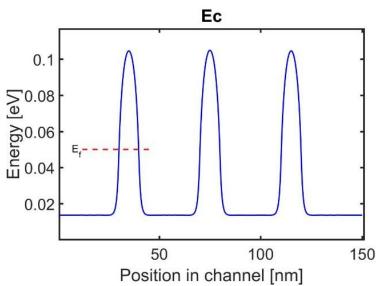
#### **ELECTROSTATICS**



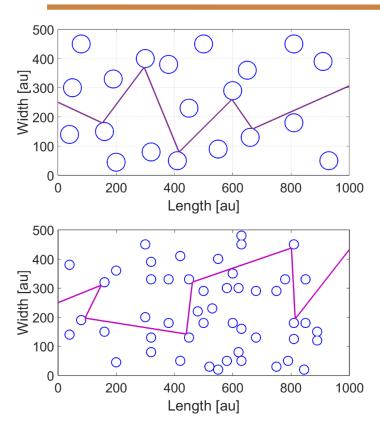
**TRANSPORT** 

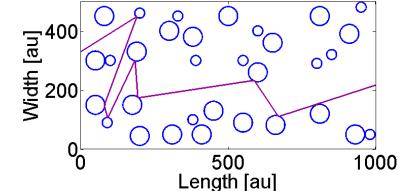
Obtain the actual potential profile for specific doping distributions





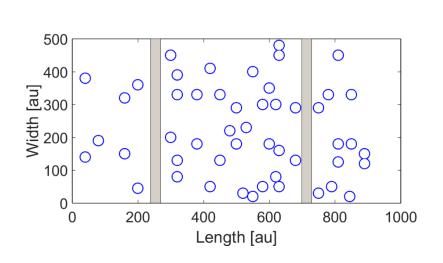
#### Extension to 2D

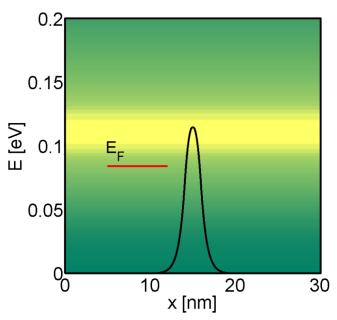




- Extend to larger geometries, where NEGF cannot reach
- ➤ Envision 100nm x 1000nm domains
- Nano-inclusions of various sizes
- > Extend to nano-inclusions, grain boundaries dislocations, etc.
- Electron AND phonon transport

# Incorporate quantum tunneling





Effect of tunnelling through a barrier

- Incorporate quantum tunneling
- Basic idea:
  - Solve 1D NEGF for simplified cases
  - Provide a probalitity of going through the barrier when an electron reaches a barrier in MC

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#### Conclusions

- Techniques for electronic transport in nanocomposites
- Quantum mechanical (NEGF)
- Semiclassical Monte Carlo
- Extend to large geometries
- Perform realistic simulations
- Incorporate all important transport effects

#### **Acknowledgements:**

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ERC StG: NANOthermMA